THE BOOK WAS DRENCHED

INTERMEDIATE
PRACTICAL PHYSICS

BY

VISSA APPA RAO, M.A., L.T.

ANDHRA UNIVERSITY
WALTAIR, SOUTH INDIA

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To

My Teachers

R. Ll. JONES

and

Rao Bahadur

M. C. S. ANANTAPADMANABHA RAO

PREFACE

The aim of this book is to provide a suitable course of practical physics for the students of the Intermediate classes of the South Indian Universities. The book had its origin in the notes prepared by me from time to time with a view to meet the difficulties which the students were experiencing in the practical classes. The earlier notes were revised in the light of later experience and the whole has been recently rearranged and rewritten to give it the form of a book. Under each exercise are given an account of the elements of the subject, the derivation, from first principles, of the formulae required for the experimental work and a description of the experiment followed by a practical example, which gives detailed observations from an actual experiment. The practical examples may not only assist students in recording and working out their own observations but may also indicate the degree of accuracy which can be expected in such trials.

My grateful acknowledgements are due to Dr. S. Bhagavantam, Ramachandra Deo Professor of Physics and Principal, Andhra University Colleges, at all stages—in preparing the manuscript for the press, in going through proofs and in making valuable suggestions.

My thanks are also due to Mr. S. Jagannadha Rao, Draftsman, Instruments Manufacturing Section, Andhra University, for the care with which he drew the diagrams and also to several others who were helpful to me in various ways.

Rajahmundry, V. APPA RAO.
25-12-1941.

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CHAPTER I

UNITS AND MEASUREMENT

THE SCOPE OF PHYSICS.

Physics is an experimental science involving the measurement of quantities such as length, mass, time, velocity, force, temperature, etc. These are called physical quantities. Length, mass and time are the simplest quantities amongst them and are, therefore, called fundamental quantities. Definite relations exist between all the other physical quantities and one or more of these three. Quantities which are derived from one or more of the three fundamental quantities are called derived quantities.

SYSTEMS OF MEASUREMENT.

There are two systems of measurement. The first is the centimetre, gram, second or the C.G.S. system. These are the units of length, mass and time, respectively. The second is the foot, pound, second or the F.P.S. system, the foot, the pound and the second being the corresponding units in which the three fundamental quantities are measured.

The expression of a physical quantity consists of two parts. The first is the unit employed and the second is the measure or the number of times the unit is contained in the quantity; for example, 3 feet, 20 grams, 30 minutes, etc. The omission of the unit in such expressions makes no meaning.

A convenient length between two fine marks, made on a bar of platinum at the temperature of melting ice, preserved in Paris, is arbitrarily chosen as the standard of length and is called a metre. Its subdivisions and multiples are in the decimal system and are, therefore, very convenient in calculations. Another standard of length in common use is the

yard. Its subdivisions are not in the decimal notation and so are not convenient in calculations.

The kilogram is the standard of mass in the metric system. It is the mass of a piece of platinum preserved in Paris by the French Government. This is the standard kilogram and is equal in mass to a litre of pure water at 4°C., the temperature of its maximum density.

A solar day would have been a convenient standard of time, if it were of constant duration throughout the year. But it is not so, because the movement of the earth round the sun is not uniform throughout the year. Therefore, *the mean solar day*, which is the mean length of the solar days in a year, is taken as the standard. $\frac{1}{24 \times 60 \times 60}$ or $\frac{1}{86400}$th part of this time interval is the second and constitutes a convenient unit of time in the C.G.S. and F.P.S. systems of measurement.

MEASUREMENT OF LENGTH.

A. Stepping. Apparatus required.—Dividers with sharp points, compass and millimetre scale.

Draw a circle of a convenient radius on a plane sheet of paper. Open the dividers so that the distance between the pointers is not more than 2 or 3 mm. Start at any point a of the circle (fig. 1). The joint of the two arms of the dividers must be stiff enough to keep the distance steady. Turn the dividers lightly on one of the points b, so that the other end occupies the position a_1, on the circumference. Continue this and carefully step off the whole length of the circumference, marking out every tenth step as a guide to correct counting. A fraction of the step may be left off at the end. Note it. Step out and count

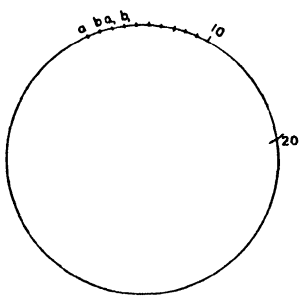


FIG. 1.

again with ab unchanged. You get the same number of steps though the fraction left out may slightly differ.

FIG. 2.

Draw a long straight line AB (fig. 2) on the sheet of paper and starting at A, step off the same number of times along the line carefully. Find the length of the step by applying it to the millimetre scale and estimating to a tenth of a scale division. Disturb the step and then adjust it to cover the fraction left out on the circumference and apply it to the straight line in continuation of the steps already marked. Let the last point be B_1. Apply the fraction in the second case and let B_2 be the point. Measure AB_1 and AB_2 to a tenth of a millimetre by estimation and take the mean length as the length of the circumference. This is evidently the length of the perimeter of the inscribed polygon of n equal sides each of length ab. This perimeter of the equiangular polygon approximates to the length of the circumference and the difference gets smaller and smaller as ab is made smaller and smaller, i.e. if n is large. It is, therefore, necessary that ab should be very small but at the same time of convenient length. Multiply the length ab (determined already) by n and add the length of the fraction of the step left out. Compare this with the mean length of the circumference obtained above. You will generally find a difference. How do you account for it? What is the probable error in the determination of the length of a single step? What fraction of the length is this error? What happens then if ab is multiplied by n? Which method of measuring the circumference do you prefer, restepping on a straight line AB and measuring along with a

scale, or multiplying the length of a single step by the number of steps ?

B. Finger and thread method. Apparatus required.—A fine thread and a millimetre scale.

Cut a convenient length of the thread. Put one end over a mark on the circumference. Press it with a finger nail and pass a finger of the other hand along the thread and along the circumference to a distance of 2 or 3 mm. Slide the first finger to the second and move the second once again. Continue the procedure and pass the thread carefully along the whole length of the circumference. Mark the point on the thread with a pencil, when it comes back to the starting point. Stretch the thread on the edge of the scale and note the length up to the mark. This is the length of the circumference. This method gives fairly satisfactory results. What is the effect of keeping the string under varying tensions ?

In each of the above cases, measure the diameter of the circle and find *the ratio of the length of the circumference to that of the diameter.* The number of times the diameter is contained in the circumference is a constant and is represented by the letter π. Draw two or three circles of different diameters and find the mean ratio.

Practical example :—

(*a*) *Measurement of Length :* Length of the step ab = 2·0 mm.

$$\left.\begin{array}{l} AB_1 = 26{\cdot}76 \text{ cm.} \\ AB_2 = 26{\cdot}70 \text{ ,,} \end{array}\right\} 26{\cdot}73 \text{ cm. is the mean circumference.}$$

n = 139·5; $n \times ab$ = 27·90 cm.

If ab were 1·9 mm., $n \times ab$ would be 1·4 cm. smaller.

The length of the step lies between 1·9 mm. and 2·0 mm.

Diameter of circle = 8·57 cm. in some direction.

,, ,, = 8·57 ,, in a direction at right angles to the first.

Mean = 8·57 cm.

(*b*) *Value of* π :

Diameter in cm.	Circumference in cm.	π
8·57	26·73 (stepping)	3·12
8·57	27·00 (thread)	3·15
8·51	26·46 (stepping)	3·11
6·24	19·46 ,,	3·12

Mean constant = 3·13

Notice that the value obtained for π is nearly the same in all cases and compare the mean value 3·13 with the standard value 3·142.

MEASUREMENT OF AREAS.

A. Graphical method.—Draw the figure on a graph paper divided into tenths of an inch. The area of the smallest square is therefore 0·01 sq. in. Divide the figure into a convenient number of parts. Count the number of small squares in each part. Neglect fractions smaller than a half and count parts greater than a half as full squares. Sum up and the area of the figure is obtained in square inches, if the sum is divided by 100. If greater accuracy is required, estimate the various fractions in the several parts of the figure to the first decimal place and add up each carefully and then sum up. The difference in areas obtained by the two methods may not generally be much, as the errors in the first method are mostly compensating.

Draw a circle with a convenient radius r on the graph paper. Count the squares and find the area as explained above. Divide this area by the area of the square with r as side. The ratio gives the value of $\frac{\pi r^2}{r^2}$ which is equal to π. Repeat the observations with a circle of a different radius and find the mean value of π.

Practical example :—

In the following observations, the circles were drawn on a graph paper. The area and radius are expressed in inches for convenience.

Circle.	Area of circle.	Area of square.	Ratio.
1.	8·88 sq. in.	2·82 sq. in.	3·15
2.	4·65 ,,	1·47 ,,	3·16

Notice that the mean ratio which is equal to π comes out to be 3·155. The maximum error is about 0·6%.

B. By the method of weighing.—The method of weighing is detailed in a later section dealing with the balance. The graph paper is practically uniform in thickness and is homogeneous in composition. Equal areas of the paper will be found to have very nearly equal masses. It can, therefore, be safely assumed that areas on the paper are proportional to the masses of the paper within the areas.

Cut the irregular figure, whose area is required, along its boundary on the graph paper carefully with a sharp scissors. Weigh the paper to a centigramme. Let its weight be M gm. From inside the irregular figure, cut a rectangle as big as possible and weigh it. Let its weight be m gm. Calculate the area of the rectangle. Let it be a sq. in. The area of the irregular figure is then $\frac{\text{M}\times a}{m}$ sq. in.

To find π by the method of weighing draw two circles again. Cut the circles carefully, fold them and weigh them. Cut a square (with r as side) within each of the circles and weigh. Find the ratio of the weight of the circle to the weight of the corresponding square and take the mean. This gives π. This method is based on weighing and hence leads to accurate results, if the cutting and weighing is done carefully.

Practical example :—

(*a*) *Area of an irregular figure* :

Area by counting squares $= 7{\cdot}86$ sq. in.

Area by weighing : Wt. of whole figure $= 0{\cdot}81$ gm.

Wt. of 2″ square $= 0{\cdot}415$ gm.

$$\text{Area} = \frac{0{\cdot}81}{0{\cdot}415}\times 4 = 7{\cdot}79 \text{ sq. in.}$$

(*b*) *Value of π :*

(1) Weight of a circle of 1·5 in. radius = 0·740 gm.
 ,, square of 1·5 in. side = 0·235 ,,
 π = ratio of weights = 3·149.

(2) Weight of a circle of 2 in. radius = 1·304 gm.
 ,, square of 2 in. side = 0·415 ,,
 π = ratio of weights = 3·142.

Mean value of π = 3·146.

THE VERNIER.

The length of a body can be determined to the first decimal place of a centimetre directly with a millimetre scale. If the body projects over a millimetre mark, the fraction can be estimated to the nearest tenth by the eye. This estimate, however, is not very reliable and there is the probability that it may be too high or too low. It is not easy to graduate an edge into tenths of a millimetre and much more so into hundredths. Hence a more satisfactory method of measuring the length to the second and if need be to the third decimal place of a centimetre is necessary. This is accomplished in practice by the two devices of the vernier and the screw.

The vernier is a small auxiliary scale sliding along a fixed main scale and reads accurately to a fraction of the smallest division of the scale. This device was invented by Pierre Vernier of Burgandy in the first half of the seventeenth century. If it is desired to read to the first decimal place of the least division of the scale (fig. 3) the edge of the auxiliary

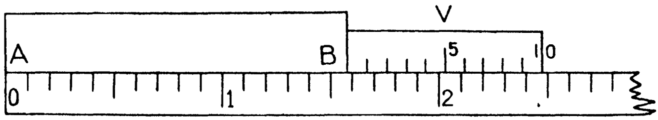


FIG. 3.

scale V, equal in length to nine small divisions of the scale, is divided into ten equal parts. The vernier V can be fixed at any desired point along the main scale. One end of V is

generally marked with an arrow and is called the zero of the vernier.

Let AB be the body whose length has to be determined. The scale is divided into millimetres. The end A coincides with zero of the scale and B projects beyond 1·5 cm. by a fraction of a millimetre, and this distance x between the 1·5 cm. mark and the zero of the vernier is to be measured. As the eye is passed along, the distance between the successive scale divisions and the corresponding divisions on the vernier gets smaller and smaller and finally the distance becomes zero and the seventh division of the vernier coincides with 2·2 cm. mark of the scale. A division on the scale is greater than a division on the vernier by 0·1 mm. Therefore, the distance between 1·6 cm. mark of the scale and the first division on the vernier is short of x by 0·1 mm. The distance between the 1·7 cm. mark and the second division on the vernier is $x-0{\cdot}2$ mm. and so on. The difference between the 2·2 cm. mark and the seventh division on the vernier is [$x-(0{\cdot}1\times 7)$ mm.] $= x-0{\cdot}7$ mm. and this is zero, as the two marks coincide. We, therefore, have $x = 0{\cdot}7$ mm. or 0·07 cm. and the length of the body is 1·57 cm.

The least count of the vernier is the least length which the vernier can read and is generally the difference between the length of the smallest division of the scale and the length of the smallest division of the vernier. The least count in the above case is 0·1 mm. or 0·01 cm.

The fraction x is therefore obtained by multiplying the least count with the number of the division on the vernier which coincides with a division on the scale. This is the rule of the vernier.

It might sometimes happen that no division on the scale exactly coincides with a division on the vernier. The choice lies always between two coincidences and the nearer one is to be considered.

If the scale is graduated to half millimetres, how do you graduate the vernier to read to 0·01 cm.? Is the length of the vernier now longer or shorter than that in the above?

Practical example :—

To construct a vernier that would read to 0·001 *cm.*

A number of possible ways are given below.

No.	Smallest division on the scale in millimetres.	Length of the vernier in scale divisions and in millimetres.	No. of divisions on the vernier and the length of each in millimetres.	Least count. (Difference between columns 2 and 4.)
1	0·5	49 = 24·5 mm.	50; $\frac{1}{50}\times 24{\cdot}5$ 0·49 mm.	0·01 mm.
2	0·25	24 = 6·0 ,,	25; $\frac{1}{25}\times 6{\cdot}0$ 0·24 mm.	0·01 ,,
3	0·20	19 = 3·8 ,,	20; $\frac{1}{20}\times 3{\cdot}8$ 0·19 mm.	0·01 ,,
4	0·10	9 = 0·9 ,,	10; $\frac{1}{10}\times 0{\cdot}9$ 0·09 mm.	0·01 ,,
5	1·0	99 = 99 ,,	100; $\frac{1}{100}\times 99$ 0·99 mm.	0·01 ,,

Note how the length of the vernier changes with the fineness of the scale. In the third and fourth cases, the main scale is too fine to be accurately drawn and in the last case the vernier is too long. So, in practice, these cases are not met with. Out of the other two, the first is the better, as the graduation of the scale is more easy and the vernier is not inconveniently long.

THE VERNIER CALLIPERS.

The principle of the vernier is applied in this instrument, which is generally employed for all ordinary work in the laboratory to read to 0·01 cm. The scale is engraved on a steel rectangular bar (fig. 4) from one end of which projects a fixed jaw at right angles to the scale as shown

in the figure. Another jaw slides along the scale and can be fixed at any point by means of a screw. The vernier scales are engraved on this movable piece and the sides of

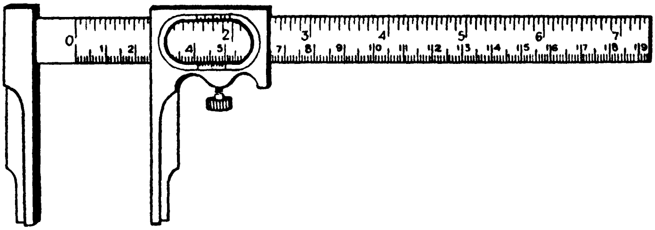


FIG. 4.

the jaws facing each other are parallel planes. When the jaws are in contact, the zero of the scale and the zero of the vernier coincide. The body whose length is to be measured is lightly gripped between the two jaws and the movable jaw is screwed in. The distance between the jaws is the length of the body and it is the distance through which the movable jaw has slided on the scale. This is equal to the distance between the zero of the scale and the zero of the vernier. So the reading of the zero of the vernier on the scale gives the length of the body. The fraction of the scale reading is read according to the rule of the vernier already explained. The other edge of the steel bar is divided into inches and fractions of an inch. For more accurate work callipers reading to 0·001 cm. is available.

CIRCULAR VERNIER.

The circumference of a circle can be subdivided into a number of equal parts. The greater the radius of the circle the finer can be the subdivision and the greater will be the length of the arc subtending a given angle. Angles are measured to a degree with circular protractors which are generally of a decimetre in diameter. Any finer subdivision makes the circle inconveniently large, and so angles cannot

be directly measured to less than 0·5 degree. Smaller fractions are measured with the help of a vernier, an auxiliary concentric arc of a suitable length moving alongside the graduated circle (fig. 5).

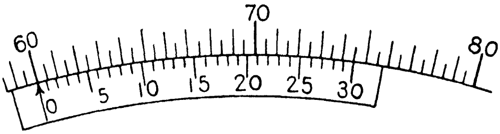


FIG. 5.

In the instrument called the spectrometer the circle is generally divided to 0·5 degree. The vernier arc subtending an angle of 29 half degrees is divided into thirty equal parts. The least count of the vernier is therefore one minute of arc. In certain instruments, used in more accurate work, the circle is bigger and is graduated to 0·25° and the vernier is divided to read to 15 seconds of the arc. How is the vernier to be constructed in this case?

THE SCREW.*

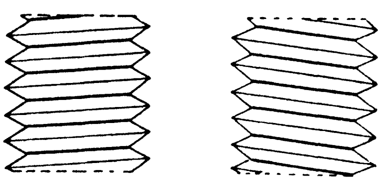
In this instrument, a screw moves in a nut, which is at rest relatively to the screw. The nut is hollow and short, and the screw is solid and long. The inner surface of the nut is cut into a spiral channel so as to correspond exactly to the spiral ridge on the solid cylinder or rod. The screw cut into the nut is called the concave or the internal screw and the screw on the rod is called the convex or the external screw.

---

* Accurate screw cutting is a very difficult operation. The screw was perfected to a high degree of accuracy by Sir Joseph Whitworth who invented a very accurate method of cutting varied standards of screws, which could be replaced at any time. The advent of the Whitworth standard screws very much improved the method of accurate measurement and made the standardization of machines possible, by facilitating the repair and replacement of the different parts of machines. Accurate scales are drawn with a dividing machine which consists chiefly of an accurate screw. Any slow motion in a modern machine is affected with the help of a screw.

When a screw moves in a nut, the motion of any point on the screw is a combination of a rotation and translation. The translation or advance of the screw is perpendicular to the plane of rotation and is parallel to the axis of rotation or the length of the screw. Any point on the screw, therefore, moves in a spiral round the length of the screw. The translation or advance of the screw for a complete rotation is called the pitch of the screw, and is equal to the distance between corresponding points of the successive turns of the thread. For a known fraction of a rotation the screw moves through the same fraction of the pitch, and by managing to have the pitch and the fraction as small as practicable, very small lengths can be measured with the screw.

A screw is either right-handed (fig. 6*a*) or left-handed (fig. 6*b*). When looked at in the direction of translation, while the screw is advancing, the rotation is right-handed in one and left-handed in the other. In one the thread slopes downwards from the right towards the left and in the other downwards from the left towards the right.



(*a*) (*b*)

FIG. 6.

Man is by nature right-handed. Driving in a screw into, say, a block of wood requires a greater effort than drawing it out. Hence a right-handed screw is more convenient. All the screws with which we generally come across are right-handed.

Is the screw, which an Indian goldsmith uses in the ornaments he makes, a right-handed one or a left-handed one? How does he make the screw? Is it by cutting into a rod?

The screw-gauge and the spherometer are the two practical applications of the screw and are employed for the measurement of small lengths.

THE SCREW-GAUGE.

This appliance is alternatively called a micrometer gauge or a micrometer callipers. It is made of steel and consists of a rod A on which a screw is cut (fig. 7). The rod is fixed

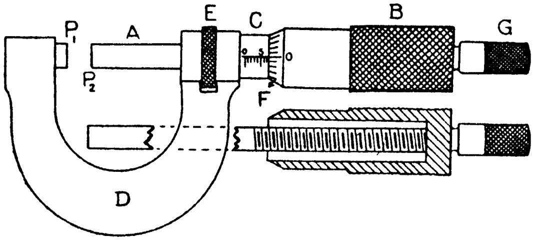


FIG. 7.

to one end of a hollow cap B shown in section. The screw moves in a nut inside a hollow cylinder C and a scale is engraved on the outside of C along a line parallel to the axis of the screw. The hollow cylinder is rigidly attached to a bent arm D. To one end of this arm is fixed a plug P_1 which has a carefully worked plane face parallel to the end face P_2 of the screw rod which is also a plane. The cap B has a bevelled edge F graduated into 50 equal parts. The scale in this case is divided into millimetres and half-millimetres. When the two plane faces P_1 and P_2 are in contact, the zero of the linear scale and the zero mark of the edge coincide. The pitch of the screw is 0·5 mm. Since the edge of the cap is graduated into 50 equal parts, the cap can be accurately moved to a fiftieth of a turn, for which the screw advances or recedes by $\frac{1}{50}$ of 0·5 or 0·01 mm. This distance is the least measurable advance of the screw and is the least count of the instrument. This instrument is used to find the thickness of thin plates, the diameter of wires, etc.

Undue pressure in handling the cap to turn the screw damages the screw thread and so the screw is turned by

handling a safety clutch G, which slips round when the pressure applied exceeds a safety limit. E is a grip which, when slightly turned, grips the screw and registers the reading of the screw-gauge.

When the two plane faces are brought into contact by working the safety head, a division on the bevelled edge coincides with a mark on the graduated scale. This point on the scale is its zero mark and this mark on the edge is the zero of the edge; and the distance between the plane faces is zero. But in a particular instrument there may be zero error as when mark 1 on the edge lies on the divided line and not the mark 0. This mark 1 is then the real zero of the edge. The real zero is one division above the marked zero. The zero error thus corresponds to one division of the edge and is $+0{\cdot}01$ mm. The change in the zero or the zero error is due to the wear and tear of the instrument. Turn the screw so that division 10 on the edge lies on the graduated line. The screw moves through a distance corresponding to $10-1$ or 9 divisions only of the edge. We have to conclude that the plane surfaces are $0{\cdot}09$ mm. apart. The nominal reading on the edge is 10 and the corrected reading is 9 : so the correction is $-0{\cdot}01$ mm.

If, in another instrument, the division 49 coincides with the zero of the scale when the plane faces are in contact, 49 is the real zero mark and lies one division below the marked zero. The zero error is, therefore, $-0{\cdot}01$ mm. and the correction is $+0{\cdot}01$ mm. If the real zero is above the marked zero, the correction is negative and if it is below the marked zero, the correction is positive. The error and correction are opposite in sign.

Practical example :—

To measure the diameter of a wire :

Pitch of the screw $= 0{\cdot}5$ mm.; least count $= 0{\cdot}01$ mm.; zero error $= +0{\cdot}01$ mm.

Reading on the scale in mm.	Reading on the edge.	
	—	|
0·5	23	24
..	24	24
..	25	26
..	25	25
..	23	25
..	25	24

Mean edge reading for one set = 24·2
,, ,, ,, another set = 24·7
Mean edge reading = 24·5
Observed mean diameter = 0·745 mm.
Corrected mean diameter = 0·735 mm.

Six sets of readings at different points of the length of the wire are taken. At each point readings are taken for two diameters in directions at right angles to each other. The mean of the twelve readings is taken as the observed diameter.

THE SPHEROMETER.

This is essentially a micrometer screw. A is the screw ending in a point (fig. 8). The screw works in the nut D,

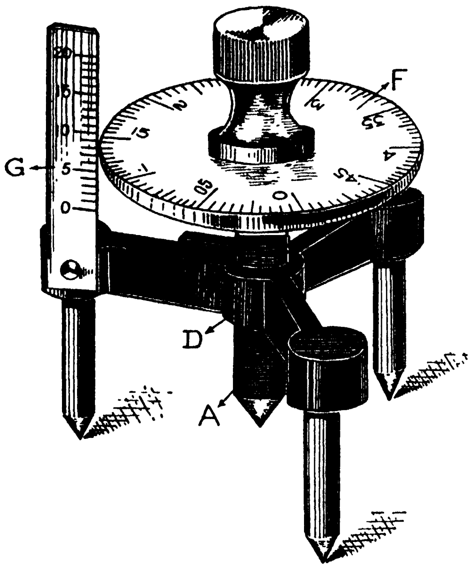


FIG. 8.

which forms the centre of a tripod supporting the screw. The ends of the legs lie on the three corners of an equilateral

triangle and the end A points to the centre of the triangle. F is a metal disc rigidly attached to the screw and its edge is divided into a number of equal parts and this enables the measurement of a fraction of a rotation of the screw. G is a scale attached to one of the legs and is parallel to the length of the screw. The movement of the screw is measured along the scale. A plane glass plate is supplied with the instrument and the instrument is mounted on it. The ends of the three legs lie in the plane of the glass and the distance between this plane and the end of the screw is measured with the spherometer.

The pitch of the screw and the least count are first determined. The instrument is mounted on the glass plate. Suppose the point A is above the plane of the plate. Screw it down until A touches the glass plate. If, in doing so, the screw is turned too much, it will be below the plane of the ends of the three legs and the instrument can be rotated about the point A. When A just touches the plane, the instrument can be just rotated about A; and then all the four points are in the plane of the plate. A very delicate touch will be found enough to rotate the instrument. When that is so, read the division on the disc which is against the zero of the scale. Repeat the observation twice and the mean of the three readings x is the zero reading of the instrument and the distance between the point A and the plane of the glass plate is zero.

This instrument may be used to measure the thickness of a plate of glass as follows. Raise the screw and push in the plate between the screw and the big glass plate. Lower the screw until it just touches the top surface of the small plate. This adjustment is to be delicately made as explained above and the mean of three readings is to be taken. Complete turns of the screw are registered on the linear scale G and can as well be counted as a check. The fraction of a turn

is read on the disc F. If F is divided into 100 equal parts and the pitch of the screw is one millimetre and x is 4, then $4 \times 0{\cdot}01$ or $0{\cdot}04$ mm. is to be deducted from the reading of the instrument to get the thickness of the glass plate. Similarly, measure the thickness at three or four points on the surface of the small glass plate and take the mean thickness of the plate.

The spherometer is mainly intended to measure the radius of curvature of a curved surface which is either concave or convex. For this purpose the instrument is mounted on the curved surface and the screw moved until it just touches the surface at A (fig. 9, *a* and *b*). Note the reading. The difference between this and the zero reading gives the elevation or depression AH. C, C_1 and C_2 are the three ends of the legs of the tripod at the corners of an equilateral triangle (fig. 9*c*). CC_1C_2 is the circumscribing circle. H is the centre of the circle and CD is its diameter. CAD (fig. 9, *a* and *b*) is a section of the surface containing the screw point A and the diameter CD of the circle, the plane of the circle CC_1C_2 being at right angles to that of the paper. O is the centre of the sphere of which the surface forms a part. OA (fig. 9, *a* and *b*) is the radius of curvature R and it can easily be expressed in terms of the radius r of the circle CC_1C_2 and AH, by the help of a well-known property of the circle. The rectangle

$$(2R - AH) \times AH = (CH)^2$$

$$\therefore\ 2R \times AH = (AH)^2 + (CH)^2$$

$$\therefore\ R = \frac{AH}{2} + \frac{(CH)^2}{2AH} = \frac{h}{2} + \frac{r^2}{2h}.$$

h = AH and is measured with the spherometer directly; r = the distance between the ends of the screw A and of any one of the three legs of the instrument when all the four points are in one plane. This distance can be directly

measured from the central screw point to the ends of the three legs separately and the mean taken.

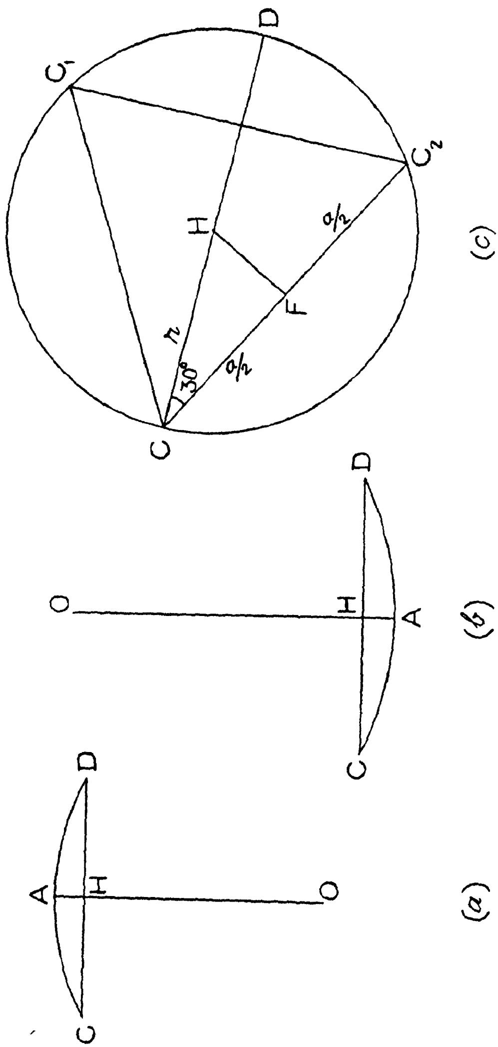


FIG. 9.

The radius can alternatively be expressed in terms of the side of the equilateral triangle. CD bisects the angle C_2CC_1

and in the triangle CHF, F is the middle point of CC_2 and HF is at right angles to CC_2. If a represents the side of the equilateral triangle, then

$$\frac{\mathrm{CF}}{\mathrm{CH}} = \frac{a/2}{r} = \cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\therefore \frac{a}{2r} = \frac{\sqrt{3}}{2} \text{ and } r = a/\sqrt{3} \text{ or } r^2 = \frac{a^2}{3}$$

$$\therefore \mathrm{R} = \frac{h}{2} + \frac{a^2}{6h}.$$

a is more easy to measure as the ends of the three legs are always in a plane whereas in measuring r directly, the central screw must be brought into the plane of the end points of the three legs before the measurements are taken.

MEASUREMENT OF MASS.

Mass is the quantity of matter in a body. It occupies space and is perceived by our senses. The weight of a

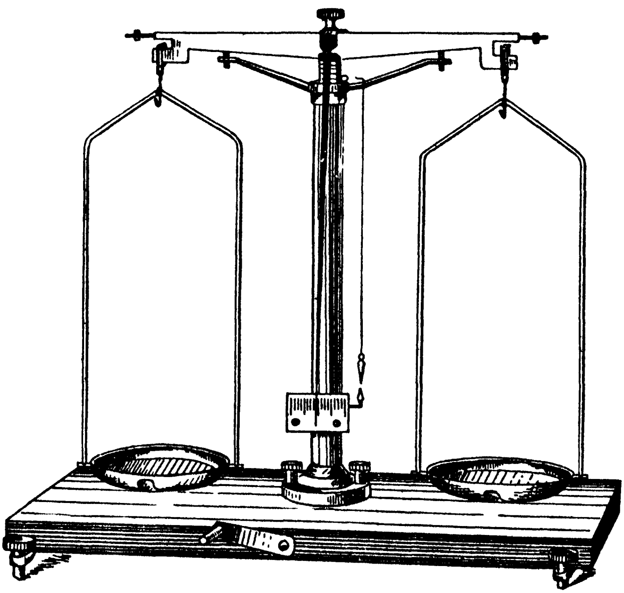


FIG. 10.

body is the force with which the body is attracted by the earth and is proportional to its mass. Two bodies are equal in mass if their weights at the same place are equal. The weights of two bodies and, therefore, their masses are compared by means of the balance.

The balance (fig. 10) consists of a beam with its fulcrum at its middle point. An agate prism with its knife-edge downwards forms the fulcrum and is the axis of suspension of the balance. The knife-edge of the beam is supported on two agate planes fixed to the top of a brass rod passing through the vertical brass pillar. At either end of the beam is fixed an agate prism with its knife-edge pointing upwards and a scale pan is suspended at either end from a small double-hooked stirrup fitted with a groove (fig. 11). Inside the groove is cemented an agate plane which rests on the knife-edge. A long pointer is screwed on to the top of the beam and moves just in front of an ivory scale fixed to the brass pillar. The end of the pointer stands at the centre of the scale when the beam rests horizontally on the central knife-edge. If there is any error, adjustment can be made by screwing in or out a small nut at either end of the beam.

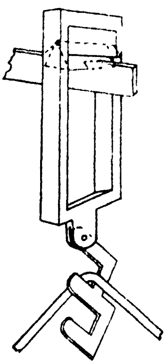


FIG. 11.

Turning the handle to the right raises the brass rod inside the pillar; the beam rests on the agate planes and is free to oscillate. Turning the handle down to the left lowers the rod and arrests the beam which is supported on the two arms of the brass pillar. The pillar is mounted on a wooden platform provided with levelling screws. A plumb line arrangement helps to keep the pillar vertical. The balance is enclosed in a wooden case with a sliding glass door in front and is provided with a catch to keep the door at a convenient height.

The following rules are to be observed when using a balance :

1. Do not touch the weights in the box but handle them with the forceps. Put back each weight into its proper place.
2. Lower the beam whenever you have to add or remove a weight; or else the knife-edges will be damaged.
3. Keep the door of the balance-case shut while observing the oscillations of the pointer; or else the currents in the outside air disturb the balance.
4. Put the body to be weighed in the left-hand pan and counterpoise it with weights in the right-hand pan.
5. Add weights from the box in the order in which they are arranged, in the decreasing order of magnitude. This is a quick way of getting at the right weight of the body.
6. Take the first big division on the left of the scale as zero but not the middle division and estimate the position of the end of the pointer to a tenth of a division on the scale, viewing directly against the centre of the scale.
7. Count the weights while they are in the pan and check them again as you put them back into the box.
8. If the pointer of the balance does not oscillate well about the centre of the scale or if the brass pillar is out of the vertical as indicated by the plumb bob, do not meddle with the balance but report the matter to the Demonstrator.

The following procedure should be adopted for obtaining the mass of a body to the nearest centigram. The first operation consists in finding the zero of the balance. For this purpose turn the handle and raise the beam. Gently beat the air above one of the scale pans with the hand if necessary, and the pointer begins to oscillate. Close the glass door. As the pointer swings within the scale judge the extreme

positions of the pointer on the scale. Take readings for a few successive turning points and lower the beam. The mean position between a turning point on the left and the next turning point on the right is nearly the place where the pointer would come to rest after the oscillations gradually die out. This point is called the resting point of the pointer. The method of finding the resting point will be understood from the practical example given below.

Place the body, whose mass has to be determined, in the left-hand pan and put a weight from the box on the right-hand one. Slightly turn the handle to the right. Generally one pan is seen clearly to be the heavier. Add on or remove weights, until a stage is reached when it is not very easy to judge which pan is the heavier. At such a stage, the pointer begins to oscillate. Close the glass door, observe the turning points and calculate the mean resting point. Lower the beam carefully and note the load in the right-hand pan. If this resting point is to the left of the zero of the balance determined already, then the weights in the right pan are heavier than the body (why?). Remove a centigram from the right-hand pan, proceed as before and find the mean resting point for the load. This resting point may be to the right of the zero of the balance. The weights then are lighter than the body. Two weights which differ by a centigram are thus obtained and the weight of the body lies between them. Select that weight which gives a resting point that is nearer to the zero of the balance and this is the weight of the body to the nearest centigram. The mass of the body is the mass of this weight, and is found correct to a centigram.

To obtain the mass of a body to a greater degree of accuracy use is made of the sensibility of the balance. The number of divisions of the scale through which the resting point moves, for a change of a centigram in the load, is called the sensitiveness of the balance. This is also called the sensibility of the

balance. It may alternatively be expressed as the weight necessary to change the resting point of the balance through one scale division. The sensitiveness of a balance varies with the load and is maximum for a particular load.

Practical example :—

A metal piece is taken and its mass is found.

Turning points : Pans empty.—

Left.	Right.
1·2	7·5
1·6	7·2
1·8	
Mean = 1·53	Mean = 7·35

Resting point is $\frac{1{\cdot}53+7{\cdot}35}{2} = 4{\cdot}44$

The middle division of the scale is 5.

Turning points : 12·05 *gm. in the right-hand pan against the body.—*

Left.	Right.
0·8	8·3
1·2	7·8
1·6	
Mean = 1·2	Mean = 8·05

Resting point is $\frac{1{\cdot}2+8{\cdot}05}{2} = 4{\cdot}6$

The body is heavier than 12·05 gm.

Turning points : 12·06 *gm. in the right pan.—*

Left.	Right.
−1·0	3·0
−0·8	3·0
−0·7	
Mean = −0·83	Mean = 3·0

Resting point is $\frac{-0{\cdot}83+3}{2} = 1{\cdot}08$

Note that two small divisions are marked to the left of the zero mark of the scale and hence negative readings are also obtained. 12·06 gm. is heavier than the body. The mass of the body lies between 12·05 gm. and 12·06 gm. 4·6 is nearer the zero of the balance than 1·08 and 12·05 gm. is, therefore, the mass of the body to the nearest centigram.

In the above example, the sensitiveness of the balance at 12 gm. load is 4·6–1·08 = 3·52 divisions per 0·01 gm. or 0·352 of a division per 0·001 gm. Using this figure, the mass of the metal piece may be obtained to the nearest milligram as follows:

Adding 10 milligrams to 12·05 gm. lowers the resting point by 3·52 divisions. Adding $\frac{0{\cdot}16}{3{\cdot}52} \times 10 = 0{\cdot}45$ milligram to 12·05 gm. brings down the resting point from 4·6 to 4·44 which is the zero of the balance. The true mass of the body is 12·05+0·00045 gm. or 12·050 gm. to the nearest milligram.

MEASUREMENT OF TIME.

The unit of time based on the mean solar day has already been defined. Accurate measurement of time is somewhat difficult and is generally not attempted in elementary physics classes. Simple mechanisms which go by the name of a clock or a watch are, however, easily available for routine measurements in a laboratory. A pendulum or a balance wheel or a tuning fork or any other mechanism which repeats its motion at regular intervals may be adapted to measure time. All such instruments must be calibrated beforehand with reference to the standard unit of time by astronomical methods. The accuracy with which an interval of time may be measured with the help of one of the above devices depends upon the accuracy with which the calibration had been carried out and upon the regularity with which the motion in question repeats itself.

An ordinary watch or a clock is generally furnished with an additional stopping and starting device and a centre seconds-hand, before it is used in a laboratory for measuring small intervals of time, generally 0·2 second. Such an instrument is called a stop-watch or a stop-clock and may be easily stopped or started at any desired instant.

CHAPTER II

DYNAMICS

MOTION.

If a body moves from a point A to another point B along a straight line AB in a certain interval of time, the body is said to be displaced from A to B and the straight line AB represents the displacement of the body both in direction as well as in magnitude. The rate of change of position of the body or its rate of displacement is called its velocity. Velocity has thus both magnitude and direction and a straight line can fully represent it.

A body may not always move with the same velocity. The rate of change of velocity is called acceleration.

Practical example :—

(*a*) *To find graphically the space traversed by a body by plotting velocity against time :*

The following are the data in a given case :

Time in seconds ..	0	1	2	3	4	5	6	7	8	9	10
Velocity in cm. per second ..	5	7	10	14	19	25	32	40	49	59	70

Plot the time on the horizontal axis, one inch representing two seconds as shown in figure 12. Plot the velocity along the ordinate, one inch representing 10 cm. per sec. Let AB be the smooth curve (?) passing through the points plotted. Let a_1 and a_2 on the X-axis represent two instants of time near each other and let p_1 and p_2 be the corresponding velocities. If the time $a_2 - a_1$ is very small, the difference in the velocities $p_2 - p_1$ will be negligible and the distance travelled will be equal to $(a_2 - a_1) \times (p_1$ or $p_2)$, if the two points a_1 and a_2 are consecutive. Then the area of the very narrow strip shaded in the figure is $(a_2 - a_1) \times (p_1$ or $p_2)$ and represents the distance travelled in that small interval of time, which is small enough for the velocity to be considered uniform. The top side of the rectangle coincides with a very small portion of the curve. The area AOCB is made up

f a number of such strips and each successive strip represents the istance travelled in each successive small interval of time. So the

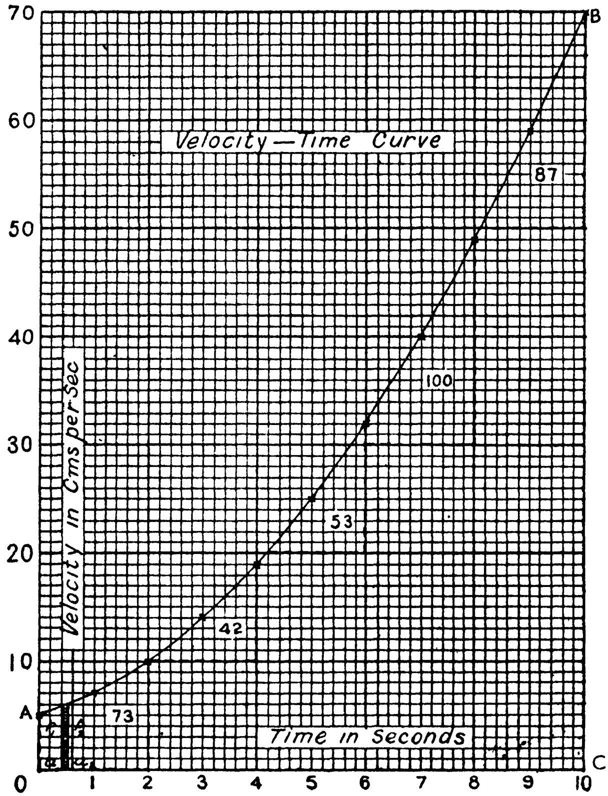


FIG. 12.

otal distance traversed by the body is represented by the area of the igure AOCB. Divide this area into a number of convenient parts as hown in the figure by dotted lines. Count the small squares, neglecting arts less than half the square and counting as whole squares those which are more than half. The area represented by one square inch s the distance travelled in 2 seconds at the rate of 10 cm. per sec. nd is equal to 20 cm.

$$1 \text{ in.} \times 1 \text{ in.} = 1 \text{ sq. in.}$$
$$\text{In 2 sec. at 10 cm. per sec.} = 20 \text{ cm.}$$
$$t \times v = s.$$

The area of the figure is found to be 14·55 sq. in. and so the distance traversed by the body in 10 sec. is $14{\cdot}55 \times 20 = 291$ cm.

The motion of the body is not one of uniform acceleration as is evident from the curve (?) and also from the values given above and the distance travelled cannot be easily calculated. Hence, the graphical method is very useful in such cases.

Again in the above case, the velocity is changing within every second. If, to the first approximation, we assume that the distance travelled in each second is numerically equal to the mean velocity of the body during that second, the sum of all these successive means gives the distance travelled by the body in 10 sec. The mean velocities in the successive seconds are 6, 8·5, 12, 16·5, 22, 28·5, 36, 44·5, 54 and 64·5 cm. per sec. Distance travelled in 10 sec. $= 6+8{\cdot}5+12+ \ . \ . \ . \ +64{\cdot}5 = 292{\cdot}5$ cm., a result which does not differ much from the one already obtained graphically. They agree within 0·5% of each other.

(*b*) *To find graphically the velocity at any instant from the distance time curve :*

In the foregoing exercise, the distances travelled in successive seconds are known and hence the distances travelled at the end of the 1st, 2nd . . . . and 10th seconds are as follows :

Time in seconds ..	1	2	3	4	5	6	7	8	9	10
Distance in cm. ..	6	14·5	26·5	43·0	65·0	93·5	129·5	174·0	228·0	292·5

Plot the distance against time. Let OA (fig. 13) be the curve. It is required to find the velocity of the body at any instant.

The velocity of a body at any instant is the ratio of the distance travelled during a small interval of time including the instant, to the interval. The interval is so small that the velocity during that interval may be considered to be practically constant. On the curve, D is a point and corresponds to an instant of time and according to the above definition, the velocity at D is given by the ratio $\frac{BC}{DC}$; BC is the distance travelled in a small interval of time DC. If these quantities are made smaller and smaller, the more and more would the side BD of the right-angled triangle BDC coincide with the curve BD. Ultimately when B coincides with D, the side BD becomes a tangent to the curve OA at the point D. Draw, therefore, a tangent DD′ to the curve at D and let DC′ be the ordinate to the curve at D. The two triangles BDC and DD′C′ are similar and hence the velocity at

D is given by the ratio $\frac{DC'}{D'C'}$. If the numerator and the denominator are expressed in centimetres and seconds respectively, the ratio expresses the velocity at the instant in cm. per sec. The velocities at different instants can similarly be found. Thus if the velocities at the end of successive seconds are found graphically, they will nearly agree with the velocities originally given in exercise (*a*) above. The results obtained are given below.

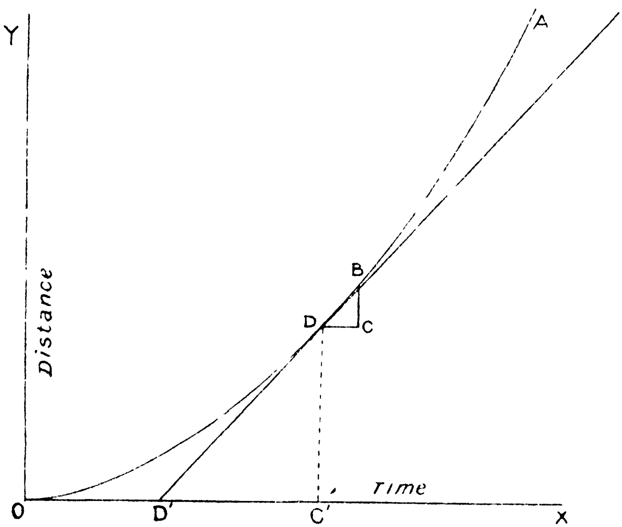


FIG. 13.

Velocity at the end of	From Graph cm./sec.	From Exercise (*a*) cm./sec.
1st second	10	7
2nd ,,	10·3	10
3rd ,,	16·6	14
4th ,,	18·7	19
5th ,,	25·0	25
6th ,,	31·2	32
7th ,,	37·0	40
8th ,,	50·0	49
9th ,,	57·0	59
10th ,,	77·0	70

In the velocity time curve of the previous exercise, the ratios like $\frac{DC'}{D'C'}$ at different points of the curve give the accelerations at the respective instants of time (why?). Calculate these accelerations.

ACCELERATION.

Velocity may change either in direction or in magnitude or in both. Acceleration which is the rate of change of velocity may be uniform or variable. We study here only the simple case of uniformly accelerated motion, along a straight line. Here, the velocity is changing only in magnitude and that uniformly: the motion is, therefore, necessarily in a straight line. A freely falling body is a familiar example.

Such motion can be conveniently studied with an apparatus called Fletcher's trolley. The apparatus consists of a heavy trolley T (fig. 14) mounted on three smooth-running aluminium wheels (W). The trolley runs freely on a rectangular hardwood plane p, along two guide lines marked parallel to its length. An iron bridge B is fixed to the plane and a long vibrating strip S, clamped to the bridge, carries

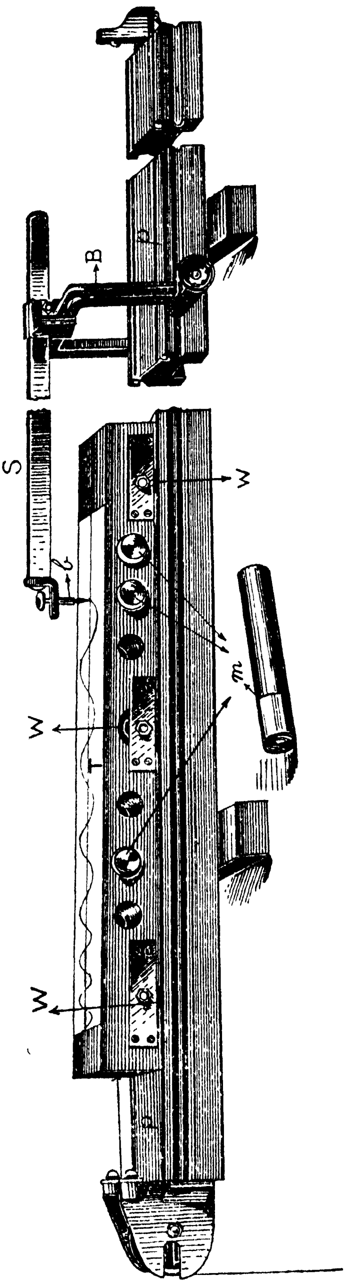


FIG. 14.

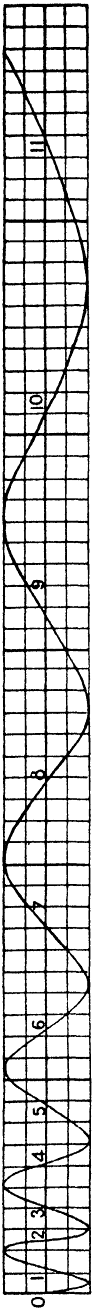


FIG. 15.

a brush *b* at its end. The length of the strip is so adjusted that it makes five complete oscillations per second. The mass of the trolley can be varied by inserting or removing iron cylindrical masses *m* which fit into the holes in the trolley.

Pin a strip of white paper, cut to size, on the flat top surface of the trolley and place it on the plane along the guide lines. Insert one or two of the wooden planks, usually supplied with the apparatus, underneath the plane and give it a certain inclination. Ink the brush and screw it down so as to make it touch the paper, holding the trolley in position as shown in the figure. Vibrate the strip and let go the trolley. The strip moves in a direction at right angles to that of the motion of the trolley and a wavy curve (fig. 15) is marked on the paper. Stop the strip and move the trolley up the incline. A straight line cutting the curve symmetrically at points 0, 1, 2, 3, etc. is drawn. The time taken by the trolley to move from the starting point zero to any of the points 1, 2, 3, etc. is obviously (?) 0·1, 0·2, 0·3, etc. of a second. Measure the distances 0–1, 0–2, 0–3, etc. with a dividers and millimetre scale. It may be that the middle line does not cut the curve symmetrically and so take observations in two sets 1–3, 1–5, etc. and 0–2, 0–4, 0–6, etc. By successive subtraction, the distances passed over by the trolley in successive intervals of 0·2 sec. are obtained. Multiplying these by 5 (?) in each case, the rate of motion in the successive intervals is obtained in cm. per sec. You find that these rates increase gradually with time. By successive subtraction

again, the increases in the velocity of the trolley in the successive intervals of 0·2 sec. are obtained and these multiplied by 5 give the increase in the velocity in each case that would take place in one second if the rate is maintained constant during the second in each case. Thus the rate of change of velocity or acceleration of the trolley is obtained in each case and the values will be found to be nearly constant. This shows that the acceleration is uniform and the mean of the different values may be taken as the mean constant. Nearly the same mean value is also obtained from the other set of observations.

Having thus determined the acceleration α from considerations of the first principles, we can now calculate it with the help of the relation $S = \frac{1}{2}\alpha t^2$ and see if we get the same value for α. In the observations made above, divide twice the distance travelled (2S) by the time square (t^2) in each case, time being calculated from the starting point. It will be found that the ratios are constant and the mean value agrees well with the mean acceleration found already.

In the above curve, at all points other than that marked O, the trolley has an initial velocity. Let 3 be the point chosen. Let the time taken by the trolley to move from the point 3 to any other odd point, say 9, be t sec. (0·6 sec.) and the time taken to the point 15 would be $2t$ sec. Let the distances travelled by the trolley be S_1 (3–9) and S_2 (3–15) in the times t and $2t$. Then

$$S_1 = ut + \tfrac{1}{2}\alpha t^2$$

$$2S_1 = 2ut + \alpha t^2$$

$$\text{and } S_2 = u.2t + \tfrac{1}{2}\alpha(2t)^2$$

$$= 2ut + 2\alpha t^2$$

$$\therefore\ S_2 - 2S_1 = \alpha t^2$$

$$\therefore\ \alpha = \frac{S_2 - 2S_1}{t^2}.$$

Therefore, choose different values for t and calculate the ratio $\frac{S_2-2S_1}{t^2}$ in each case. You will find that the ratios are nearly constant and that the mean ratio agrees well with the mean acceleration determined. This affords an easy method of finding the acceleration from the curve and we can take any point as a start and there is no restriction that the starting point for observations should coincide with the starting point of the curve. Therefore, in the case of the determination of α from the first principles, it is better to mark a convenient point as the zero.

Practical example.—

Time in *secs.*	Distance travelled in *cm.*	Distance moved in the successive intervals of time in *cm.*	Rate of motion or velocity in the intervals in *cm. per sec.*	Change of velocity in the successive intervals in *cm. per sec. per* 0·2 *sec.*	Rate of change of velocity in the intervals in *cm. per sec. per sec.*
0–0·2	1·0	1·0	5·0	..	..
0–0·4	4·17	3·17	15·85	10·85	54·25
0–0·6	9·27	5·10	25·50	9·65	48·25
0–0·8	16·42	7·15	35·75	10·25	51·25
0–1·0	25·55	9·13	45·65	9·90	49·50
0–1·2	36·75	11·20	56·00	10·35	51·75
0–1·4	49·95	13·20	66·00	10·00	50·00
0–1·6	65·20	15·25	76·25	10·25	51·25
				Mean =	50·89
0·1–0·3	2·05	2·05	10·25	..	..
0·1–0·5	6·17	4·12	20·60	10·35	51·75
0·1–0·7	12·23	6·06	30·30	9·70	48·50
0·1–0·9	20·30	8·07	40·35	10·05	50·25
0·1–1·1	30·40	10·10	50·50	10·15	50·75
0·1–1·3	42·50	12·10	60·50	10·0	50·0
0·1–1·5	56·60	14·10	70·50	10·0	50·0
				Mean =	50·21

Mean acceleration = 50·55 cm. sec^{-2}.

Time after start (t sec.)	Distance travelled (s cm.)	$\frac{2s}{t^2}$ cm. sec.$^{-2}$
0·1	0·25	50·0
0·2	1·00	50·0
0·3	2·30	51·1
0·4	4·17	52·1
0·5	6·40	51·2
0·6	9·27	51·5
0·7	12·45	50·8
0·8	16·42	51·0
0·9	20·55	50·8
1·0	25·55	51·1
1·1	30·65	50·7
1·2	36·75	51·0
1·3	42·78	50·6
1·4	49·95	51·0
1·5	56·90	50·6
1·6	65·20	51·0

Mean acceleration = 50·9 cm. sec.$^{-2}$

Starting point on the curve.	Time in secs. t.	S_1 cm.	S_2 cm.	$\frac{S_2-2S_1}{t^2}$ cm. sec.$^{-2}$
2	0·6	15·42	48·95	50·3
3	0·6	18·25	54·60	50·3
6	0·4	16·28	40·68	50·75
7	0·4	18·20	44·45	50·31

Mean acceleration = 50·4 cm. sec.$^{-2}$

The value obtained for acceleration may further be verified in the following manner. Let θ be the inclination of the plane to the horizon. Measure the height and length along the plane at any convenient point. Sin θ in the above experiment was found to be $\frac{8\cdot95}{148}$. Acceleration down the incline = $g \sin \theta$, where g is the acceleration due to gravity at the place and is equal to 978 cm. sec.$^{-2}$

$$g \sin \theta = \frac{8\cdot95}{148} \times 978 = 59\cdot10 \text{ cm. sec.}^{-2}$$

But the experimentally determined value is 50·6 cm. sec.$^{-2}$ This difference of $-(8{\cdot}5)$ cm. sec.$^{-2}$ must have been the mean retardation due to the opposing friction. The force of friction opposing the motion of the trolley, accordingly, must be $\frac{8{\cdot}5}{59{\cdot}1}$ or 0·144 of the weight of the trolley along the incline or $0{\cdot}144 \times \frac{8{\cdot}95}{148}$ of the weight of the trolley and is equal to 27·5 gm. wt. since the weight of the trolley is 3160 gm. wt. Even though the trolley moves on wheels there is some friction left still. The load necessary to just start the trolley when the plane is horizontal must be nearly this, as will be evident in the next exercise.

THE SECOND LAW OF MOTION.

The rate of change of momentum is proportional to the impressed force and takes place in the direction of the impressed force.* If a force F acts on a body of mass m for time t and if the velocity of the body changes from U to V, the change of momentum is $m(\mathrm{V}-\mathrm{U})$ units and the rate of change of momentum is $\frac{m(\mathrm{V}-\mathrm{U})}{t}$ units per second. But $\mathrm{V} = \mathrm{U}+\alpha t$, where α is the acceleration produced. Therefore, the rate of change of momentum $= m\alpha$ and according to the law this is proportional to F, i.e. $\mathrm{F} \propto m\alpha$ or $\frac{\mathrm{F}}{m\alpha} =$ constant.

If a unit force is defined as one which acting on a unit mass produces in it unit acceleration, then F, m and α each being equal to unity, the constant of proportion becomes unity and the second law can be symbolically expressed as $F = m\alpha$. The second law may therefore be expressed in one of the following ways: (i) Rate of change of velocity is

* The momentum of a body is measured by the product of its mass and velocity.

directly proportional to the impressed force, or (ii) the ratio of the impressed force to the acceleration caused is a constant for a body and measures the mass of the body, or (iii) the masses of bodies are inversely proportional to the accelerations produced by the same impressed force.

Therefore, if forces F_1, F_2 and F_3 act separately on the same body of mass m and produce accelerations α_1 , α_2 and α_3 respectively, then $\frac{F_1}{\alpha_1} = \frac{F_2}{\alpha_2} = \frac{F_3}{\alpha_3} = \text{constant} = m$. This affords us a method of comparing forces and verifying the law. Again if the same force F acts on masses m_1, m_2 and m_3 and produces accelerations α_1, α_2 and α_3, then $F = m_1\alpha_1 = m_2\alpha_2 = m_3\alpha_3$ or $\frac{m_1}{m_2} = \frac{\alpha_2}{\alpha_1}$, $\frac{m_1}{m_3} = \frac{\alpha_3}{\alpha_1}$ and $\frac{m_2}{m_3} = \frac{\alpha_3}{\alpha_2}$. Thus a number of masses can be compared and the law can as well be verified this way.

Fletcher's trolley is a convenient apparatus for verifying the second law of motion. The accessories required are a scale pan, box of weights, millimetre scale and a spring balance.

The hard wood plane of the apparatus is provided at one end with a smoothly running, light aluminium pulley (fig. 14). The trolley can be moved by placing weights in the scale pan which is supported by a string passing over the pulley, the other end of the string being attached to the hook at the end of the trolley. Keep the plane horizontal so that the pulley end just projects out of the supporting table. Put enough weights in the pan, to make the trolley move very slowly. With this weight on, trace the curve, as explained in the previous exercise. Fix another strip of paper to the top of the trolley, add an additional load of F_1 gm. to the pan and obtain the curve of motion again. Remove F_1 and place F_2 gm. and get the trace again. Proceed similarly with another additional load F_3 gm. Calculate the accelerations

from the curves using the relation $\alpha = \frac{S_2 - 2S_1}{t^2}$. Take two or three sets of observations for S_1, S_2 and t with each curve and calculate the mean accelerations α_1, α_2, α_3, and α_4.

Let W be the combined weight of the pan, the string and the load required to just move the trolley; let f be the opposing force of friction. The moving forces and the corresponding accelerations in each of the four cases are as follows:—

No.	Force (gm. wt.)	Acceleration (cm. sec.$^{-2}$)
1	$(W-f)$	α_1
2	$(W-f+F_1)$	α_2
3	$(W-f+F_2)$	α_3
4	$(W-f+F_3)$	α_4

The accelerations α_2, α_3 and α_4 are partly caused by $(W-f)$ and partly by F_1, F_2 and F_3 respectively. So we can associate the forces F_1, F_2 and F_3 respectively with accelerations $\alpha_2-\alpha_1$, $\alpha_3-\alpha_1$ and $\alpha_4-\alpha_1$. Calculate, in each of these three cases, the ratio of the force acting to the acceleration produced. This ratio will be nearly constant. Compare the mean constant, thus obtained, with the mass of the trolley as measured with a spring balance directly. Verify the law by following the other method indicated above.

Practical example.—The load put in the pan to make the trolley move very slowly is 30 gm. (compare this with the value of 27 gm. obtained in the previous exercise). The pan and the string together weigh 10 gm. This extra weight is to compensate for the extra friction due to the pulley and the string. W in the table (on page 37) is accordingly equal to 40 gm. F_1, F_2 and F_3 were 50, 100 and 150 gm. wts. respectively. g, acceleration due to gravity, is taken to be equal to 978 cm. sec.$^{-2}$ In determining α_1, it is convenient to put a load slightly over 30 gm.

No.	t sec.	S_1 cm.	S_2 cm.	$\frac{S_2 - 2S_1}{t^2} = \alpha$ cm. sec.$^{-2}$	Mean α cm. sec.$^{-2}$	Total force acting $(F+W-f)$ gm. wt.	Accl. due to F alone cm. sec.$^{-2}$	$\frac{F}{\alpha} = M$ gm. Total mass moved in gm.	Mass of trolley alone in gm.
1	1·0 1·2 1·4	8·67 11·10 13·80	22·90 30·20 38·36	5·56 5·55 5·49	5·53	$F+W-f$ $F = 0$, no extra load.	..	..	..
2	0·6 0·8 1·0	6·05 9·63 14·08	19·33 32·43 48·83	20·08 20·58 20·63	20·44	F_1+W-f $F_1 = 50$ gm. wt.	$(\alpha_2-\alpha_1)=a_1$ $= 14{\cdot}91$	$\frac{50\times 978}{14{\cdot}91} = M_1$ $= 3279{\cdot}2$	$M_1 - 90 = m_1$ $= 3189$
3	0·4 0·6	4·45 8·78	14·50 30·28	35·00 35·33	35·16	F_2+W-f $F_2 = 100$ gm. wt.	$(\alpha_3-\alpha_1)=a_2$ $= 29{\cdot}63$	$\frac{100\times 978}{29{\cdot}63} = M_2$ $= 3300$	$M_2 - 140 = m_2$ $= 3160$
4	0·4 0·6	5·58 11·30	19·05 40·65	49·31 50·14	49·72	F_3+W-f $F_3 = 150$ gm. wt.	$(\alpha_4-\alpha_1)=a_3$ $= 44{\cdot}19$	$\frac{150\times 978}{44{\cdot}19} = M_3$ $= 3319$	$M_3 - 190 = m_3$ $= 3129$

Mean mass of the trolley=3160 gm.

In each case, the mass moved is that of the trolley with the combined mass of the string, pan and the load in the pan. The mass of the trolley was found directly with a spring balance, graduated in pounds and reading to 2 oz. The reading was 7 lb. which is equal to 3175 gm. and this compares well with the value obtained above.

ATWOOD'S MACHINE.

Let m, m be two equal masses hung on either side of a frictionless light pulley by means of a fine string or a ribbon. The resultant force at any point of the string is zero and the masses remain at rest. If an external force acts on one of the masses downwards or upwards, only for an instant, an initial velocity is given to the system and the system continues to move uniformly with that velocity. If a constant force acts continuously, the system moves with a uniform acceleration. If m_1 be the additional mass put on one of the masses m, the resultant force acting on the system is m_1g dynes and the total mass moved is $(2m+m_1)$ gm. So the acceleration of the system, which is equal to the ratio of the moving force to the mass moved, may be written as $\dfrac{m_1g}{2m+m_1}$ cm. sec.$^{-2}$ The moving

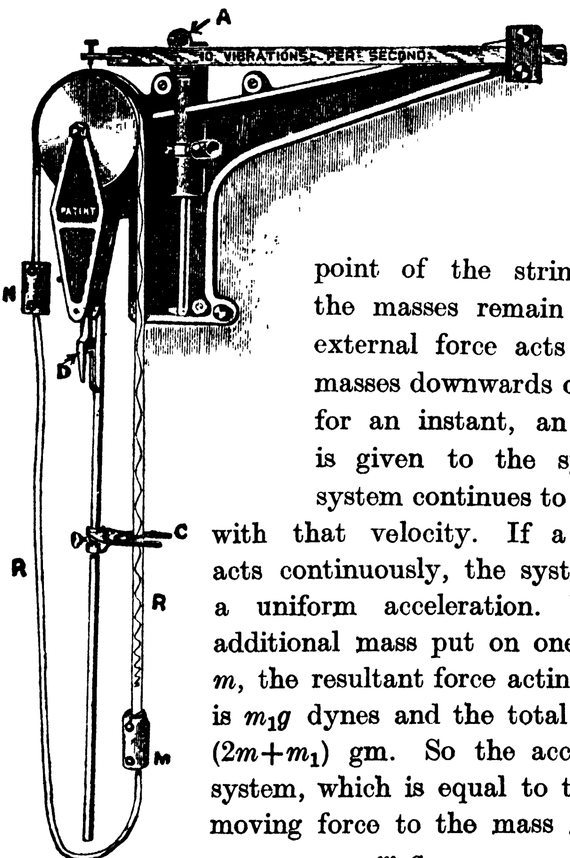


FIG. 16.

mass $(2m+m_1)$ changes its velocity by α cm. sec.$^{-1}$ in every second and the momentum of the masses changes by $(2m+m_1)\alpha$

units in every second. The rate of change of momentum is, therefore, $(2m+m_1)\alpha$ and is equal to the impressed force $m_1 g$ which is the weight of the rider m_1.

$$\therefore m_1 g = (2m+m_1)\alpha \text{ and } \alpha = \frac{m_1}{2m+m_1} g.$$

So α can be made any convenient fraction of g by suitably selecting m_1. The following conclusions are easily arrived at. Since $g = \frac{2m+m_1}{m_1}\alpha$ and m and m_1 are known, if α can be determined, the acceleration g due to gravity can also be determined. $\alpha_1 = \frac{m_2}{2m+m_2} g$ with a different rider m_2 and if m_1 and m_2 are so small that $2m+m_1$ may be practically taken to be equal to $2m+m_2$, we get $\frac{\alpha}{\alpha_1} = \frac{m_1}{m_2}$. $\frac{\alpha}{\alpha_1}$ can be practically determined and can be shown to be equal to $\frac{m_1}{m_2}$. This verifies the statement that the accelerations produced by different forces on the same mass are directly proportional to the forces (Newton's second law of motion). If another set of equal masses M, M are used with the same rider m_1, and if α_2 is the acceleration produced, $\alpha_2 = \frac{m_1}{2M+m_1}. g$ and $m_1 g = (2M+m_1)\alpha_2 = (2m+m_1)\alpha =$ constant. The accelerations produced will be found to be inversely proportional to the masses moved. This also verifies Newton's second law of motion.

The above deductions and the laws of falling bodies can be conveniently studied with the ribbon type of Atwood's machine (fig. 16). The peculiarity in this arrangement, originally due to George Atwood, is that the acceleration of the moving system can be adjusted to a convenient and measurable value. The measurements made with it are accordingly more manageable than in the case of a freely falling body. The

machine shown in the figure is of an improved design, wherein small intervals of time are accurately recorded by an oscillating spring, as in the case of the Fletcher's trolley. The pulley is of thick sheet of aluminium containing a flat rim nearly one inch wide. Equal masses M, M are hung by means of a paper ribbon RR, the masses being attached to the ribbon by a simple device. A vibrating metal strip is attached to the framework of the machine and a fine brush is attached to the free end of the strip. A is an arrester of the vibrator and D is a hinged platform, on which one of the masses is supported before start. An oblong rider and a friction-rider f are put on this mass. A handle releases the metal spring and at the same time allows the platform to fall down and the system begins to move. The inked brush is screwed down to touch the paper ribbon at the topmost point of the pulley and records the motion of the system. C is a catch, movable on a vertical steel rod, and allows the mass M to pass through when properly adjusted, but catches the oblong rider, and hence-forwards, the motion of the system becomes uniform and is recorded on the ribbon. The ribbon has an appreciable weight and as it moves round, the difference of weight of the ribbon on the moving side, gets greater and greater and adds to the moving force. To correct for this error a compensating piece of ribbon is attached, as shown in the figure. The pulley is balanced well and is mounted on frictionless ball-bearings. In spite of this, there will be some friction left, to compensate which a small friction-rider f is supplied and is placed on the mass underneath the bigger oblong rider and is not removed by the catch C. The pulley absorbs some of the kinetic energy of the moving masses and the motion is consequently retarded, as if an addition is made to the mass moved. This additional equivalent mass can be calculated from the dimensions, etc. of the pulley and is engraved on the pulley or on the framework of the

apparatus. A box of accessories is supplied along with the apparatus and contains the masses, three riders, ribbon roll, ink and brush.

The value of g can be determined with the ribbon machine, its accessories, a millimetre scale and dividers in the following manner. Arrange the apparatus and start the experiment, keeping the catch as low as possible. When C catches the oblong rider, hold the masses and stop the vibrator. A trace is obtained on the ribbon. Preserve the ribbon and repeat the experiment with the other two riders. With the scale and dividers, determine the acceleration in each case using the formula $\alpha = \frac{S_2 - 2S_1}{t^2}$ as already explained. The total mass in each case is $2m + m_1 + E$, where E is the equivalent mass of the pulley. The mass of the friction rider, being very small, is neglected.

Practical example.—(a) To determine the value of g:

$$m = 375 \text{ gm.}, \ E = 62 \text{ gm.}$$

(1) $m_1 = 5$ gm. Total mass $= 817$ gm.

$$\left.\begin{aligned} S_1 &= 7{\cdot}15 \text{ cm.} \\ S_2 &= 22{\cdot}9 \ \ \text{,,} \\ t &= 1{\cdot}2 \text{ sec.} \end{aligned}\right\} \alpha = \frac{S_2 - 2S_1}{t^2} = \frac{8{\cdot}6}{1{\cdot}44} \text{ cm. sec.}^{-2}$$

$$\therefore g = \frac{817 \times 8{\cdot}6}{1{\cdot}44 \times 5} = 976 \text{ cm. sec.}^{-2}$$

(2) $m_1 = 8$ gm.

$$\left.\begin{aligned} S_1 &= 11{\cdot}5 \text{ cm.} \\ S_2 &= 36{\cdot}3 \ \ \text{,,} \\ t &= 1{\cdot}2 \text{ sec.} \end{aligned}\right\} \alpha = \frac{13{\cdot}3}{1{\cdot}44} \text{ cm. sec.}^{-2}$$

$$\therefore g = 947 \text{ cm. sec.}^{-2}$$

(3) $m_3 = 10$ gm.

$$\left.\begin{aligned} S_1 &= 14{\cdot}55 \text{ cm.} \\ S_2 &= 45{\cdot}55 \ \ \text{,,} \\ t &= 1{\cdot}2 \text{ sec.} \end{aligned}\right\} \alpha = \frac{16{\cdot}45}{1{\cdot}44} \text{ cm. sec.}^{-2}$$

$$\therefore g = 940 \text{ cm. sec.}^{-2}$$

Mean value of $g = 954$ cm. sec.$^{-2}$

(b) To verify the second law of motion:

The total mass in each case may be regarded as practically constant, being 817, 820 and 822 gm. respectively. The ratio of the accelerations

observed is 8·6 : 13·3 : 16·45 which is the same as 5·0 : 7·73 : 9·57. According to the law, the ratio should have been as 5 : 8 : 10 which is the ratio of the respective impressed forces.

(c) *To verify that $S = vt$:*

Raise the catch to a convenient height and proceed with the experiment. The distances between corresponding points on the second-half of the trace will be found to be constant. The first half of the trace records the accelerated motion and the second half records the uniform motion of the system.

(d) *To verify that $S \propto t^2$ or that $\frac{S}{t^2}$ = constant $(\frac{1}{2}\alpha)$:*

This can be verified from any one of the traces drawn, as shown in a previous exercise on acceleration.

(e) *How do you proceed to verify the relation $v = \alpha t$?*

THE SIMPLE PENDULUM.

A heavy particle suspended from a rigid support by a fine weightless thread constitutes a simple pendulum, in theory. This cannot be realized in practice and so a heavy bob O suspended by a fine thread forms a simple pendulum. The distance SO (fig. 17) between the point of suspension S and the centre of gravity of the bob O, is called the length of the simple pendulum. If the bob is displaced through an angle θ, (position O_1) and let go, the bob moves towards its position of rest O along an arc and shoots beyond O, to an equal distance on the other side to O_2, where it is momentarily at rest. It returns towards O, and shoots again beyond O to O_1 and thus the bob oscillates about its position of rest for a long time. The maximum angle of shift θ of the pendulum, from its position of rest, on either side, is called the amplitude of the oscillation. The time between two

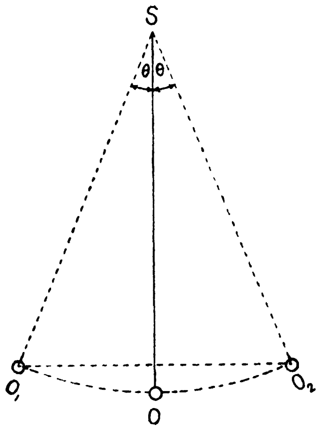


FIG. 17.

consecutive passages of the pendulum, in the same direction through any point in its path is called the time of a complete oscillation.

In starting the pendulum work is done in lifting the bob against its weight through a small height and energy is stored in the bob at O_1 and is all potential, the velocity at O_1 being momentarily zero. This potential energy is gradually transformed into kinetic and the velocity of the bob gradually increases to a maximum while passing through the position of rest O, where the energy is all kinetic, and according to the first law of motion the bob continues to move. As the length SO is constant, the bob is constrained to move along the arc of a circle and, in so doing, is raised against its weight until all the kinetic energy is spent and the velocity gradually becomes zero at O_2 where the energy is once again all potential. The same phenomenon repeats itself for some time. Friction at the point of suspension and friction due to its passage through air gradually absorb the energy of the pendulum. The oscillations are steadily damped and the amplitude becomes smaller and smaller till the pendulum is finally brought to rest.

The time of oscillation of a simple pendulum remains constant to within one part in 1,000 and the vibrations are isochronous. This is so only when the amplitude of the pendulum is small, say within 5° at the most. If θ is greater, accurate determinations show that T increases with θ. The time of oscillation is found to be independent of the shape, size or material of the bob (except for the variations in the friction due to the passage in air), but it is directly proportional to the square root of the length l of the pendulum and this is expressed by the relation $T = 2\pi\sqrt{\frac{l}{g}}$. g stands for the acceleration due to gravity. From this we find that $\frac{l}{T^2}$, which

is equal to $\frac{g}{4\pi^2}$, is a constant at any place. This constant K can be experimentally found and g can be thus obtained. This method of determining g by the simple pendulum is a ready and good laboratory method.

The relation $T \propto \sqrt{l}$ may be verified in the following manner.

Apparatus required are a retort stand (heavy), a spherical metal bob, fine thread, stop-clock reading to 0·2 sec., foot rule, metre scale, callipers, etc.

Arrange the apparatus as shown in figure 18. Displace the pendulum through a small angle from its position of rest and let it go, having noted the position of rest of the thread on the foot rule which is placed with its edge along that of the table. Care is to be taken that the pendulum moves in a single plane parallel to the edge of the table. When the thread passes the position of rest, say towards the right, start the clock and when it next passes the point in the same direction count one; when it next passes the point similarly, count two and stop the clock as you count fifty. Note the time interval. Repeat the observation and divide the mean interval by 50. This gives the time of a complete oscillation T. Find the length of the thread from the point of suspension to the top of the bob, with the metre scale. Find the diameter of the bob with the callipers and add half the value to the above length measured. This gives the length of the pendulum l. Change the length and repeat the observations

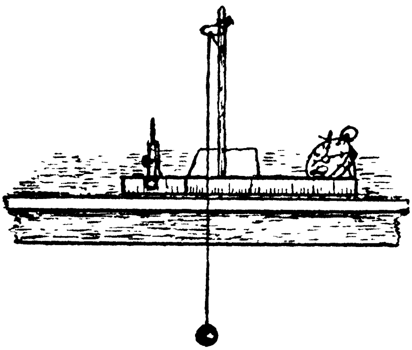


FIG. 18.

at least half a dozen times. Calculate $\frac{l}{T^2}$ in each case and it will be found to be nearly constant. Plot the l and T curve and deduce the length of the seconds pendulum from it. Note that the length of the seconds pendulum may also be deduced, as explained in the practical example, from the mean value obtained for $\frac{l}{T^2}$.

Practical example.—Diameter of the bob = 2·60 cm. The distance from the point of suspension to the edge of the foot rule was nearly 60 cm. The displacement of the thread along the scale on either side of the position of rest was never more than 4 cm. So the value of θ, the amplitude in the experiments, was never more than 4/60 or 1/15 of a radian or 4° nearly.

No.	l cm.	Time of 50 oscillations.		T seconds.	$\frac{l}{T^2}$ = K
1	157·8	2′ 6″·0 2′ 6″·4	2′ 6·2″	2·524	24·77
2	135·5	1′ 57″ 1′ 57″	1′ 57″	2·34	24·75
3	111·1	1′ 46″ 1′ 46″	1′ 46″	2·12	24·72
4	93·0	1′ 37″ 1′ 37″·2	1′ 37″·1	1·942	24·61
5	83·0	1′ 31″·8 1′ 31″·8	1′ 31″·8	1·836	24·62
6	67·6	1′ 22″·8 1′ 22″·8	1′ 22″·8	1·656	24.64

Mean K =.24·685
g = 974·5 cm. sec.$^{-2}$

$\frac{l}{T^2}$ is found to be nearly constant and the law is accordingly verified. The value of g at the place of experiment (latitude 17°) is 978·4 cm. sec.$^{-2}$ and the value obtained above is correct to within one in 250. How is the result affected if the clock goes fast ?

The results may be used to obtain the length of the seconds pendulum. A clock with a seconds pendulum ticks seconds.

The interval between consecutive ticks or escapements of the toothed wheel in such a clock is one second. Time can be counted from the number of ticks. Such a clock ticks twice in every complete oscillation and so the time of a complete oscillation of the seconds pendulum is two seconds. Therefore, its length can be readily calculated thus.

$$\frac{l}{T^2} = 24{\cdot}685 \text{ and } T = 2 \text{ sec.} \quad \therefore l = 98{\cdot}74 \text{ cm.}$$

The value obtained from the l—T curve (not shown) is 99·0 cm. This is the length of a simple pendulum which ticks seconds. The pendulum which is generally seen in clocks cannot be considered as a simple pendulum and the length of such a pendulum in a seconds clock is considerably more than 98·74 cm.

Is the interval between consecutive ticks in ordinary clocks and watches a second?

Do the hands of a clock or watch move continuously or at intervals? Why?

Is the length of the seconds pendulum the same at all places?

Why do you find $g = 981$ cm. sec.$^{-2}$ in text books in general?

What value do you give to g in accurate work in South India?

How do you adjust the length of the pendulum, if the clock is going fast?

What is that which you wind in a clock and what purpose does it serve?

Chapter III

STATICS

FORCES AT A POINT.

A force has a point of application, a direction and a magnitude. A straight line has the same three elements. If one of the ends of the line is taken as the point of application, a force can be completely represented by a straight line. We will now consider the effect of the action of a number of forces on a body. Here, a body is considered to have a very small volume and any forces acting on it are regarded as acting at a point. A single force acting on a particle cannot keep it at rest. Two forces can keep the particle in equilibrium, provided that they are equal in magnitude and act along the same straight line in opposite directions. When the two forces do not act along the same straight line, the particle moves in a direction between the two. A single force, which may be regarded as producing the resultant motion, may be found. This single force, which has the same effect on the particle or body as the two forces that act on it, is defined as the *resultant* of the two forces. The same definition can be extended to the case where more than two forces act on a body. The forces which can thus be replaced by their resultant are called the *components*. When three forces act on a body and keep it at rest, then the resultant of any two of the forces must be equal in magnitude and opposite in direction, to the third force.

If two forces acting on a particle be represented in magnitude and direction by the adjacent sides of a parallelogram, the resultant force is represented in magnitude and direction by the diagonal passing through the point of intersection of the

two sides. This is called *the law of parallelogram of forces.* If three forces acting on a body be represented in magnitude and direction by the three sides of a triangle taken in order, the forces are in equilibrium. This is called *the law of triangle of forces. Lami's theorem,* which is an alternative way of expressing the law of triangle of forces, states that if three forces acting on a particle are in equilibrium, each force is proportional to the sine of the angle between the directions of the other two forces.

Any two forces acting on a body must be in a single plane. Their resultant also must be in the same plane. If a third force also acts on the body and keeps it at rest, this third force must be equal in magnitude and act in the same straight line as the resultant but in the opposite direction and so must be in the same plane again. Therefore all the three forces acting on a body and keeping it at rest must be co-planar forces. Such are the forces mentioned above.

The above laws may be verified in the following manner. Apparatus required are retort stands, a drawing board, two ball-bearing light aluminium pulleys, a set of weights, string,

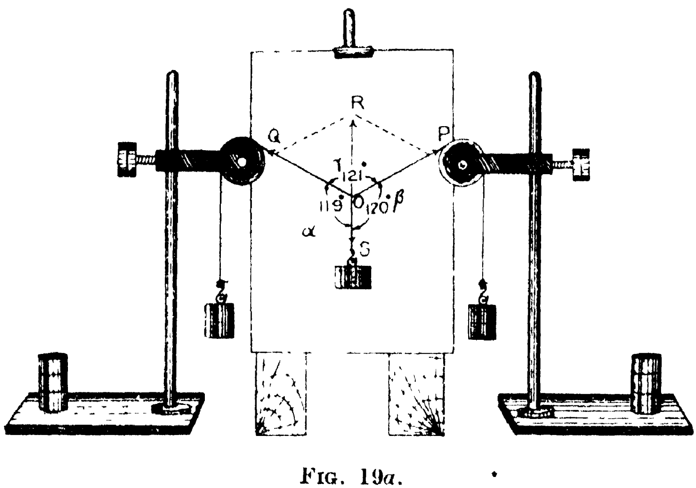


FIG. 19*a*.

metal hooks of S shape, scale pan, instrument box, foot rule, etc.

Pin a sheet of white paper on the drawing board and arrange the apparatus as shown in figure 19*a*, keeping the board just behind and parallel to the plane in which the three forces are acting. Carefully trace * the directions of the forces P, Q, and S on the board, noting the value of the forces in grams weight. Change the hanging weights and get a different arrangement as in figure 19*b*. Trace the directions and note down the forces as before. Again lower the pulley on the left hand side and adjust the load in the scale pan passing over the pulley and get a position as shown in figure 19*c* such that the direction of Q is horizontal Trace the directions as before and note the value of the forces Find the weight of the scale pan correct to a gram. In each case select any two forces P and Q. Cut off lengths OP and OQ, to represent the forces, adopting a convenient scale. Construct the parallelogram OPRQ with OP and OQ as adjacent sides. Measure the length of the diagonal OR and calculate what force it represents according to the scale adopted. It will be found that this is nearly equal to the magnitude of the third force S. Produce RO and it will very nearly coincide with OS. OR will thus be found to be equal in magnitude and opposite in direction to OS. The point O is at rest under the action of the three forces OP, OQ and OS. If the point O were impressed by the two forces OR and OS only, it can also be maintained at rest.

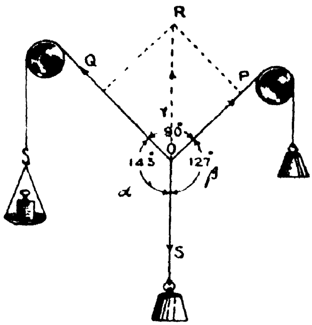


Fig. 19*b*.

* Keep a set square at right angles to the board along the string and trace on the board the direction of the force with a pointed pencil.

Therefore, the single force OR has the same effect as the two forces OP and OQ. This force OR is the resultant of the two forces OP and OQ and is represented by the diagonal of the parallelogram with OP and OQ as its adjacent sides.

Consider the triangle OQR in the figures. QR is equal in length to OP and is parallel to it, and so represents the force P. OQ is already drawn to represent the force Q. OR represents the resultant of the forces P and Q and is equal in magnitude but opposite in direction to the third force S. So RO represents S. Hence the sides of the triangle OQ, QR and RO taken in order, represent the three forces Q, P and S respectively and the point O is at rest. We, accordingly, verify the law of the triangle of forces.

In the triangle OQR,

$$\frac{OQ}{\sin QRO} = \frac{QR}{\sin QOR} = \frac{RO}{\sin OQR};$$

but $\sin QRO = \sin ROP = \sin \beta$

and $\sin QOR = \sin \alpha$ and $\sin OQR = \sin \gamma$

$$\therefore \frac{Q}{\sin \beta} = \frac{P}{\sin \alpha} = \frac{S}{\sin \gamma}.$$

Therefore, the ratio of any force to the sine of the angle between the directions of the other two forces is constant.

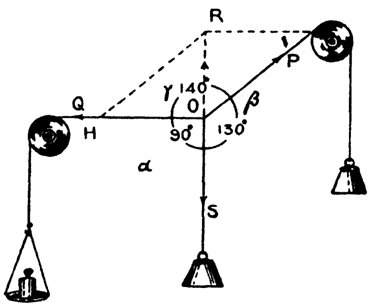


FIG. 19c.

In one of the three above arrangements (fig. 19c), Q is horizontal and S is vertical. If the force Q were absent, the string POS would be hanging vertically. If the horizontal force Q is now applied at O, the length of the string PO is deflected out of the vertical, through an angle θ. Apply Lami's theorem and show that $Q = S \tan \theta$ and verify it by calculation.

Keep S constant, vary Q and O by raising and lowering the pulley. As the value of P is not wanted, the string OP may permanently be tied to a retort stand. A circular protractor placed about the point O will measure the angle θ in each case. Calculate $\frac{Q}{\tan\theta}$ in each case and it will be found to be nearly a constant. This constant is equal to the controlling force S. Thus, we find that *the tangent of the angle of deflection is proportional to the deflecting force Q.*

Is the tension of the string OP the same in the various cases?

How do you calculate its value in any case?

Can the tension in any case be less than the force S?

Practical example.—

(*a*) *Parallelogram Law:* Scale adopted in I and II was one cm. to 20 gm. wt. and in III, it was one cm. to 40 gm. wt. In each of the three cases, the diagonal OR when produced backwards passed very nearly through the direction of the third force. The maximum angle of deviation was 1°. All the forces were in gm. wt. and the angles in degrees.

No.	Calculated value of the resultant from the length of the diagonal according to the scale adopted. in gm. wt.	Equilibrant or the third force. in gm. wt.
I	196	200
II	405	400
III	197	200

(*b*) *Lami's Theorem:*—

No.	P	Q	S	α	β	γ	$\frac{P}{\sin\alpha}$	$\frac{Q}{\sin\beta}$	$\frac{S}{\sin\gamma}$
	in gm. wt.			in degrees					
I	200	200	200	119	120	121	228·7	231·0	233·3
II	250	307	400	143	127	90	415·4	384·4	400·0
III	300	227	200	90	130	140	300·0	296·3	311·1

(c) *Tangent Law :*

No.	Q gm. wt.	S gm. wt.	θ	$\frac{Q}{\tan\ \theta}$
1	227	200	50°	190·5
2	112	,,	31°	194·0
3	144	,,	36°	198·0
4	174	,,	42°	193·2
5	207	,,	46°	199·9

PARALLEL FORCES.

A solid body which can keep its shape and size unaltered under the action of impressed forces is called a rigid body. The effect of a force on such a body is the same, whatever be the point of application of the force. When a rigid body is acted on by a number of forces and is in equilibrium, the forces do not produce either translation or rotation. If the directions of all the applied forces meet at a single point, rotation cannot be produced and when the number of the concurrent forces is three, the condition of equilibrium has already been stated in the previous exercise.

Let us now consider the simple case of a rigid body acted on by a number of parallel forces, some *like* and some *unlike*. Two parallel forces are said to be *like* when their directions are the same and said to be *unlike* when their directions are opposite. The tendency of a force to rotate a rigid body about a fixed point is measured by the turning effect and is called the moment of the force about the point. The moment varies directly as the magnitude of the force F and the perpendicular distance d of the line of action of the force from the fixed point. If the turning power of a unit force at a unit perpendicular distance from the point is defined as the unit of moment, then the constant of variation becomes unity and the measure of the moment of the force about the point is the product $F \times d$. Let O (fig. 20) be the fixed point in the

rigid body. Let a force F be applied at the point A in the direction AB. The moment of the force about O is, according to the above definition, measured by $F \times d$.

$$F \times d = OA \times \frac{d}{OA} \times F = OA \times F \cos \theta.$$

So the moment of the force about O can also be measured by the product of the distance between the point chosen and the point of application of the force and the resolved component of the force along the perpendicular at the point of action to the line joining the point chosen and the point of application. The moment of a force about a point is considered to be negative if the force tends to rotate the body in the clockwise direction, and positive if it tends to rotate it in the anticlockwise direction.

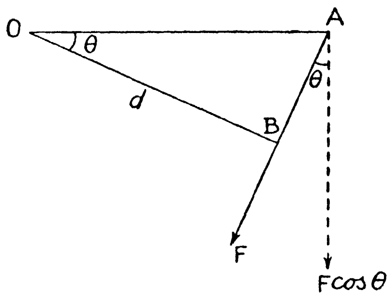


FIG. 20.

When a number of parallel forces acting on a body are in equilibrium, two conditions hold good. These are: (i) the algebraic sum of the forces is zero, and (ii) the algebraic sum of the moments of the forces about any point in the plane of the forces is zero. These conditions may be verified in the following manner.

Apparatus required are a metre scale, a number of weights, two spring balances (graduated in grams), wire hooks, etc.

Weigh the metre scale with the hooks slipped on to it. Let the hooks be so thin that each weighs less than a gram. Arrange the apparatus as shown in figure 21. By adjusting the points of application of the forces on the scale, it can be made horizontal as judged with the eye, from a distance. This facilitates the reading of the perpendicular distances of

the lines of action of the forces from the point chosen, directly from the scale. The rigid body is the scale itself and it may

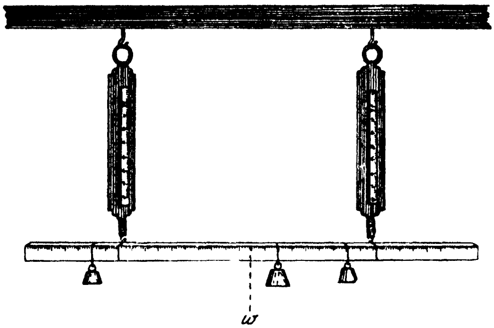


FIG. 21.

be kept at rest in an inclined position as well, but the distances are then to be measured with another scale. Note the value of the forces and the points on the scale at which they are acting. Moments can be taken about any point in the plane of the forces which is the vertical plane in this particular case. For convenience of measurement, the points may be chosen on the scale itself. Select 2 or 3 points, one of them being a point at which one of the forces acts. Write down the moments of the forces about the points chosen with their proper signs and find their algebraic sum in each case. The point chosen is supposed to be fixed and the scale capable of rotation about it. As the body is at rest, obviously, there can be no resultant force acting on it and for the same reason no resultant turning moment either.

Practical example.—

The spring balances read to 10 gm. and to a maximum of 500 gm. The metre scale weighed 100 gm. The upward forces were the tensions in the spring balances and their sum was 500 gm. wt. The sum of the downward forces including the weight of the scale was also 500 gm. wt. and the algebraic sum was zero.

Force in gm. wt.	Point of action on scale.	Points about which moments are taken.			Moments of the forces in gm. wt. cm. units.		
		a	b	c	a	b	c
		cm.	cm.	cm.			
100 ↓	18·6 cm.	0	25	50	– 1860	640	3140
250 ↑	23·5 ,,	..	..	..	5875	– 375	– 6625
100 ↓	50·0 ,,	..	..	..	– 5000	– 2500	0
200 ↓	54·6 ,,	..	..	..	– 10920	– 5920	– 920
100 ↓	71·2 ,,	..	..	..	– 7120	– 4620	– 2120
250 ↑	76·0 ,,	..	..	..	19000	12750	6500
Algebraic sum of moments:					– 25	– 25	– 25

SIMPLE MACHINES.

A machine is a contrivance in which a force applied at one point is available at another, with its magnitude and direction often changed. Machines like a steam engine or a bicycle are complicated in their structure. They can be analyzed into simpler parts called simple machines and these can be classified under a few heads, like the lever, the inclined plane, and the pulley. The last two will be studied in some detail in the following sections.

In a machine a force called *power* is applied at a point and work is done against a resisting force called *weight* or *resistance*. This power must be carefully distinguished from the usual term which means the rate at which an agent can work. The ratio of weight to power is called the mechanical advantage of the machine. This is, in many cases, greater than unity. The advantage of a machine is only in the power applied but not in the work done. According to the principle of the conservation of energy, the work done on the machine must be equal to the work done by the machine. Hence the power P multiplied by the distance through which its point of application is displaced must be equal to the product of the weight W and the displacement of the point of action of

the weight. This means that the displacements are inversely as the corresponding forces. This statement is known as *the statical principle of work.*

THE INCLINED PLANE.

A rigid plane surface inclined to the horizontal at an angle less than a right angle is called an inclined plane. A body can be pulled or rolled along the length of the plane by the application of a force less than the weight of the body. Let ABC (fig. 22) be a vertical section of the inclined plane. A weight W can be kept in equilibrium at any point on the incline by the application of a force P. Three cases arise. These are (*a*) when the direction of P is along the plane, (*b*) when the direction of P is horizontal, and (*c*) when the direction of P makes any angle θ with the plane.

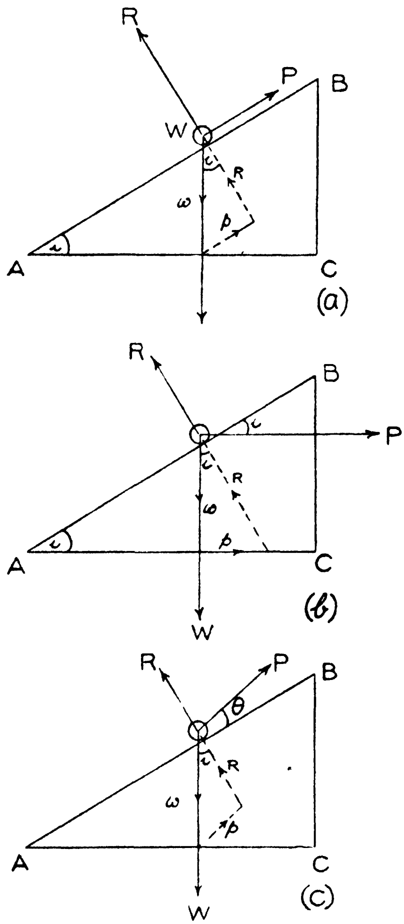


FIG. 22.

Let i be the angle of inclination of the plane. In any case, the body is in equilibrium under the action of three coplanar forces, the power P, the reaction R and the weight W, and they can be represented by the three sides of a triangle taken in order as shown in the figures.

In fig. (a), let the forces be resolved in two directions at right angles to each other, one of the directions being along and the other at right angles to the plane. Since the forces are in equilibrium, the algebraic sum of the components in each direction must be zero. R has no component along AB and P has none at right angles to AB. W sin i is the component of W along BA and W cos i along RW.

$$\therefore \mathrm{P}-\mathrm{W} \sin i = 0 \text{ and } \mathrm{R}-\mathrm{W} \cos i = 0$$

$$\therefore \mathrm{P} = \mathrm{W} \sin i \text{ and } \frac{\mathrm{W}}{\mathrm{P}} = \frac{1}{\sin i} = \frac{\mathrm{AB}}{\mathrm{BC}} = \frac{l}{h},$$

where l is the length of the plane (AB) and h is the height (BC) of the plane.

In fig. (b), resolving and equating similarly, we get P cos i— W sin $i = 0$ where P cos i is the component of P up the plane.

$$\therefore \mathrm{P} = \mathrm{W} \tan i \text{ or } \frac{\mathrm{W}}{\mathrm{P}} = \frac{\mathrm{AC}}{\mathrm{BC}}.$$

This is the tangent law of forces. We also have P sin i+ W cos i—R = 0, if we consider the forces at right angles to the plane.

In fig. (c), considering the resolved components parallel to AB, we have P cos θ—W sin i = 0.

$$\therefore \quad \frac{\mathrm{W}}{\mathrm{P}} = \frac{\cos \theta}{\sin i} \qquad \ldots \qquad \ldots \qquad \ldots \quad (1)$$

Considering the components perpendicular to AB, we have

$$\mathrm{R}+\mathrm{P} \sin \theta-\mathrm{W} \cos i = 0 \qquad \ldots \qquad \ldots \quad (2)$$

$\frac{\mathrm{W}}{\mathrm{P}} = \frac{\cos \theta}{\sin i}$ is a general expression and the ratio $\frac{\mathrm{W}}{\mathrm{P}}$ in the previous two cases can be deduced from this. Putting $\theta = 0$ corresponds to fig. (a) and $\theta = i$ corresponds to fig. (b). P may be on either side of AB and this does not affect the value $\frac{\mathrm{W}}{\mathrm{P}}$ but only alters the value of the reaction R.

The condition of equilibrium that $\frac{P}{\sin i}$ *is a constant, when the power acts parallel to the plane, may be verified in the following manner.*

Apparatus required are an inclined plane, a roller weight, scale pan, box of weights, foot rule, dividers, clamp, etc.

To reduce resistance due to friction, a glass plate G (fig. 23) is fixed on the top face of the incline. The plane AB is hinged at the bottom edge to a horizontal wooden plank AC, which is secured to the working table by a metal clamp. A ball bearing aluminium pulley is fixed to the top end of the plane. Through a slot cut parallel to the length of the vertical wooden plank F, passes a thumb screw E, by means of which, the plank F can be clamped to the incline. A heavy brass roller W is supported by the weight of the scale pan and its contents. Clamp F at a convenient height. Put weights P_1 in the pan just enough to draw the roller upwards—the motion being as slow as possible (why?). See that the length of the string is parallel to the edge of the plane (why?). Adjust the weights now to P_2 so that the roller just rolls downwards.* Take the mean of P_1 and P_2. Add the weight

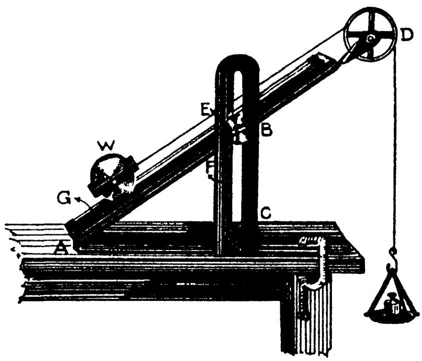


FIG. 23.

* Let f be the frictional resistance between the glass plate and the roller. When the roller moves up, friction resists motion and acts downwards. If p is the load in the pan required to balance the weight if there were no friction, P_1 must be in excess of p by f and $P_1 = p+f$. When the roller moves down the incline, the frictional force again resists the motion and so acts up the plane and assists the pull exerted by the load in the pan and P_2 is less than p by f and $P_2 = p-f$. The mean of P_1 and P_2 is equal to p. The resistance due to friction is thus eliminated.

of the pan to it and P the power required to balance the weight is obtained.

Measure the height BC (h), the vertical distance between the lower edge of the upper plank and the upper edge of the lower plank, at any convenient point B, with a millimetre scale. Measure the length AB (l) along the edge. Calculate the ratio $\frac{P}{\sin i}$ which is equal to $\frac{P \times l}{h}$. Alter the inclination i, by clamping F at a different height and repeat the observations. Take a number of sets of observations. You find that $\frac{P}{\sin i}$ is nearly constant. Weigh the roller with a spring balance and compare it with the mean constant obtained experimentally. Hence for a given weight, the power required to balance it on the inclined plane is proportional to sine i or the slope of the plane. Note how the mechanical advantage decreases as the inclination increases. What is the lower limit for the mechanical advantage?

Apply the principle of work and show that the work done is the same whether (i) the roller is lifted vertically against its weight through the height h (work $= W \times h$), or (ii) when it is rolled against the component P of the weight down the incline through a length l to reach the height h (work $=$ $P \times l$). If the roller is to be raised through the same height in either of the above two ways in the same time, it is obvious that the rate of displacement of the roller along the plane is greater than that along the vertical. This idea is sometimes expressed as *what is gained in power is lost in speed* and this is only another way of stating the statical principle of work and the principle of conservation of energy.

Is it more advantageous to apply the power parallel to the incline or in a horizontal direction?

What is the maximum angle of inclination in the latter case beyond which there is no mechanical advantage?

Practical example.—Scale pan weighed 80 gm. (w). A spring balance reading to 500 gm. in 50 steps was used. W = 840 gm. (Two spring balances were employed.) Weights are expressed in gm. wt.

No.	P_1	P_2	Mean P'	Total power $P'+w=P$	h cm.	l cm.	$\frac{P}{\sin i} = \frac{P \times l}{h}$	$P_1 - P_2 = 2f$
1	382	359	370·5	450·5	21·7	40·8	847·0	23
2	332	309	320·5	400·5	19·2	40·8	851·1	23
3	300	279	289·5	369·5	17·7	40·8	851·8	21
4	255	232	243·5	323·5	15·6	40·8	846·1	23
5	202	181	191·5	271·5	12·9	40·8	858·8	21
						Mean	851 gm. wt.	22·2 gm. wt.

Half the difference $P_1 - P_2$ is the mean frictional force between the roller and the glass plate and was 11 gm. wt. nearly, in the above case.

THE PULLEY.

The pulley consists of a rigid circular wheel with a uniform groove cut in its outer edge. It moves freely round a central axis, which rests in sockets in an outside frame, called the block. In such a pulley (fig. 24a), the friction at the points of contact of the axis and the frame is considerable. In fig. 24b is given a sketch of a ball-bearing pulley and is of an improved design, where the friction at the bearings is considerably reduced. The axle is fixed to the outer framework and passes through the central cylindrical part of the wheel, called the hub. Steel balls of equal and suitable size are symmetrically arranged between the axle and the hub. If the framework is fixed, the pulley is said to be a fixed pulley. The power is applied to lift a weight, by means of a rope or string passing over the groove of the pulley. The power is

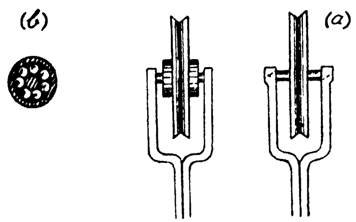


FIG. 24.

equal to the weight in magnitude, but it can be applied in any direction to suit our convenience. A fixed pulley is used to draw water at a well as it is more convenient (why?) to apply the force in a downward direction than upward, as would be the case if there were no pulley. The magnitude of the power applied is smaller than that of the weight lifted, if a movable pulley is used. The mechanical advantage increases as the number of movable pulleys employed increases.

The mechanical advantage with a single movable pulley (i) when the strings are parallel, and (ii) when the strings are inclined at an angle θ to the vertical may be determined in the following manner.

Apparatus required are two ball-bearing pulleys in block, a set of hecto-gram masses, a scale pan, box of weights, string, etc.

Weigh the movable pulley to the nearest gram in a balance. Arrange the apparatus as shown in figure 25 (*a*), with the

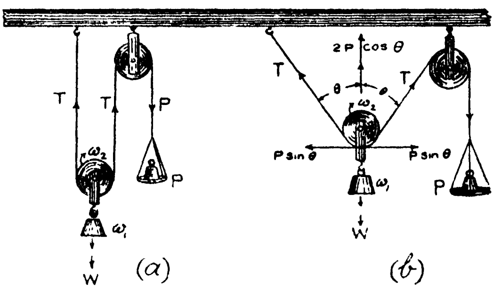


Fig. 25.

strings parallel and in the same vertical plane. Load the pan till the weight W just moves up. Remove some of the load and adjust again till the weight just goes down. Take the mean load and add the weight of the pan and the sum gives the power P. The weight W is the sum of the hanging weight (w_1) and the weight of the pulley (w_2). When a force is applied by means of a string which passes over a smooth peg or a pulley, the force is transmitted

from point to point of the string unaltered and the tension of the string is the same throughout and is equal to the power acting. The parallel forces T and T act upwards, W acts downwards and they are in equilibrium. $T+T-W=0$ and $W=2T=2P$ and $\frac{W}{P}=2$. Change the hanging mass w_1 and repeat the observations a number of times. You find that the ratio $\frac{W}{P}$ is always nearly 2.

The principle of work may be verified in the following manner. The power and weight balance each other in all positions. Measure the heights of the bottoms of the pan and w_1 from the table. Displace P through a distance and measure the heights again. The difference in the readings gives the heights moved. Let the weight be moved up d_1 cm. and let the power consequently go down d_2 cm. $W \times d_1$ units of potential energy is gained by the weight and the power P has lost $P \times d_2$ units of its potential energy. You will find that $P \times d_2 = W \times d_1$. This happens if d_2 is equal to $2d_1$. Compare the speeds with which W and P move and verify the statement *what is gained in power is lost in speed.*

The case of inclined strings is shown in fig. 25 (*b*). The strings are inclined but lie in the same vertical plane. As the tension is the same, the strings must be equally inclined to the vertical (?). Resolve the tensions along the vertical and the horizontal. The horizontal components are each $T \sin \theta$ and are opposite in direction. The resultant force is therefore $2T \cos \theta$ upwards along the vertical and this balances W. The relation $W = 2P \cos \theta$ should therefore hold good. Find the power P to balance the weight W (w_1+w_2).* Trace

* If weights are put in the pan to make it just move up and down and the mean taken, the angle θ is different in the two cases and introduces a considerable error. Since the error due to friction is much less with the ball-bearing pulleys, a single observation for each position will suffice.

the directions of the strings on a paper pinned to a vertical board placed just behind the arrangement. Produce the traces and measure the angle between the strings with a protractor. Half this angle is θ. Change the inclination and observe again. Work out $\frac{W}{P}$ and $2 \cos \theta$ in each case. You find that they are nearly equal. Note how the mechanical advantage varies with θ and how, when θ is zero, the advantage is a maximum and is equal to 2.

Practical example.—Wt. of movable pulley = 76 gm., Wt. of pan = 8 gm. Forces are expressed as gm. wt. and lengths in cm.

(*a*) *Strings, parallel.*

Wt. in pan just to move up W. gm.	Wt. in pan just to prevent W moving down. gm.	P_1 gm.	P_2 gm.	Mean P gm.	$W = w_1 + w_2$ gm.	$\frac{W}{P}$	d_1 cm.	d_2 cm.	$\frac{d_2}{d_1}$
130	126	138	134	136	276	2·03			
							17·2	34·5	2·006
230	225	238	233	235·5	476	2·02			
							21·0	42·5	2·024
284	278	292	286	289	576	1·99			

(*b*) *Strings, inclined.*

Weight in pan gm.	P gm.	2θ	W gm.	$2 \cos \theta$	W/P
350	358	73°	576	1·61	1·62
390	398	85°	576	1·47	1·45

Chapter IV

HYDROSTATICS

DENSITY AND SPECIFIC GRAVITY.

Density is the mass per unit volume. The density of water at ordinary temperatures is very nearly one gram per cubic centimetre. In the English system of units, it is very nearly 62·5 lb. per cubic foot. Water is easily procured anywhere and the densities of all other substances are compared with the density of water. The density of a body compared with that of water is called its relative density or specific gravity.

$$\text{Sp. Gr. of a body} = \frac{\text{density of the body}}{\text{density of water}}$$

$$= \frac{\text{mass of unit volume of body}}{\text{mass of unit volume of water}}$$

$$= \frac{\text{mass of any volume of the body}}{\text{mass of the same volume of water}}.$$

Glass bottles of known volume, fitted with ground and perforated stoppers, are used in the determination of the density of liquids and the specific gravity of heavy and insoluble solids available in small pieces or in powder.

The density of water at the laboratory temperature may be determined in the following manner. Take a density bottle, dry it and find its mass when empty, with a balance, to the nearest centigram. Let it be m_1 gm. Fill the bottle carefully with clean distilled water. Insert the stopper gently. Clean and dry the outside. Weigh again and let the mass be m_2 gm. Density of water is equal to $\dfrac{m_2 - m_1}{\text{volume of bottle}}$ gm. per c.c.

At the ordinary laboratory temperature (25°C.) the density is very nearly 0·996 gm. per c.c. Repeat with clean tap water or well water and note that the density is not appreciably different (perhaps 0·997 gm. per c.c.). In all ordinary work, the density of water is taken as 1 gm. per c.c. and this gives an error of only about 3 in 1,000, which is quite small.

The relative density of any other liquid, say kerosene, may be found in the following manner. Clean and dry the specific gravity bottle and fill it with kerosene. Clean the outside of the bottle again. Find the mass of the bottle filled with kerosene to the nearest centigram. Let it be m_3 gm. If m_1 gm. is the weight of the empty bottle, (m_3-m_1) gm. is the weight of the given volume of kerosene. Density of kerosene is equal to $\dfrac{m_3-m_1}{\text{vol. of bottle}}$ gm. per c.c. (m_2-m_1) and (m_3-m_1) are the masses of the same volume of water and kerosene respectively. Sp. gr. of kerosene is, therefore, equal to $\dfrac{m_3-m_1}{m_2-m_1}$, a number or ratio.

You find that the density of kerosene expressed as above in grams per cubic centimetre is numerically the same as its specific gravity. This is because the density of water in the C.G.S. system of units is 1 gm. per c.c. If x gm./c.c. be the density of a body, its relative density will be equal to $\dfrac{x \text{ gm. per c.c.}}{1 \text{ gm. per c.c.}}$ which is x. In the F.P.S. system, the density of water is not numerically unity but as stated already is very nearly 62·5 lb. per cubic foot. Hence the density of any body in the F.P.S. system is numerically different from its density in the C.G.S. system. But the specific gravity of any body is the same whether densities are expressed in the C.G.S. system or in the F.P.S. system. For example, 0·81 gm. per c.c. is the density of kerosene in the C.G.S. system; $\dfrac{0{\cdot}81 \text{ gm./c.c}}{1 \text{ gm./c.c.}}$ which is 0·81 is its relative density. Let the

5

density of kerosene be now expressed in the F.P.S. system, remembering that 1 lb. = 453·6 gm. and that one cubic foot = 28320 c.c.

$$\text{Density of kerosene} = \frac{0{\cdot}81}{453{\cdot}6} \times 28320 \text{ lb. per c.ft.}$$

$\therefore$ relative density of kerosene

$$= \frac{0{\cdot}81 \times 28320}{453{\cdot}6 \times 62{\cdot}5} = 0{\cdot}81 \text{ (approx.)}$$

As the relative density of a body is only the number of times the body is denser than water, it must be the same in whatever units the densities may be expressed.

The specific gravity of a heavy body in the form of small pieces may be determined in the following manner. The weight of the body in air is found as usual with a balance and the weight of water equal in volume to that of the solid is found out with the help of the specific gravity bottle by displacement.

w gm. = wt. of solid in air.

w_1 gm. = wt. of solid in air + wt. of sp. gr. bottle filled with water only.

w_2 gm. = wt. of bottle filled partly with the solid and the rest with water.

Then $w_1 - w_2$ gives the weight of water displaced by the solid. The solid displaces its own volume of water, if care is taken that no air bubbles adhere to the solid under water. Sp. gr. of the solid is $\dfrac{w}{w_1 - w_2}$.

PRESSURE WITHIN A FLUID.

Let A (fig. 26) be a metal cube placed on the surface B of a table. The weight of the cube acts on the surface vertically downwards and since there is equilibrium, the table reacts at each point with an equal and opposite force. A downward thrust due to the weight of the cube acts on

the surface and is balanced by an equal upward thrust. The cube is homogeneous and of uniform height. Hence the thrust is uniformly distributed over the surface. *Thrust per unit area is called the pressure at a point on the surface* and so the pressure is the same at every point of the surface. If A is a metal cone, the weight on different unit areas on the surface B is different and therefore the thrust is not uniformly distributed. *Pressure at a point, when uniform, is measured by the total thrust divided by the area on which the thrust is acting; when not uniform, it is measured by the ratio of the thrust over a very small area surrounding the point to the area, the area being so small that the thrust can be taken to be uniformly distributed over the area.* Pressure is expressed in grams weight per square centimetre in the C.G.S. system and in pounds weight per square inch in the F.P.S. system. Pressure is caused in one of two ways. It may be due to the weight of a body as in the above case or it may be due to the elasticity of the body exerting pressure as in the case of a compressed spring. In either case, pressure is transmitted from point to point by the elasticity of the body. Air has weight. The atmosphere surrounding us exerts pressure. The weight of the whole column of the atmosphere over each unit area in the open is the atmospheric pressure. This pressure is communicated to the inside of a room by the elasticity of the atmosphere.

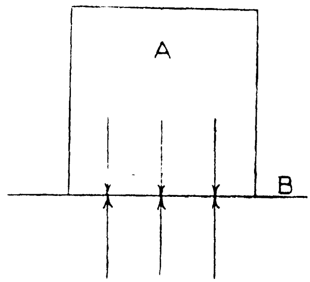


FIG. 26.

It is common experience that water jets out forcibly through a hole at the side of a vessel containing water and that it jets out more and more forcibly if the hole is bored lower and lower. It is obvious that the liquid exerts a lateral thrust

on the sides of the containing vessel and that this thrust increases with the depth. Consider any point inside a liquid at rest. As explained already, a set of two equal and opposite vertical thrusts acts on the point. As the point is subject to lateral thrusts also and as it has no lateral motion, a set of two equal and opposite lateral forces must similarly act on it. The point is at rest under the action of these two sets of forces.

Consider a small area a sq. cm. (fig. 27) at B, at a depth h_1 cm. inside a liquid at rest, of density d gm. per c.c. The downward thrust on a sq. cm. at B is due to the weight of a column h_1 cm. of the liquid on the area a sq. cm. The volume of this column is h_1a c.c. and its weight is h_1ad gm. Therefore, the thrust per unit area or pressure at a point at B is $\frac{h_1ad}{a} = h_1d$ gm. wt. per sq. cm. Similarly, the pressure at a point on the surface A at the depth h_2 cm. is h_2d gm. wt. per sq. cm. It is clear, therefore, that *the pressure at a point inside a liquid or fluid at rest is directly proportional to the depth of the fluid and to the density.* As the elasticity of a fluid is independent of direction, the pressure at any point is communicated equally in all directions and the lateral pressure at any point on A is, therefore, also h_2d gm. wt. per sq. cm. If A_1 is another point in the fluid at the same depth h_2 cm., in the same horizontal plane as the surface A, it is obvious that the pressure at that point A_1 is equal to the pressure at A.

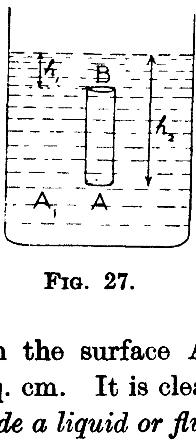


FIG. 27.

We shall now compute the thrust on the vertical side of a vessel containing a liquid. Let the height of the contained liquid be h cm. = AB and let C be at a height $\frac{h}{2}$ cm. from the bottom B (fig. 28).

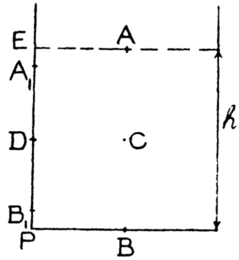


FIG. 28.

Pressure at A = pressure at E = 0.

pr. at C = pr. at D = $\frac{h}{2}d$. gm. wt. per sq. cm.

pr. at B = pr. at P = hd. gm. wt. per sq. cm.

pr. at C = $\frac{\text{pr. at A}+\text{pr. at B}}{2}$, or pr. at D = $\frac{\text{pr. at E}+\text{pr. at P}}{2}$.

Consider another set of points A_1, B_1 one centimetre below E and above P respectively.

pr. at A_1 = $1 \times d$ gm. wt. per sq. cm.

" " B_1 = $(h-1)\ d$. " " "

$\therefore$ mean of pressures at A_1 and B_1

$$= \frac{(1+h-1)d}{2} \text{gm. wt. per sq. cm.}$$

$$= \frac{h}{2} \times d \quad \text{"} \quad \text{"}$$

$$= \text{pr. at D.}$$

Similarly, by considering pairs of points, one of which is x cm. below E and the other x cm. above P, the mean pressure can always be shown to be that at D, the middle point on the vertical side. Therefore, it is clear that the pressure on the vertical side EDP gradually increases from E to P and that the mean pressure is that at D. The thrust on the vertical

side is therefore the mean pressure multiplied by the area of the vertical side EDP.

Now consider the cylindrical column of the liquid AB (fig. 29) on a unit area. At any point of its length, the transverse section of the column is a circle. Lateral thrusts of equal value act at every point of the circle and are together in equilibrium. So these lateral thrusts need not be considered. The other forces that must be considered are the downward thrust of the liquid on B, the weight of the column AB and the upward thrust of the liquid on A, the bottom surface of the column of the liquid. Under the action of these three forces, the column is at rest. Therefore, the two downward forces must be together equal and opposite to the upward thrust. *So the difference in the thrusts or the resultant upward thrust is equal to the weight of the column of the liquid AB.*

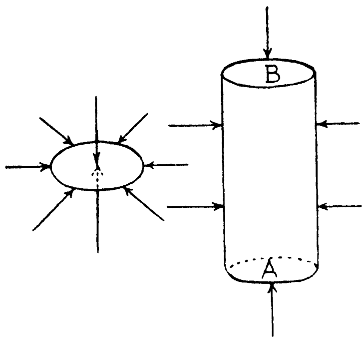


FIG. 29.

Let M be the mass of any portion of the liquid at rest. The lateral thrusts acting on M must cancel one another and have no resultant, as the mass has no lateral motion. Again, all the downward and upward forces acting on M must be similarly in equilibrium; one of these forces is the weight of M which acts vertically downwards. Since M is at rest, this weight must be balanced by a resultant upward thrust and this is due to the liquid surrounding M.

If, instead of considering a portion of the liquid, we consider any body M immersed under the liquid, the lateral thrusts and the upward and downward thrusts due to the surrounding liquid remain as before. The lateral thrusts are still in

equilibrium. *The resultant upward thrust** due to the surrounding liquid and *equal to the weight* of the liquid of the size M, i.e. of a volume *of the liquid displaced* by the body M, acts vertically on the body and the weight of the body acts vertically downwards. The resultant action of these two forces determines the motion of the body.

If the weight of the body M is greater than the resultant upward thrust which the body under the liquid experiences, the body sinks with a force equal to the difference. If these two forces are equal to each other in magnitude, the body is at rest at any depth of the liquid and behaves just like a portion of the liquid. If the resultant upward thrust is greater than the weight, the body moves up like a cork under water under the action of a force equal to their difference and comes to rest and floats, part of the body being immersed under the liquid. The downward thrust due to the liquid is now absent and the upward thrust, which is equal to the weight of the liquid displaced, is equal and opposite to the weight of the body.

PRINCIPLE OF ARCHIMEDES.

When a body is immersed in a fluid, it experiences an upward thrust which is equal to the weight of the fluid displaced. This statement was first made by Archimedes, the philosopher of Syracuse, more than two thousand years ago.

This principle may be verified with the help of a metal cylinder, callipers, balance, hydrostatic stool, box of weights, a beaker of water, a beaker of kerosene and fine thread.

1. Find the weight of the cylinder w_1, with the balance by the method of oscillations, correct to a centigram. Suspend the cylinder by a piece of thread to the lower hook of the left

---

* It is clear from this that when a body is immersed in a liquid, it experiences a resultant upward thrust which is equal to the weight of the liquid displaced by the body, (Archimedes' Principle).

and stirrup, so that the cylinder is well immersed under water in a glass beaker supported on the hydrostatic stool which lies astride the pan below. Put an equal length of thread in the right hand pan. Find the weight w_2 of the cylinder under water by the method of oscillations. The cylinder should freely move under water and should have no air bubbles sticking to it.

$w_1 - w_2$ = loss of weight under water.
= the resultant upward thrust or buoyancy.
= wt. of displaced water.
= wt. of water of volume equal to the volume of the cylinder, according to the principle of Archimedes.

The volume of the cylinder can be directly determined and the weight of an equal volume of water ascertained. For this purpose, find the mean length, l cm. of the cylinder and the mean diameter, $2r$ cm., with the callipers. The volume of the cylinder is $\pi r^2 l$ c.c. and the weight of this volume of water is $\pi r^2 l$ gm. It will be found that $(w_1 - w_2)$ gm. and $\pi r^2 l$ gm. are very nearly the same and these quantities have been determined independently. The statement, that the loss of weight under water is equal to the weight of the water displaced, is thus verified.

2. Find the weight w_3 gm. of the cylinder in another liquid, kerosene. The loss of weight under kerosene is $(w_1 - w_3)$ gm. The weight of the volume of kerosene, equal to that of the cylinder, is $\pi r^2 ld$ gm., where d gm. per c.c. is the density of kerosene. It will be found that $(w_1 - w_3)$ gm. and $\pi r^2 ld$ gm. are very nearly equal. Hence, the principle is once again verified.

The following applications of the principle are of interest.

1. The principle of Archimedes affords us a very good method of determining volumes of solids, regular or irregular

in shape. If x gm. is the loss of weight of a solid under water, the volume of the solid is x c.c. (?)

2. The specific gravity of a liquid may be determined with the help of this principle. Weigh a given solid in air, in water and in the given liquid Let m_1 gm. and m_2 gm. be the losses in weight of the body under water and under the liquid respectively. m_1 gm. is the weight of water equal in volume to that of the solid and m_2 gm. is the weight of the same volume of the liquid. Thus the weights of equal volumes of water and the liquid are obtained.

$$\text{Specific gravity of the liquid} = \frac{\text{loss of weight in liquid}}{\text{loss of weight in water}} = \frac{m_2 \text{ gm.}}{m_1 \text{ gm.}}$$

3. The specific gravity S of a solid heavier than water may be found as follows.

$$S = \frac{\text{wt. of body in air}}{\text{wt. of an equal volume of water}} = \frac{\text{wt. in air}}{\text{loss of weight in water}}.$$

4. The specific gravity of a heavy solid soluble in water may be found in the following manner.

Weigh the solid in air. Let the weight be w_1 gm. Weigh the solid in a liquid in which it does not dissolve and find the loss in weight, w gm. The specific gravity s of the liquid is previously determined by a specific gravity bottle or is given.

w gm. = loss of weight in the liquid = weight of liquid of volume equal to that of the solid immersed.

$$s = \frac{\text{weight of a given volume of liquid}}{\text{weight of the same volume of water}}$$

$$= \frac{\text{loss of weight of the solid in the liquid}}{\text{loss of weight of the solid in water}},$$ if the given volume is that of the solid.

$\therefore$ loss of weight of the solid in water $= \dfrac{w \text{ gm.}}{s}$.

$$\text{Sp. gr. of solid, S} = \frac{\text{wt. in air}}{\text{loss of weight in water}} = \frac{w_1}{w/s} = \frac{w_1}{w}s.$$

5. The specific gravity of a solid lighter than water may be found as follows. A heavy solid that can sink the light solid under water is required.

Let w = wt. of solid in air.

w_1 = wt. of solid in air + wt. of sinker under water.

w_2 = wt. of both together under water.

w_2 is smaller than w_1, because when the light solid is under water, it experiences a loss in weight.

$w_1 - w_2$ = loss of weight of the light solid or the weight of the volume of water displaced by the solid.

$$\frac{w}{w_1 - w_2} = \text{sp. gr. of the solid.}$$

6. The percentage of copper in a gold ring may be found in the following manner. Gold rings are made generally out of sovereigns and contain a certain amount of copper. Copper may be added while making the ring. Archimedes' Principle can be applied to find out the percentage of this copper. This was the problem about King Hiero's crown which Archimedes solved.

Weigh the ring in air and in water and find the specific gravity s of the ring. s gm. per c.c. is the density of the ring. s will be smaller than 19·3, the specific gravity of pure gold (?)

Let w = wt. of ring in air.

x = wt. of copper in the ring.

We shall assume the specific gravity of copper to be 8·9. The weight of gold in the ring is $(w-x)$ gm. and its volume is $\dfrac{w-x}{19 \cdot 3}$ c.c. The weight of copper in the ring is x gm. and its volume is $\dfrac{x}{8 \cdot 9}$ c.c. Total volume or volume of the ring is

$\left[\frac{w-x}{19\cdot3}+\frac{x}{8\cdot9}\right]$ c.c. But, volume of ring $\times$ density $=$ weight of ring. $\left[\frac{w-x}{19\cdot3}+\frac{x}{8\cdot9}\right]s = w.$

From this, x can be calculated and $\frac{x}{w}\times 100$ is the percentage of copper in the ring.

In the above determinations, the following points are to be noted.

(*a*) Whenever a solid is suspended by a thread under a liquid, an equal length of the thread is to be put in the other scale pan.

(*b*) The body must be freely moving under the liquid and must have no air bubbles sticking to it.

(*c*) All weights have to be determined to the nearest centigram.

THE TEST TUBE FLOAT.

A flat bottomed test tube of uniform section is taken. A narrow strip of squared paper divided into tenths of an inch is pasted inside the tube parallel to the length with its zero touching the bottom of the tube. The strip is so pasted that the divisions are seen from outside. The test tube is loaded at the bottom with lead-shot and, if necessary, the shot may be fixed in position by pouring in melted paraffin wax. Such an apparatus is called a test tube float.

The test tube floats to different heights in different liquids. According to the principle of Archimedes, the weight of the test tube is equal to the weight of the volume of the liquid displaced, as explained already. Since the sectional area of the tube is uniform, the volumes of the displaced liquids are proportional to the depths of immersion. Let h_1 cm. be the depth of immersion in a liquid of density d_1 gm./c.c. Let h_2 cm. be the depth of immersion in a liquid of density

d_2 gm./c.c. and let w gm. be the weight and a sq. cm. the sectional area, of the tube.

Then $w = h_1 a d_1 = h_2 a d_2 =$ weight of liquid displaced.

$\therefore\ h_1 d_1 = h_2 d_2$

$\therefore\ \dfrac{h_1}{h_2} = \dfrac{d_2}{d_1}.$

The depths of immersion are thus inversely proportional to the densities of the liquids and the ratio of the densities of any two liquids can thus be readily determined. The heights h_1 and h_2 are read to the tenth of a scale division by estimation.

If a narrower test tube of half the section, $\frac{a}{2}$ sq. cm., is selected and if the weight of the tube w is the same, the height of immersion in any liquid is doubled, because,

$$w = h_1 a d = 2h_1 \cdot \frac{a}{2} \cdot d.$$

The difference in the heights of immersion for any two liquids is therefore also doubled. Thus the sensitiveness of the instrument increases with the narrowness of the tube. As test tubes are generally not sufficiently narrow, the results obtained with such floats are not very accurate.

To compare the densities of two given liquids, put the float in the heavier liquid (why?) and add lead shot till it just floats vertically. Read the height of immersion h_1. Put the float in the other liquid and find the height of immersion h_2. Calculate the ratio $\frac{h_1}{h_2}$. Float the tube again in the heavier liquid and add a few more shot. Read the height. Put the float in the other liquid and read the height again. Calculate the ratio of heights. Repeat the observation with a little more of the shot in the float and find the ratio. The mean of the three ratios gives the mean density of one liquid relative to the other. If one of the liquids is water, the specific gravity of the other liquid is easily obtained. Whenever the float is transferred from one liquid to the other,

care has to be taken to clean and dry the outside of the tube.

BALANCING COLUMNS.

The U tube and the Hare's apparatus are examples in which the principle of balancing columns is employed.

The apparatus required for comparing the densities of two liquids are a U tube on stand, a beaker of water, a beaker of kerosene, and two funnels.

The U tube is mounted vertically on a wooden stand (fig. 30). A millimetre scale graduated along either edge is screwed on to the stand in the middle, parallel to the length of the limbs of the tube.

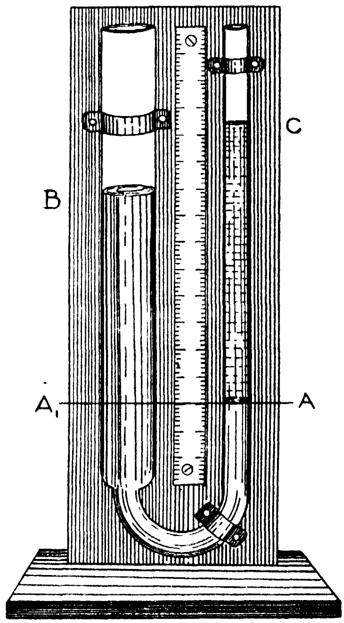


FIG. 30.

Pour water through one of the limbs. It stands at the same height in both the limbs. Then, pour kerosene into the other limb. A surface of separation A forms where the two liquids meet and moves down as kerosene is added. In the figure, BA_1A is water and CA is kerosene. But in a liquid at rest, the pressures at two points are equal, if they are on the same horizontal. So the pressures at A and A_1 are equal. Let a_1 and a_2 be the sectional areas of the limbs and d_1 and d_2 be the densities of the liquids in the limbs AC and A_1B respectively.

$$\text{Pressure at A} = \frac{\text{wt. of liquid column AC}}{\text{sectional area at A}}$$

$$= \frac{AC \times a_1 \times d_1}{a_1} = AC \times d_1 \text{ gm. wt. per sq. cm.}$$

$$\text{Pressure at } A_1 = \frac{A_1B \times a_2 \times d_2}{a_2} = A_1B \times d_2 \text{ gm. wt. per sq. cm.}$$

$$\therefore\ AC \times d_1 = A_1B \times d_2 \text{ and if } AC = h_1 \text{ and } A_1B = h_2$$

$$h_1d_1 = h_2d_2 \text{ and } \frac{h_2}{h_1} = \frac{d_1}{d_2}.$$

The pressure due to the column h_1 cm. of kerosene balances the pressure due to h_2 cm. of water and their *densities are inversely proportional to the heights of the balancing columns.* These balancing heights are, as shown above, independent of the sectional area of the two limbs, because pressure at any point is thrust on each unit area.

Slowly add more and more of kerosene into the limb AC and take six sets of balancing heights. The mean of the ratios of the observed heights gives the specific gravity of kerosene. Let not the common level A be too low (?). If A gets very near the bottom, add water into B. The above method is employed for liquids that do not mix.

If two liquids that mix with each other are given, a heavy liquid that does not mix with either of them, say mercury, is required. Mercury is first poured into the tube to a small height. The two liquids are now poured one in each limb until the surfaces of mercury lie in the same horizontal as judged on the scale. Arguing as before, the pressures of the two columns of liquids balance each other and their heights are inversely proportional to their densities. It may be noted that this method gives accurate results only when the adjustment of the equality in level of the mercury surfaces is perfect. Any small error in this produces a large error in the result (?).

In the case of free surfaces of liquids that wet the tube, the lowest point of the meniscus is to be read on the scale. If the surface of the liquid is convex, the top point of the meniscus is read.

Practical example.—

Reading of			Balancing ht.		Specific gravity of kerosene $= \frac{h_2}{h_1}$
Top surface of kerosene	Common surface.	Top surface of water.	Kerosene h_1.	Water h_2.	
13·9 cm.	9·95 cm.	13·0 cm.	3·95 cm.	3·05 cm.	0·772
15·1 „	7·0 „	13·7 „	8·1 „	6·7 „	0·827
16·8 „	3·3 „	14·6 „	13·5 „	11·3 „	0·837
17·7 „	1·3 „	15·1 „	16·4 „	13·8 „	0·842
23·0 „	4·6 „	20·2 „	18·4 „	15·6 „	0·848
24·2 „	2·3 „	20·7 „	21·9 „	18·4 „	0·840

Mean sp. gr. of kerosene = 0·83.

With miscible liquids, mercury can be dispensed with if the U tube is inverted as in *Hare's apparatus.* ABDC (fig. 31) is the inverted U tube with an opening at the top. A rubber tube is slipped on to the projection at the opening and P is a pinch cock. The lower ends of the tube dip into two beakers A and C containing the two liquids of densities d_1 and d_2 respectively. Levels of the liquid columns are read on a vertical millimetre scale alongside the tube.

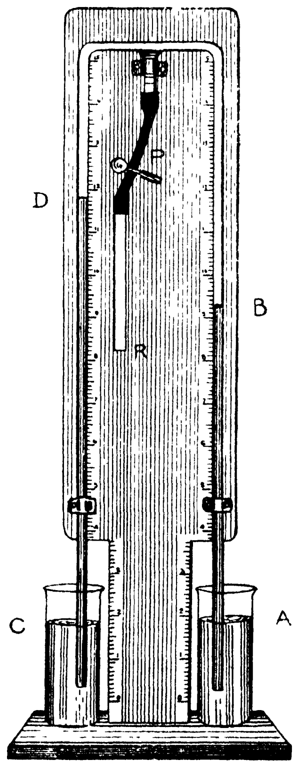


FIG. 31.

Release the pinch cock and suck gently. Close the cock. The pressure of the confined air falls. To balance the difference of pressure between the atmosphere and the confined air, the liquids rise in each tube and equilibrium is restored. Consider the point C at the bottom of the liquid column DC. The pressure of the confined air and the pressure

due to the column DC (h_2 cm.) act together at C vertically downwards and the atmospheric pressure acts vertically upwards and C is at rest. If p and π respectively represent the pressures of the confined air and atmosphere, we have $p+h_2d_2 = \pi$. Similarly considering the point A we have $p+h_1d_1 = \pi$ where AB $= h_1$ cm.

$$\therefore \pi - p = h_1d_1 = h_2d_2 \text{ and } \frac{h_1}{h_2} = \frac{d_2}{d_1}.$$

The densities are inversely as the balancing heights of the liquids. The balancing heights are independent of the section of the limbs of the apparatus.

Suck up slowly and gradually. At each stage, take a set of readings and calculate the ratio $\frac{d_2}{d_1}$. A mean of seven or eight values gives the density of one liquid relative to the other.

Practical example.—Copper sulphate solution and water are taken.—

Reading of surface of liquid column.		Reading of surface of water column.		Height of Liquid column.	Height of Water column.	Sp. gr. of solution.
Lower.	Upper.	Lower.	Upper.			
4·2 cm.	51·7 cm.	4·0 cm.	56·5 cm.	47·5 cm.	52·5 cm.	1·105
4·2 „	48·8 „	4·0 „	53·0 „	44·6 „	49·0 „	1·098
4·2 „	45·9 „	4·0 „	49·8 „	41·7 „	45·8 „	1·098
4·2 „	42·25 „	4·0 „	45·85 „	38·05 „	41·85 „	1·100
4·3 „	38·15 „	4·1 „	41·5 „	33·85 „	37·4 „	1·105
4·3 „	33·5 „	4·1 „	36·2 „	29·2 „	32·1 „	1·099
4·3 „	25·1 „	4·1 „	27·0 „	20·8 „	22·9 „	1·101

Mean sp. gr. of solution = 1·101

HYDROMETERS.

Hydrometers are of two kinds, namely: (i) of constant weight and variable immersion, and (ii) of constant immersion and variable weight. The instruments of the first class are sub-divided into (*a*) those in which the graduations on the stem are of equal length, and (*b*) those in which the gradua-

tions on the stem are of equal differences of specific gravities. Baume's hydrometer, common hydrometer and Nicholson's hydrometer are examples of the three types respectively.

We shall now describe how a *Baume's hydrometer* is graduated. Consider a flat bottomed test tube float, weighted at the bottom. Let the length be divided into, say, 65 equal parts. Let the tube be so loaded that it stands at mark 45 when floated in water. Let s_{45}, s_{44}, etc. be the densities of liquids in which the float sinks to marks 45, 44, etc. respectively. But the densities are inversely as the heights of immersion.

$$\therefore \frac{s_{44}}{s_{45}} = \frac{45}{44} \text{ or } s_{44} = \frac{45}{44} \text{ because } s_{45} = 1.$$

$$\text{Again } s_{43} = \frac{45}{43}, s_{42} = \frac{45}{42} \text{ etc.}$$

The specific gravity corresponding to each mark on the tube can thus be calculated.

$$\text{Again } \frac{1}{s_{45}} - \frac{1}{s_{44}} = 1 - \frac{44}{45} = \frac{1}{45}$$

$$\frac{1}{s_{44}} - \frac{1}{s_{43}} = \frac{44}{45} - \frac{43}{45} = \frac{1}{45} \text{ and so on.}$$

Thus the reciprocals of the specific gravity numbers are in arithmetical progression, the common difference being $\frac{1}{45}$. Therefore, the calculated specific gravity numbers s_{45}, s_{44}, etc. are in harmonic progression.

$$\text{Again, } s_{46} = \frac{45}{46}, s_{47} = \frac{45}{47}, s_{48} = \frac{45}{48}, \text{ etc.}$$

and

$$s_{45} - s_{46} = 1 - \frac{45}{46} = \frac{1}{46}$$

$$s_{46} - s_{47} = \frac{45}{46} - \frac{45}{47} = \left(1 - \frac{1}{46}\right) - \left(1 - \frac{2}{47}\right)$$

$$= \left(\frac{2}{47} - \frac{1}{46}\right) \text{ and this is less than } \frac{1}{46}.$$

Thus, the difference in the specific gravity numbers corresponding to successive divisions on the tube increases as we go down the tube.

Spirits, oils, solutions and other common liquids have their specific gravities ranging between 0·700 and 2·000. These values correspond to the divisions 64·3 and 22·5 of the test tube float (?). These divisions mark the limits of the working length of the tube. So, up to the division 22, the tube may be of any convenient shape, if its size and weight remain the same. Floats having a weighted bottom, a hollow body and a uniform stem are called hydrometers. In a Baume's hydrometer the stem is divided into equal lengths and the instrument can be used for the determination of specific gravities of all common liquids.

The difference between the specific gravity numbers corresponding to two successive marks on the stem is a measure of the sensitiveness of the hydrometer. The smaller the difference, the greater is the sensitiveness of the hydrometer. Again, *the sensitiveness of a hydrometer* is not constant throughout its length but, as shown above, *decreases as we go down the stem.* If the sectional area of the stem is reduced to half its former value, the volume and weight of the float remaining the same, the length of the float will be doubled. Twice the number of graduations of the same length as before can be marked on the stem and the sensitiveness of the instrument is doubled at every point of its length. Therefore, by keeping the sectional area of the stem at a convenient value, we can have a suitable graduation and thus obtain the requisite accuracy. But the stem may then be too long. This difficulty is avoided if two hydrometers are employed and so weighted that one serves to find the specific gravity of liquids lighter than water and the other of liquids heavier than water. If greater accuracy is desired, three or four hydrometers are employed to cover the same range. Different

quantities of mercury are added to the bottom bulb and the instruments are so adjusted that one sinks to the top reading on the stem in the same liquid in which the next floats to its bottom reading.

The *common hydrometer* will now be described. The stem is graduated into marks of equal specific gravity differences. These graduations are marked on paper which is inserted in position into the hollow stem. The length corresponding to the same difference in specific gravity decreases as we go down the stem, the graduations becoming closer and closer. The specific gravity of the liquid in which this instrument floats to a particular point on the stem is marked against it and the specific gravity is thus directly read off. Consider once again the flat-bottomed test-tube 65 divisions long. Let x_0, the 65th division, correspond to a specific gravity of 0·7000. This is adjusted by loading the tube properly. The problem before us now is that of determining the heights x_1, x_2, x_3, etc. from the bottom of the tube where the specific gravity numbers 0·800, 0·900, 1·000, etc. are to be marked.

According to the law of flotation, if W is the weight of the hydrometer and a is the sectional area of the stem, we have

$$
\begin{aligned}
W &= 65 \times a \times 0{\cdot}7 \\
&= x_1 \times a \times 0{\cdot}8 \\
&= x_2 \times a \times 0{\cdot}9 \\
&= x_3 \times a \times 1{\cdot}0, \text{ etc.}
\end{aligned}
$$

$$
\begin{aligned}
\therefore\ x_0 &= 65 \times \frac{7}{7} \text{ and } \frac{1}{x_0} = \frac{7}{7} \times \frac{1}{65} \\
x_1 &= 65 \times \frac{7}{8} \text{ and } \frac{1}{x_1} = \frac{8}{7} \times \frac{1}{65} \\
x_2 &= 65 \times \frac{7}{9} \text{ and } \frac{1}{x_2} = \frac{9}{7} \times \frac{1}{65} \\
x_3 &= 65 \times \frac{7}{10} \text{ and } \frac{1}{x_3} = \frac{10}{7} \times \frac{1}{65}
\end{aligned}
$$

$$
\therefore\ \frac{1}{x_1} - \frac{1}{x_0} = \frac{1}{7} \times \frac{1}{65} = \frac{1}{x_2} - \frac{1}{x_1} = \frac{1}{x_3} - \frac{1}{x_2}, \text{ etc.}
$$

The reciprocals of the distances x_1, x_2, are in arithmetical progression (A.P.) and hence the distances are in harmonic progression (H.P.).

Again, $x_0 - x_1 = 65 \times \frac{1}{8} = 65 \times \frac{9}{72}$

and $\quad x_1 - x_2 = 65 \left(\frac{7}{8} - \frac{7}{9}\right) = 65 \times \frac{7}{72}$

$\therefore$ $x_0 - x_1$ is greater than $x_1 - x_2$ and so on.

Thus it is clear that the length of the stem corresponding to the same specific gravity difference decreases gradually as we go down the stem. This is another way of stating that *the sensitiveness of the instrument decreases as we go down the stem.* A single hydrometer, as has been already explained, is incapable of giving accurate measurements and so the whole range of specific gravities 0·7 to 2·0 is distributed over a series of four or five hydrometers, the scales being continuous.

Practical example.—

Baume's hydrometer for liquids lighter than water is taken. It floats to the division 10 in water and has forty divisions of equal length 10–50 marked on it. If the bulb, etc. at the bottom were elongated into a stem of the same sectional area, the total weight and volume remaining the same, then the parent tube of which the present instrument is a later development is obtained. Let x divisions of the parent tube be the equivalent length of the instrument below the mark 10. Floated in kerosene of specific gravity 0·788, the instrument stood at division 46.

Then, if W is the weight of the hydrometer and a the sectional area of its stem, we have

$$\begin{aligned} W &= x \times a \times 1{\cdot}00 \text{ in water} \\ &= (x + 36^*) \times a \times 0{\cdot}788 \text{ in kerosene} \\ \therefore\ x &= (x+36) \times 0{\cdot}788 = 0{\cdot}788x + 36 \times 0{\cdot}788 \\ \text{or } x &= 133{\cdot}9 \text{ divisions.} \end{aligned}$$

---

* Why 36?

The equivalent length of the hydrometer below the mark 10 being known, the specific gravity number corresponding to any division on the scale can be easily calculated and tabulated. Such a table is supplied with Baume's instruments which are so made that the specific gravity number corresponding to any particular division, say 20, is always the same.

	Calculated.	From tables.	Difference.
Sp. gr. at 50 $= \frac{x}{x+40} = \frac{133 \cdot 9}{173 \cdot 9}$	0·7696		
40 $= \frac{133 \cdot 9}{163 \cdot 9}$ ..	0·8164	0·817	0·0006
30 $= \frac{133 \cdot 9}{153 \cdot 9}$ ..	0·8696	0·871	0·0014
20 $= \frac{133 \cdot 9}{143 \cdot 9}$ ..	0·9301	0·928	0·0021

Practical example.—

A *common hydrometer* of range 1·000 to 1·200 was taken. The length of the stem between these two density marks was found to be 13·62 cm. Consider the parent tube again. Let x cm., from the bottom of such a tube, be equivalent to the hydrometer up to the mark 1·2 on the stem. Then,

$$W = x \times a \times 1 \cdot 2 = (x + 13 \cdot 62) \times a \times 1 \cdot 0$$

assuming that these two marks, 1·0 and 1·2, are correct.

$$\therefore \; 0 \cdot 2x = 13 \cdot 62 \text{ or } x = 68 \cdot 1 \text{ cm.}$$

Obviously, for the range 1·0 to 1·2, the ratio of the equivalent length x to the length between the marks is a constant (5). The distance l of any density mark on the stem from the bottom of the parent or equivalent tube can be readily calculated. Consider the specific gravity mark 1·10.

Then $\frac{1 \cdot 10}{1 \cdot 20} = \frac{68 \cdot 1}{l}$ because densities are inversely as the heights of immersion of the equivalent tube.

$$\therefore \; l = 74 \cdot 29 \text{ cm. or } 68 \cdot 1 + 6 \cdot 19 \text{ cm.}$$

So the specific gravity number 1·100 has to be marked 6·19 cm. above the number 1·200 and it is 0·62 cm. below the middle point of the stem which is 6·81 cm. above the mark 1·200. In this way,

the distances on the stem from the mark 1·200 to any given specific gravity number can be calculated and these calculated distances can be compared with those obtained by direct measurement on the graduated stem. Results of such an experiment are given in the following table :—

Sp. gr.	Distance from 1·200 mark calculated. (cm.)	Observed distance on the stem. (cm.)	l in cm.
1·180	1·12	1·12	69·22
1·160	2·30	2·30	70·40
1·140	3·54	3·53	71·63
1·120	4·82	4·80	72·90
1·100	6·18	6·19	74·28
1·080	7·54	7·54	75·64
1·060	8·99	8·99	77·09

That the distances l are in H.P. is verified thus:

$$\frac{1}{68{\cdot}10} - \frac{1}{69{\cdot}22} = 0{\cdot}000238$$

$$\frac{1}{69{\cdot}22} - \frac{1}{70{\cdot}40} = 0{\cdot}000242$$

$$\frac{1}{74{\cdot}28} - \frac{1}{75{\cdot}64} = 0{\cdot}000242$$

Reciprocals of l are in A.P., the common difference being 0·00024. Therefore, when the specific gravity numbers are in A.P. the corresponding distances on the equivalent tube from its bottom are in H.P.

Nicholson's hydrometer is of the shape shown in fig. 32. It consists of a heavy conical bottom A with a flat top, a hollow body B, a narrow stem C with a fine mark and a pan D in which weights are placed. The ends of B and A are conical and this makes for easy sinking in liquids.

The hydrometer, a box of weights, a tall jar containing a liquid, usually water, and a card-board disc E with a slit cut through the middle, are the *apparatus required* to find the weight w of a solid. Disc E covers the mouth of the jar and prevents weights from falling accidentally into the liquid below.

The slit at the centre of the disc is wide enough to allow a free play to the stem C. Put weights w_1 gm. in pan D until the hydrometer floats vertically in water up to the mark C on the stem. Let no air bubbles stick to the surface of the instrument. Remove the weights, put the solid body in the pan D and add weights w_2 gm., until the hydrometer sinks once again to the mark. The weight of the hydrometer W together with that in the pan is equal to the weight of the water displaced which is the same in either case, as the volume of the displaced liquid is equal to that of the hydrometer up to the same mark on the stem.

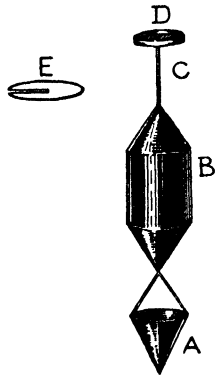


FIG. 32.

$$\therefore \text{W}+w_1 = \text{W}+w+w_2$$

$$\text{Wt. of the body} = w = w_1 - w_2 \text{ gm.}$$

The weight of the body should not be more than w_1 gm. (why?) and w_1 gm. is therefore the maximum load which the hydrometer can weigh. As the stem C is narrow, the weight can be determined to a centigram. Notice that while the hydrometer is floating to a point on the stem, it sinks through an appreciable length h cm. on the stem, if an additional load of a centigram is put in the pan D. The volume of immersion of h cm. of the stem is $\pi r^2 h$ c.c., where r is the radius of section of the stem. If r is 0·1 cm., $\pi r^2 h = \pi h \times 0{\cdot}01$ c.c., and the weight of the liquid of this additional volume of immersion is 0·01 gm. (?), the additional load put into the pan.

$$\therefore \pi h \times 0{\cdot}01 = 0{\cdot}01 \text{ or } \pi h = 1 \text{ cm. and } h = 3{\cdot}2 \text{ mm.}$$

Therefore, an addition of a centigram sinks the hydrometer appreciably (about 3 mm.) on the stem, if the diameter of the stem is 2 mm. as is generally the case. This length of

immersion of the stem per centigram is a measure of the sensitiveness of the hydrometer.

To sink the float to the same mark in a liquid, the same weight is required. This principle of the Nicholson's hydrometer has been known to the world from a very long time and was employed in weighing big cannon, elephants and such other heavy bodies.

The *specific gravity of a solid* may be found in the following manner. Find the weight w gm. of the body in air as before. Put the body on the top of the conical tray A. Adjust weights w_3 in D so that the instrument sinks to the mark. w_3 will be found to be greater than w_2 as the body suffers a loss of weight which is equal to (w_3-w_2) gm. This loss will be smaller than w gm. if the solid is heavier than water bulk for bulk and greater if lighter. In the latter case, the light solid has to be tied down to the heavy tray A by a fine thread.

$$\text{Weight of solid in air} = w \text{ gm.}$$

$$\text{Weight of displaced water} = w_3 - w_2 \text{ gm.}$$

$$\text{Sp. gr. of solid} = \frac{w}{w_3-w_2}.$$

If the solid is soluble in water, the loss of weight in a liquid in which the solid does not dissolve is to be determined. Knowing the specific gravity of the liquid, the loss of weight in water can be calculated and hence the specific gravity of the solid can be determined.*

If the loss in weight of the same solid is found in water and another liquid, the *specific gravity of the liquid* is easily determined. However, the specific gravity of a liquid may be found by means of the Nicholson's hydrometer by the following *alternative method.* Find the weight W of the

---

* In practice it is better to take two or three different sets of observations. For this purpose the weight of the hydrometer is increased each time by adding a few lead shot to the pan D and thus different sets of values for w_1, w_2 and w_3 are obtained.

hydrometer with a balance. Let W_1 and W_2 gm. be the weights in the pan D required to sink the hydrometer to the mark in water and in the liquid respectively. Then

$$W+W_1 = \text{wt. of water displaced.}$$
$$W+W_2 = \text{wt. of liquid displaced.}$$

Thus the weights of equal volumes of water and the liquid are obtained and the sp. gr. of liquid $= \dfrac{W+W_2}{W+W_1}$.

For liquids heavier than water, W_2 is greater than W_1. In the case of light liquids like kerosene the hydrometer by itself may be found to be too heavy to float and in such cases lighter hydrometers should be used. If such a light hydrometer is used with liquids heavier than water, the additional weight to be placed on the top pan D will be so great as to make the instrument top-heavy. The instrument cannot then float vertically in the liquid and hence will be of no use. There is a maximum limit to the weight that can be placed in the pan if the instrument is to float vertically and this limit is nearly reached in water itself, in the case of the hydrometer which does not sink in light liquids like kerosene. Care must be taken to place the weights in the pan symmetrically about the central axis of the hydrometer so that it does not lean to a side and touch the sides of the jar containing the liquid.

It is, therefore, clear that a light hydrometer is required for the determination of the specific gravities of liquids lighter than water and a heavier one for liquids heavier than water. The latter instrument is also useful in the determination of the specific gravity of a solid.

THE BAROMETER.

The barometer is an instrument by which the pressure of the atmosphere is measured. This pressure is commonly measured in inches or centimetres of mercury. The mercurial

column of the barometer exerts a pressure and balances the atmospheric pressure. The height of this balancing column may as well be measured by using liquids like glycerine and water, but mercury is chosen chiefly because of its high density and of the short barometric column which it consequently gives.

The mercury barometer is of two kinds, namely, the syphon barometer and the cistern barometer. In the second, a uniform glass tube, generally one inch in diameter and 36 inches long, closed at one end, is filled with pure, clean and dry mercury and the open end is immersed in a cistern of mercury. The vertical distance between the top levels of mercury in the tube and the cistern gives the barometric height.

The mercury level in the tube rises and falls with the atmospheric pressure and so the surface of the mercury in the cistern falls and rises respectively, through a small height. Therefore, if a fixed scale is used to measure the barometric height (as it is generally done), the zero of the scale cannot always coincide with the mercury surface in the cistern and so the reading on the scale, of the top level of mercury in the tube, cannot always give the correct barometric height. This error is altogether avoided in the Fortin's barometer, where the mercury surface in the cistern can be raised or lowered at will and can be always made to coincide with the zero of the fixed scale.

Therefore, the instrument commonly used for finding the atmospheric pressure in all laboratories and observatories is the Fortin's Standard Barometer. This instrument (fig. 33*a*) consists essentially of a glass cistern H and a glass tube K. The lower part of the cistern is enclosed in brass. The bottom of the cistern is of soft leather and the level of mercury in the cistern can be raised or lowered by working the screw C, at the bottom. Small lengths of the tube K are visible at

the bottom and at the top, and the rest of the tube is enclosed in a brass outer tube which forms the scale of the instrument. The two edges of the slot at the top of the brass tube are graduated, the one in inches and the other in centimetres. The zero of both these scales is in a level with the fine point of an inverted ivory cone I which is fixed in the inner surface of the top of the cistern which is attached to the brass tube. The vernier F can be moved along the graduated edges, by working the side screw E. The zero marks, on either side of the vernier, are on the same horizontal. The whole instrument hangs freely in a vertical position from the hook projecting from a wooden plank fixed alongside the wall. Three screws B, as shown in the figure, keep the barometer in position. G_1 and G_2 are milk glass pieces fixed just behind the mercury surfaces in the tube and in the cistern. These glass pieces reflect diffused light and illuminate the space above the mercury surfaces, and thus facilitate accurate adjustment. The brass lid of the cistern H can be screwed down air-tight by the three screws A, but these have to be kept loose when the instrument is in use, so that the changes in the atmospheric pressure may be readily communicated to the inside

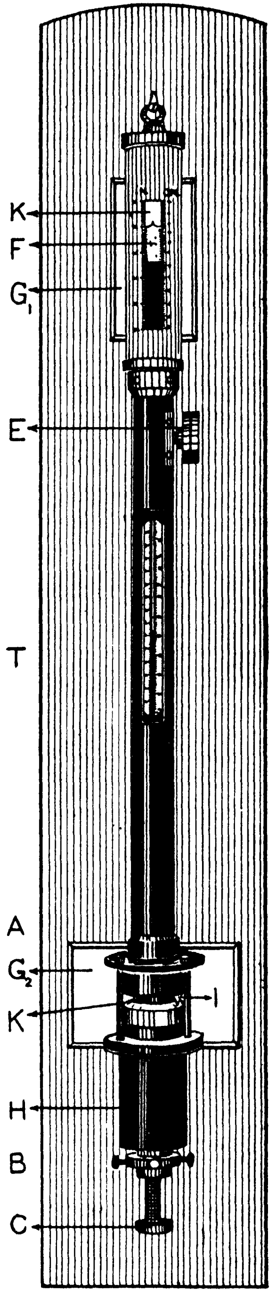


FIG. 33*a*.

of the barometer. A thermometer T is attached to the instrument and gives the temperature at the time of observation.

On the inch scale an inch is divided into 20 parts and a length of 24 such parts is divided into 25 equal parts on the corresponding edge of the vernier. So the vernier can read to $\frac{1}{20} \times \frac{1}{25}$ or 0·002 inch. On the centimetre scale, millimetres are marked and 19 mm. length of the corresponding edge on the vernier is divided into 20 equal parts so that the least count of the vernier is $\frac{1}{10} \times \frac{1}{20} = 0{\cdot}005$ cm. Thus, the two sides read very nearly to the same degree of accuracy (0·002″ = 0·005 cm. very nearly).

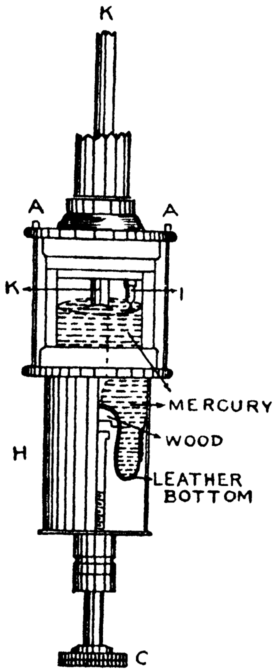


FIG. 33*b*.

The instrument is hung in position vertically and so if the zero of the scales is made to coincide with the level of the mercury in the cistern and the zero of the verniers with the level of mercury in the tube, then the readings of the zero of the verniers on the scales give the barometric height on the two scales. To secure these adjustments, raise or lower the screw C (fig. 33*b*) until the reflected image of the ivory index just appears to touch the fine point of the index or until the clear surface between the tip of the index and the mercury surface, as seen in the diffused light reflected by the white background G_2, just vanishes. The zero edge of the vernier cannot touch the mercury surface in the tube. Hence the error

due to parallax is to be avoided in this adjustment. To this end, a brass plate D (fig. 33c *) is provided. It is connected with the vernier V and slides in a corresponding slot in the back of the upper portion of the enclosing brass tube of the barometer. The lower edge of this plate is in the same horizontal as the zero or the lower edge of the vernier plate. The line of sight is horizontal, as shown in fig. 33c, only when the lower edge of D is just visible. With the eye always in such a position, lower or raise the vernier (and the back plate) until the clear white space between the top surface of mercury and the lower edges of the vernier and the back plate, as seen against the white background of the glass plate G_1, just vanishes at the centre. As the top surface of the mercury in the tube is slightly convex, the milk glass surface G_1 is now visible at the sides, and the horizontal line of sight touches the topmost point of the meniscus.

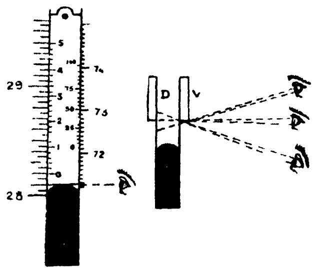


FIG. 33c.

Having made these adjustments, read the position of the zero of the vernier on both the scales. Disturb the adjustments by working the screws C and E (fig. 33a) and take the

* Fig. 33c.—The centimetre scale reads to 0·005 cm. Suppose the 5th division on the vernier coincides with a division on the scale; the fraction of the centimetre given by the vernier is 0·025. Since the scale is read to the first decimal place of the centimetre (i.e. to a mm.), the second and third decimal places are readily given by the above fraction and so 25 is marked against the 5th division of the vernier as shown in the figure. Similarly 50, 75 and 100 are marked against the 10th, 15th and 20th division on the vernier, respectively. Again, each 5th division on the other vernier is successively marked 1, 2, 3, 4 and 5, and this indicates that 0·010″, 0·020″, 0·030″, etc. have to be successively added to the scale reading, when the corresponding division on the vernier coincides with a division on the scale. This marking of the two verniers facilitates the reading of the barometer.

readings again after readjustment. Take the mean of three successive observations as the barometric height.

Practical example.—

The instrument used was the one commonly found in the observatories of the Indian Meteorological Department. The limits of graduation on the two scales are 26·5″ to 32·5″ and 67·2 cm. to 82·5 cm. These are practically the limits of the usual barometric height.

Anantapur.—11-30 A.M. Height of the place above sea-level is nearly 1,100 ft. Latitude is nearly 14°. Attached thermometer reads 30°C.

Scale reading.		Vernier reading.		Observed barometric height.	
cm.	inches	cm.	inches	cm.	inches
73·2	28·80	0·035	0·042	73·235	28·842
73·2	28·80	0·030	0·040	73·230	28·840
73·2	28·80	0·030	0·040	73·230	28·840

Mean observed height is 73·232 cm. or 28·841 inches.

Take 73·232 cm. as the corrected barometric height. Then the atmospheric pressure is the weight of 73·232 c.c. of mercury per sq. cm. If 13·6 gm. per c.c. is the density of mercury and 978 cm. sec.$^{-2}$ is the acceleration due to gravity at the place, then the pressure of the atmosphere is $73{\cdot}232 \times 13{\cdot}6 \times 978 = 0{\cdot}9738 \times 10^6$ dynes per sq. cm. or 0·9738 megadyne per sq. cm. (Weight of mass of one gram is g dynes and hence is 978 dynes at the place.)

The observed barometric height has to be corrected for certain errors. The pressures at different places, when tabulated for comparison, are generally expressed not in absolute units in dynes per sq. cm. but in centimetres of mercury. The pressure due to H_0 cm. of mercury at any place is $H_0 \times d \times g$ dynes per sq. cm., where H_0 is the barometric height, d is the density of mercury at 0°C. and g is the acceleration due to gravity at the place. The observed height h is the length, when the temperature of the scale and that of mercury is t°C. as registered by the attached thermometer. Hence two corrections are necessary, one for the

expansion of the scale and the other for the expansion of the mercurial column. h cm. of scale at t°C. would become, say, H cm. at 0°C. Hence H cm. would be the length corrected for the expansion of the scale only. Again, this H cm. of mercury column at t°C. would become, say, H_0 cm. at 0°C. Then H_0 cm. is the barometric height corrected both for the expansion of scale and mercury. The two heights h at t°C. and H_0 at 0°C. exert the same pressure and hence are equivalent. In other words, the observed barometric height h cm. would become H_0 cm. on a scale kept at 0°C. if the temperature of the mercury were also 0°C. If H_0 and H'_0 are the heights thus corrected at two different places, the ratio of the absolute values of the pressures at the two places is $\frac{H_0 \times d \times g_1}{H'_0 \times d \times g_2}$, where g_1 and g_2 are the values of acceleration due to gravity at the places. This ratio is equal to $\frac{H_0}{H'_0}$ only if g_1 and g_2 are equal. As they are generally not (?), these heights have to be corrected for the change in g, with reference to g_{45}, the acceleration at sea-level and at latitude 45°, which is chosen as the standard. Let $H_0 \times g_1 = H_1 \times g_{45}$ and $H'_0 \times g_2 = H_2 \times g_{45}$, where H_1 and H_2 are the reduced corresponding equivalent heights. Thus H_1/H_2 gives the ratio of the absolute values of the atmospheric pressure at the two different places. The correction due to the expansion of the scale and mercury, in South India, is generally of the magnitude -3 to -4 millimetres, and the attached thermometer generally reads 25° to 30°C. Again $-1{\cdot}5$ mm. is generally the correction required for the variation in g and so the total correction is roughly -5 mm., i.e. $H_1 = h - 0{\cdot}5$ cm. roughly, in South India.

Expressions* from which the observed barometric height can be accurately corrected for the variation in temperature and acceleration due to gravity are given below.

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* These expressions are taken from Barton's Mechanics of Fluids.

A. Temperature Correction.—

(i) *Millimetre scale :*

$$h-a = H_0 \begin{cases} h = \text{observed height in mm.} \\ H_0 = \text{corrected height in mm.} \end{cases}$$

and $a = -(h \times 0{\cdot}000162 \times t)$ mm.

$t = {}^\circ$C. registered by the attached thermometer.

The temperature at which the metric scale is correct is 0°C.

(ii) *Inch scale :*

The temperature at which the inch scale is correct is 62°F. and the density of mercury is known at the standard temperature 32°F. or 0°C.

$a = -\{h \times (0{\cdot}00009t - 0{\cdot}00255)\}$ inches.

$h =$ observed height in inches.

$t =$ temperature in °F. registered by the attached thermometer.

B. Gravity Correction.—

$$H_0 - b = H_1 \begin{cases} H_0 = \text{ht. corrected for temperature.} \\ H_1 = \text{ht. corrected further for the variation in gravity.} \\ b = \text{value to be subtracted.} \end{cases}$$

(i) *Millimetre scale :*

$b = -h(0{\cdot}0026 \cos 2\lambda + 0{\cdot}00000031 \times m)$ mm.

$h =$ observed height in millimetres.

$\lambda =$ latitude of the place.

$m =$ height of place in metres, above sea-level.

(ii) *Inch scale :*

$b = -h(0{\cdot}0026 \cos 2\lambda + 0{\cdot}0000000944 \times f)$ inches.

$h =$ height in inches observed.

$\lambda =$ latitude of the place.

$f =$ height above sea-level in feet.

This b would be positive, generally, for all places whose latitudes are more than 45° and negative for all those below

45°. The errors due to the mercurial vapour pressure, capillarity and the wrong setting of the zero of the scale are very small and can be neglected in all ordinary work.

Practical example.—

A. (i) $h = 732{\cdot}32$ mm., $t = 30°$C., $\lambda = 14°$.

$\therefore \quad a = -(732{\cdot}32 \times 0{\cdot}000162 \times 30) = -3{\cdot}56$ mm.

$\therefore \; H_0 = (732{\cdot}32 - 3{\cdot}56)$ mm. $= 728{\cdot}76$ mm.

B. (i) $b = -\{732{\cdot}3\,(0{\cdot}0026 \cos 28° + 0{\cdot}00000031 \times 1100 \times 0{\cdot}3048)\}$mm.

$= -(1{\cdot}681 + 0{\cdot}076)$ mm.

The first and the second terms may be denoted as b_1 and b_2. They are of opposite signs at places where λ is more than 45°.

$b = -1{\cdot}757$ mm. and $H_1 = H_0 - b = 727{\cdot}00$ mm.

Total correction is $-(a+b) = -5{\cdot}32$ mm.

Work out the correction using the expressions for the inch scale and using the observed value in inches.

($h = 28{\cdot}841$ inches and $H_1 = 28{\cdot}622$ inches).

The readings of the barometer during different hours of the day will be observed to be different and will regularly pass through a maximum and a minimum at definite hours of the day. This diurnal variation may be studied by taking successive observations with the barometer during different days. An idea can be had of this variation from the following table obtained by taking readings at Madras on two successive days in the month of February.

First day.		Second day.	
Hour.	Ht. corrected for temperature in mm.	Hour.	Ht. corrected for temperature in inches.
12 noon	762·205	11 A.M.	30·046
12-30 P.M.	761·816	11-30 "	30·030
1-0 "	761·655	12 noon	30·011
1-30 "	761·157	12-30 P.M.	29·987
2-0 "	760·710	1-0 "	29·976
2-30 "	760·460	1-30 "	29·967
3-0 "	759·913	2-0 "	29·944
3-30 "	759·715	2-30 "	29·925
4-0 "	759·714 Minimum	3-0 "	29·912
4-30 "	759·715	3-30 "	29·909 } Minimum
5-0 "	759·718	4-0 "	29·909 } Minimum
		4-30 "	29·912

A maximum is generally reached twice in a day, at about 10 A.M. and at about 10 P.M. Similarly a minimum is reached twice in a day, at about 4 P.M. and 4 A.M.

What does a sudden fall in the atmospheric pressure generally indicate? What is the principle of an aneroid barometer? Is it as reliable as the above instrument? Such aneroid barometers are generally hung in drawing-rooms. These are called weather prophets and they indicate the state of the weather, Stormy, Rain, Change, Fair and Dry, being the different stages marked on them, the barometer gradually reading higher and higher pressures in the order.

Aviators have reached a height of nearly 45,000 ft. above the sea-level in aeroplanes. This height is very much higher than that of the highest known mountain peak. Professor Piccard reached nearly a height of $10\frac{1}{2}$ miles in his stato-sphere-balloon. How are these heights recorded?

Why does the pressure of the atmosphere decrease as the height of a place above sea-level increases? Why is the pressure higher as we go down a mine shaft? What is the value of the standard atmospheric pressure in centimetres and in inches? Calculate the pressure in absolute units in both the systems.

Place.	Ht. above sea-level in ft. (nearly)	Pressure in inches of mercury (nearly)
Himalayan peaks ..	29,000	8·0
Mt. Blanc	15,800	15·0
Ootacamund ..	7,230	22·8
Kodaikanal.. ..	7,000	22·8
Bangalore	3,150	26·5
Anantapur	1,100	28·7
Madras	0	29·95

A column of 76 cm. of mercury balances the pressure due to the atmosphere. The density of air at N.T.P. is 1·293 gm.

per litre. The densities of balancing fluids are inversely proportional to the balancing heights and so the height H of the atmosphere, if it had the same density 1·293 gm. per litre all throughout its height, would be given by the following equation:

$$76 \times 13{\cdot}595 = H \times 0{\cdot}001293 \text{ or } H = 799{,}088 \text{ cm. or 8 kilometres nearly.}$$

This height is called the height of the 'homogeneous atmosphere'.

BOYLE'S LAW.

The volume of a gas varies inversely as its pressure when the mass and temperature of the gas are kept constant. This law may be verified with the apparatus shown in the figure (fig. 34). This apparatus consists of two glass tubes BC and AD supported on a vertical stand with a heavy cast iron base. The tube BC is closed at the top B and AD is open. These two are connected by a length of rubber pressure tubing. The rubber tubing and the lower parts of the two glass tubes contain mercury. BC is the confined air column and the volume is proportional to the length BC, as the cross-section is uniform throughout the length of the tube. The levels C and A of the mercury in the tubes are read on a vertical metre scale, screwed on to the stand. The right-hand side tube can be moved up and down, parallel to the scale and can be clamped in any position, by means of the screw S, on to the vertical iron rod R.

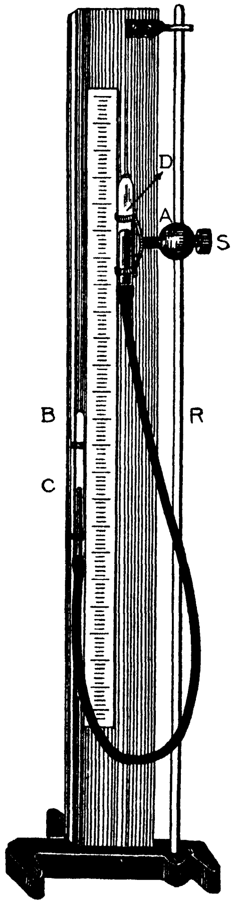

FIG. 34.

Read the barometer and let the height be h cm. Let b cm. be the reading of the top B, on the scale, allowance being made for the thickness of the tube B. Lower AD as much as possible and read the levels of mercury in the two tubes on the scale, a cm. for the open and c cm. for the closed tube. Gradually raise the open tube and at each stage clamp it and take readings of the mercury levels. Let one of the readings be such that c and a are in the same horizontal. Raise the tube AD as high as possible and take a dozen sets of readings. $(b-c)$ gives in cm. the volume of the confined gas and $(a-c)$ gives the difference in pressure between that of the confined air and the outside atmosphere. This is to be added (?) to the atmospheric pressure if the level of A is higher than that of C and has to be subtracted (?) from the atmospheric pressure if it is lower and in each case, the pressure of the confined air is obtained. The confined air is at atmospheric pressure when the level of mercury in the two limbs is the same. The products of the pressure and volume in each case will be found to be nearly constant. This verifies Boyle's law. The following alternative method may as well be followed: The product of the pressure and volume when the gas is at atmospheric pressure may be assumed to be constant. This product is divided by the observed pressure in all other cases. The corresponding volume in each case is thus calculated. This calculated volume agrees well with the observed volume in each case. With the help of a thermometer hung nearby it may be observed that the temperature does not appreciably change during the time of the experiment (?).

When the values of p and v are plotted on a convenient scale and a smooth curve drawn passing evenly through the points, the curve graphically represents the relation between the two quantities. This curve is called a rectangular hyperbola. What would p be according to the law if v were infinitely large and *vice versa* ?

Practical example.—

		Barometric ht. observed in the beginning (h)	73·23 cm.
Temperature throughout	30·5°C.	Barometric ht. observed at the end of the experiment ..	73·23 ,,
		Mean (h) during the experiment	73·23 ,,

Top of closed tube reads, allowing for the thickness of the glass top and for the shape of the top } 88·8 cm. (b)

Mercury level.		Pressure difference. $a-c$	Pressure of confined air. $h+(a-c)$	Vol. of confined air *observed.* $b-c$	Pressure multiplied by the volume of the gas at atmospheric pressure.	Vol. of the gas *calculated.* (Alternative method.)
Open tube. a	Closed tube. c					
cm.	cm.	cm.	cm.	cm.		cm.
11·2	26·8	− 15·6	57·63	62·0	3573	61·5
18·7	30·6	− 11·9	61·33	58·2	3570	57·8
22·5	32·5	− 10·0	63·23	56·2	3559	56·05
27·7	34·9	− 7·2	66·03	53·9	3558	53·7
33·2	37·35	− 4·15	69·08	51·45	3554	51·3
40·4	40·4	0	73·23	48·4	3544	..
46·3	42·7	3·6	76·83	46·1	3542	46·1
50·0	44·05	5·95	79·18	44·75	3543	44·76
56·7	46·45	10·25	83·48	42·35	3535	42·45
63·8	48·75	15·05	88·28	40·05	3536	40·15
72·0	51·25	20·75	93·98	37·55	3530	37·7
84·4	54·65	29·75	102·98	34·15	3518	34·4
99·4	58·2	41·2	114·43	30·6	3492	31·0
					max. error 2·3%	max. error 1·3%

Taking the pressure along the Y axis and the volume along the X axis, the results of the foregoing table may be plotted on a graph paper.

In the above experiment, if the mass of the confined air is greater than what it is, i.e. if the length BC is longer, how would the constant $p \times v$ be altered ?

Boyle's law may also be verified with a simpler apparatus as follows : Take a glass tube uniform in section, nearly 2 mm.

in diameter and 60 cm. long. Suck up a length of about 15 cm. of pure dry mercury into the tube. Tilt the length into position, as shown in fig. 35(*c*), and close the right-hand side end by melting the glass. Cool the end. Put the tube in the positions shown in figs. 35(*a*) to 35(*e*) successively. Erect a drawing board vertically, with a graph paper divided into 0·1″ fixed on to it, and hold the tube in position in each case and read the effective height (?) *h* of the mercurial column, estimating to 0·01″ with the eye. The volume V in centimetres of the confined air can be measured in the positions represented by fig. (*b*) and fig. (*d*) with a foot rule, whose edge is divided into 0·1″. Other positions similar to fig. (*b*) and fig. (*d*) can be had by inclining the tube at different angles. Take 8 or 9 sets of readings for *h* and V and if π is the atmospheric pressure in inches of mercury, p, the pressure of the confined air is equal to $\pi+h$ or $\pi-h$, as shown in the figures. Calculate the products $p_1 \times V_1$, $p_2 \times V_2$, etc. and you find that they are nearly the same.

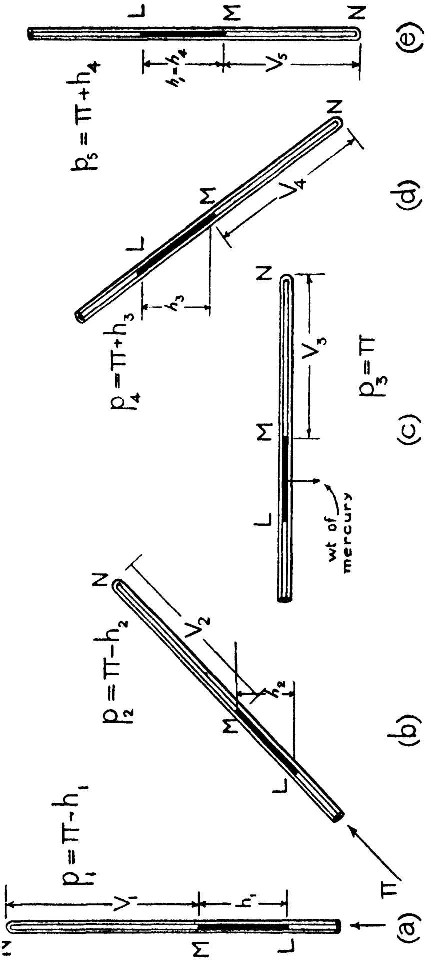

FIG. 35.

In this experiment the inside of the thistle tubing must be well cleaned with caustic soda solution, with dilute nitric acid and finally with distilled water and dried. The mercury used must be very clean and should not adhere to the sides. The tube should be handled very gently during the experiment and never given any violent shake, for otherwise the length of the mercury column would then break into parts.

CHAPTER V

THERMOMETRY AND THERMAL EXPANSION

THERMOMETER.

The thermometer is an instrument employed for measuring temperature. The quality of heat or degree of hotness of a body is called its temperature. When a hot body is in contact with a cold one, the former loses, and the latter gains, temperature until the temperatures are equalized. Heat is said to pass in these circumstances from the hot to the cold body.

The sense of touch enables us roughly to arrange bodies at different temperatures in an ascending or descending order of scale of hotness, but it cannot always be relied upon, and the degrees or intervals of temperature cannot be accurately determined by its help. Hence a property of matter, which varies regularly and continuously with temperature and which always remains the same at the same temperature, is to be chosen to measure the degree of heat. Thermal expansion is one such property. The change of volume of mercury in glass is commonly chosen to determine the change of temperature. The mercury thermometer is thus commonly used in the measurement of temperature.

This instrument consists of a uniform, capillary glass tube or stem, one end of which is blown into a bulb, generally cylindrical in shape. The bulb and a part of the stem are filled with clean, dry mercury and the other end of the stem is hermetically sealed. The temperature of melting ice and that of steam, issuing out of water boiling under standard atmospheric pressure, are arbitrarily chosen as the fixed

temperatures, for purposes of comparison and reference. The points at which mercury stands at this lower and upper fixed temperatures are marked on the stem; and the distance between the two marks, called the *fundamental interval*, is divided into a number of equal parts. In the centigrade system this number is 100. If the bore of the thermometer is uniform, the volumes of the capillary tube between consecutive marks will be equal.

The lower fixed point of a centigrade thermometer is checked in the following manner:

A double walled copper vessel, with a hole at the bottom and supported on legs, pounded ice and the thermometer are the *apparatus required.*

Fill the vessel (fig. 36) to nearly half the height with small pebbles or pieces of brick. Fill the other half with pounded ice and keep the thermometer in position as shown in the figure, so that the mercury thread is just visible. Wait for about ten minutes and read the position of the thread on the graduated stem, estimating to the tenth of a division of the scale. This point a on the stem is the position which the thread occupies whenever the temperature is that of melting ice and marks the zero on the centigrade scale. If a is lower than the zero already marked on the stem by a fraction of a scale division, say x, the error is $-x$ division and if higher, the error is $+x$ division. (Saw-dust is packed between the two walls of the copper vessel. What purpose does it serve?)

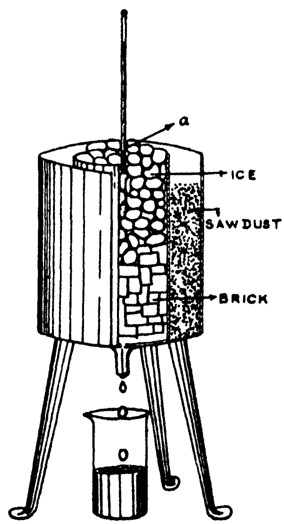

FIG. 36.

The upper fixed point of a centigrade thermometer is tested in the following manner :

The *apparatus required* are a hypsometer, the thermometer, tripod stand, burner, etc.

The hypsometer (fig. 37) is a double chambered cylindrical copper vessel. Into its inner chamber is inserted the thermometer through a cork fitted to the tubulure *t* of the lid. The steam escapes through a side opening and the condensed water in the outer chamber drops down into the water below, through the holes *h*.

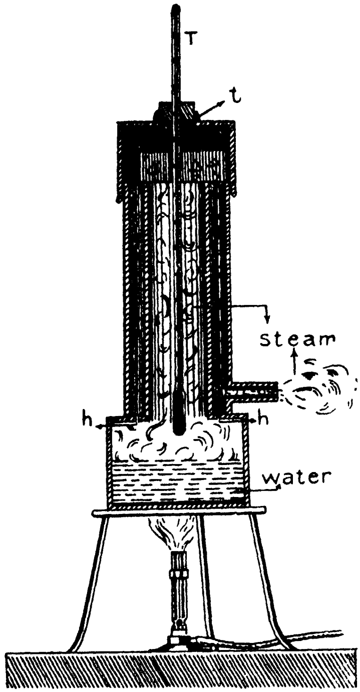

FIG. 37.

Arrange the apparatus as shown in the figure with the thermometer in position. When the steam issues out freely from the side tube for a few minutes, the reading of the mercury thread on the graduated stem gets steady. Note the reading. This is the observed boiling point. Expose more of the stem to the outside air, wait for a few minutes and note the steady reading again. Similarly expose different lengths of the mercury thread and note the steady readings. What do you infer from the readings? When is the maximum reading obtained? Why do the readings change? Which of them gives the correct temperature of the steam bath?

Having thus determined the correct reading, note the barometric height and read the attached thermometer. Correct the barometric height for the expansion of the scale and

mercury. Assuming that a fall or rise of 26·8 mm. in the atmospheric pressure reduces or raises the boiling point of water by 1°C when the pressure is nearly 760 mm., calculate what the boiling point ought to be under the corrected pressure and find the error in the upper fixed point. The boiling point is taken to be 100°C at 76·0 cm. pressure.

Do these errors pertain only to the fixed points? How do they affect the reading of the thermometer at other intermediate temperatures? (The zero of the Fahrenheit thermometer is much lower in temperature than that of the centigrade thermometer. Why is this temperature which is lower than that of melting ice chosen as the starting point on the Fahrenheit scale?)

What is the general average temperature of your laboratory? If a standard scale is to be marked on a steel bar, for use in your laboratory for accurate work, at what temperature would you like the graduations to be correct?

Why are two chambers provided in each of the vessels used in the two tests made above and why is the thermometer inserted into the inner?

If we have two thermometers having the same bore and if the capacity of the bulb of one of them A is greater than that of the other B, the expansion of mercury in A will be greater than that in B, for the same rise of temperature, and the distance between the two fixed temperatures will be greater on the stem of A. The distance between two consecutive degree marks on A will be greater than that on B, and can therefore be further subdivided either to fifths or tenths of a degree. Such a thermometer would register temperatures more accurately and is called a sensitive thermometer. The sensitiveness can as well be increased by having the bore slightly narrower and keeping the capacity of the bulb the same. In practice it is found convenient to combine both these requisites. Place an ordinary centigrade thermometer and a sensitive thermometer side by side and examine. What do you notice

regarding the range of temperature on the sensitive thermometer? Why is the scale of a centigrade thermometer generally extended to 10° on either side of the fixed points? Why are the bulbs of thermometers subjected to the process of annealing before they are filled and graduated? Why is Jena glass employed in the manufacture of thermometers? How does a clinical thermometer differ from an ordinary thermometer?

Practical example.—

Length of stem exposed to outside air.	Thermometer reading (Steam).
0 degree upwards	98·1°C
20 ,, ,,	98·3°C
40 ,, ,,	98·5°C
60 ,, ,,	98·7°C
80 ,, ,,	98·9°C
Nearly no exposure	99·0°C

It is clear that, the greater the length of the mercurial column that is exposed to the colder air, the lower is the reading registered by the thermometer, while the temperature of the steam remains the same. Hence 99·0°C is the required reading of the temperature of the steam. The observed barometric height was 732·55 mm. and the attached thermometer read 31°C. The scale is of brass and is correct at 0°C. So at 31°C the distance between two millimetre marks is $(1+0{\cdot}0000189 \times 31)$ mm. and 732·55 such distances are $732{\cdot}55\ (1+0{\cdot}0000189\times 31)$ mm. $= 732{\cdot}98$ mm. $= h$. This is the height of the barometer corrected for the expansion of the scale.

The atmospheric pressure at the time of the experiment is equivalent to the pressure due to a column of 732·98 mm. of mercury at 31°C. But it is usual to express the pressure in terms of mercury at 0°C. So, the column of mercury at 0°C, which would balance a column or exert the same pressure as h mm. of mercury at 31°C, is to be determined. If this is denoted by h_0, we have $h \times d_{31} = h_0 \times d_0$ where d_{31} and d_0 are the corresponding densities of mercury at the temperatures indicated. $d_{31} = d_0\ (1 - 0{\cdot}000182 \times 31)$, where 0·000182 is the coefficient of absolute expansion of mercury and also the coefficient of decrease of density of mercury with the increase of temperature. We get $h_0 = 728{\cdot}84$ mm. This is the height corrected for the

expansions of the scale and mercury. This is lower than the standard pressure by 31·16 mm. The calculated boiling point must therefore be $\left(100-\frac{31 \cdot 16}{26 \cdot 8}\right)$°C $= 98 \cdot 84$°C. This is what a correct thermometer would register under the given conditions. The thermometer under test, therefore, reads 0·16°C too high. Hence the error is $+0 \cdot 16$°C and the correction, to be added to the observed reading to get the correct reading, is $-0 \cdot 16$°C.

MELTING POINT.

The temperature at which a solid begins and continues to change its state to a liquid is called the melting point of the solid. This is a constant for the solid and is the same as the temperature at which the melted liquid solidifies when cooled. *A good method of determining this constant temperature for a solid is as follows :*

A test tube fitted with a perforated cork, a thermometer and a stirrer fitted into the cork, a watch, paraffin, Bunsen burner, retort stand, etc., are *the apparatus required.*

Heat the paraffin in the test tube gently in a water bath and let the temperature rise a few degrees above the melting point. Stop heating and allow the melted liquid to cool. Stir the liquid continuously. Note the temperature at the end of every half minute in the beginning and afterwards at the end of every minute or more as required. Proceed until it is some time after the whole liquid has solidified. Plot the time on the x axis and the temperature on the y axis. You find that a portion of the curve is parallel to the x axis. The corresponding value of y gives the melting point. This value can also be inferred by inspecting the observations made.

BOILING POINT.

The temperature at which a liquid begins and continues to change its state to a vapour *throughout the mass of the liquid* is called the boiling point of the liquid. The pressure of the

saturated vapour of any liquid at its boiling point is equal to the atmospheric pressure and this fact is utilized *to determine the boiling point of a given liquid, say alcohol.*

A bent tube with the short arm closed, dry mercury, alcohol thermometer, water bath, etc., are the *apparatus required.*

Fill mercury into the tube as in fig. 38(a), leaving no air bubbles in the shorter arm. Put in a few drops of alcohol * into the open tube and by tilting carefully, let the liquid occupy the top of the shorter arm as in fig. 38(b) without air bubbles in it. Arrange as in fig. 38(c). Heat the water

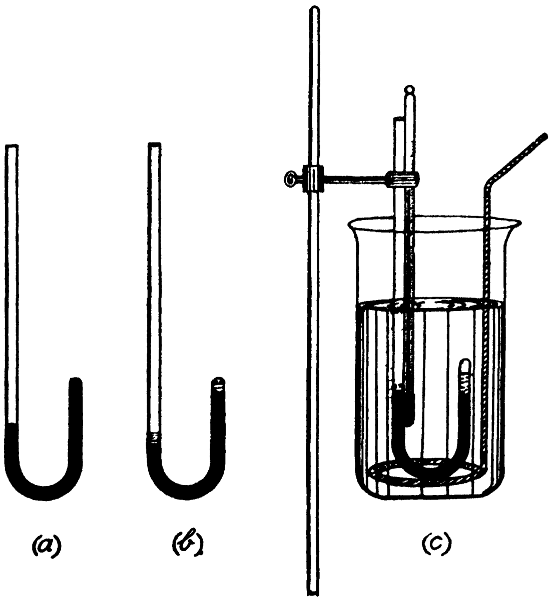

FIG. 38.

bath and stir it. At a stage the level of mercury in the shorter arm begins to get down and just a bubble of vapour appears over the liquid. (Why ?) Lower the flame and heat it gently, stirring the bath well. After a time the levels in

* Do not take the alcohol near a flame. It is liable to catch fire.

the two arms would lie in the same horizontal. Then note the reading t_1°C on the thermometer. The level of mercury in the shorter arm continues to go down. Remove the flame. The level in the shorter arm rises and once again the two levels will be in the same horizontal. Note this temperature t_2°C. The two temperatures noted differ slightly. (Why?) The mean gives the boiling point of the liquid. Repeat the observations once again and take the mean of all the observations.

What is the pressure of the vapour as it just appears in the shorter limb compared with the atmospheric pressure? What is the pressure of the vapour at the boiling point of the liquid?

Practical example.—

Temperature when the levels are in the same horizontal.

	Temperature rising.	Temperature falling.
	77·0°C	76·8°C
	76·8°C	76·8°C
Mean.	76·9°C	76·8°C

Mean boiling point for ethyl alcohol = 76·85°C or 76·9°C.

Additional observation.—

The maximum difference in the mercury levels that could be obtained was 9 cm. and the temperature was then 80·5°C. So, the maximum vapour pressure of the alcohol at 80·5°C was $(\pi+9)$ cm. where π = atmospheric pressure and it was 73·1 cm.

The maximum vapour pressure exerted by alcohol at 80·5°C is accordingly 82·1 cm. of mercury.

What happens if the temperature of the bath is raised further?

How do you roughly determine the maximum tension of the vapour with the apparatus for a few degrees on either side of the boiling point?

VARIATION OF BOILING POINT WITH PRESSURE.

A bent tube filled as in fig. 38(*b*) of the previous exercise, thermometer (T), a T tube, mercury manometer (M), a millimetre scale (C), air-pump (A), water bath (B), stirrer (S), etc., are *the apparatus required* for determining the boiling point of alcohol at different pressures below an atmosphere.

Arrange the apparatus as in fig. 39. D is the drying apparatus containing glass wool soaked in strong sulphuric acid.

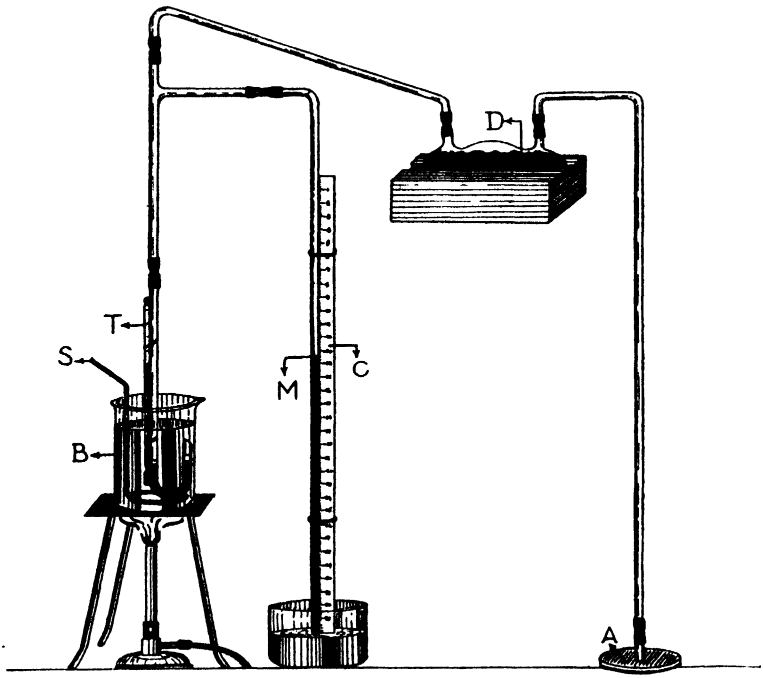

FIG. 39.

The pressure in the left-hand limb of the bent tube can be lowered by working the pump. The rubber joints in the arrangement must be secured air-tight. In the beginning, the level of mercury in the manometric tube would be the same as that in the outer vessel, and as the pressure is gradually reduced the level in M will rise, and this height of the mercurial

column deducted from the barometric height gives the incumbent pressure. Observe the barometric height. Raise the temperature of the bath, stirring it well, until the levels of mercury in the two limbs are the same. Note the temperature of the bath. The pressure is the atmospheric pressure. Lower the temperature of the bath by removing the flame. At any desired temperature try and heat the bath with a reduced flame, so that the heat supplied nearly compensates for the loss of heat due to radiation and the temperature of the bath is steady for a minute or two, to allow alcohol to take up the temperature of the bath. Work the pump and lower the pressure so that the levels of mercury in the two limbs are again the same at this temperature.

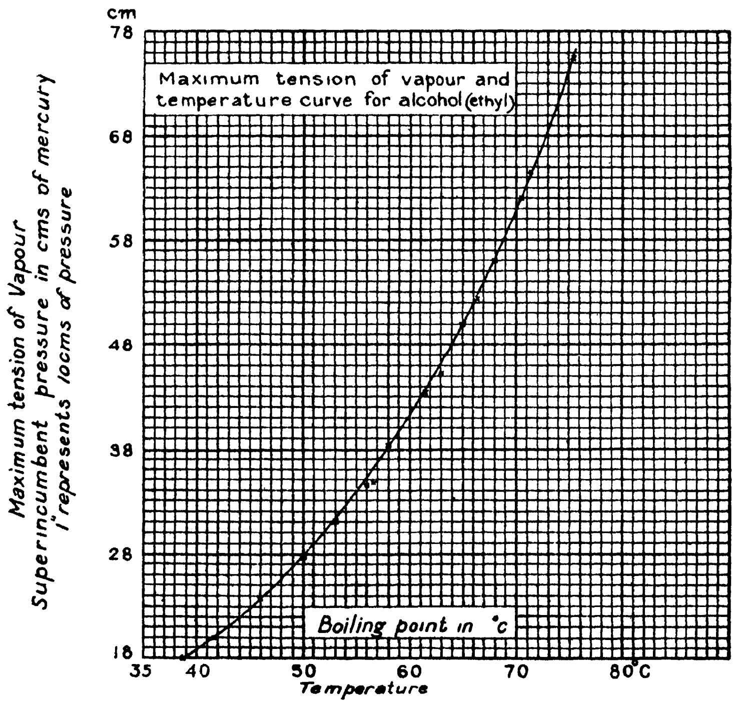

FIG. 40.

Note the levels of mercury in the tube M and in the outer vessel. Read the steady temperature. Cool the bath gradually and take observations as stated above at each stage. Stop when the pressure falls to nearly half an atmosphere. A corresponding set of readings may also be taken when the temperature of the bath is rising. Tabulate the readings and plot temperature against pressure. Draw a smooth curve (fig. 40). Read from the curve the pressures at 40°, 50°, 60° and 70°C. Compare them with those given in the standard tables. How does the pressure of a small air bubble at the top of the liquid in the shorter limb of the bent tube affect the results?

Practical example.—

The stroke of the pump was adjusted at each stage and the levels of mercury in the two limbs judged to be the same. A Fluess air-pump was employed. Barometric height, corrected for temperature, was 75·30 cm.

Temperature or Boiling Point °C.	Manometer readings.		Mercury column corrected to 0°C. cm.	Incumbent pressure in cm. of mercury at 0°C.
	Top cm.	Bottom cm.		
56·5	59·0	99·8	40·6	34·7
61·4	67·5	99·7	32·0	43·3
63·0	69·5	99·7	30·0	45·3
66·3	76·5	99·6	23·0	52·3
66·5	77·0	99·6	22·5	52·8
68·0	80·0	99·5	19·4	55·9
71·3	88·5	99·4	10·8	64·5
75·5	99·2	99·2	0	75·3
70·5	86·0	99·4	13·3	62·0
68·0	80·0	99·5	19·4	55·9
65·0	74·0	99·6	25·5	49·8
61·5	67·5	99·7	32·0	43·3
58·3	62·7	99·7	36·8	38·5
56·0	58·8	99·8	40·8	34·5
53·0	55·3	99·8	44·3	31·0
50·1	52·0	99·8	47·5	27·8
46·0	48·0	99·9	51·6	23·7
41·8	44·4	99·9	55·2	20·1
38·8	42·4	99·9	57·2	18·1

Temp. °C.	Saturation pressure from the curve: cm. of mercury.	Saturation pressure for Ethyl alcohol from tables: cm. of mercury.	Observed excess: cm. of mercury.
40	19·0	13·4	5·6
50	27·7	22·0	5·7
60	40·9	35·0	5·9
70	60·8	54·0	6·8
75·7	76·0		

Boiling point at standard pressure is 75·7°C from the curve and the value from the tables is 78·4°C. A small air bubble has been left over in filling mercury in the special tube in the beginning, and the excess observed is thus accounted for.

EXPANSION.

All bodies, solids, liquids or gases, generally expand when heated and contract when cooled. Very few bodies, like cast-iron, expand when cooled. Expansion is not confined to a single dimension but takes place in all the three. Length in any direction, area in any plane and volume as a whole, all increase. These three are called the linear, the superficial and the cubical expansions. In the case of fluids cubical expansion only need be considered (?).

If the temperature of a solid is raised through a degree centigrade, it is always found that the increase in length is a constant fraction of its length at a standard temperature chosen, say 0°C. This fraction is therefore the ratio of the increase in the length of the body for a rise of 1°C to the length at 0°C. This ratio is called the coefficient of linear expansion of the solid. This fraction is very small and the increase in length of a bar or rod of a solid, say one yard or two long, is so very small that it cannot be directly measured with a scale. Special methods, such as those described in the following, must be employed.

I. We may magnify the increase in length mechanically by a lever. The ratio of the longer to the shorter arm of

the lever gives the magnification: the end of the longer arm is read on a graduated scale. II. The increase in length may directly be measured by the help of (i) vernier and scale, as of a travelling microscope, or (ii) a screw gauge or a spherometer. The first method is simple but not so accurate as the second.

COEFFICIENT OF LINEAR EXPANSION OF A SOLID.

Linear expansion apparatus (fig. 41), steam generator, a half and a full metre scale, Bunsen stand, thermometer, dividers, etc., are the *apparatus required.*

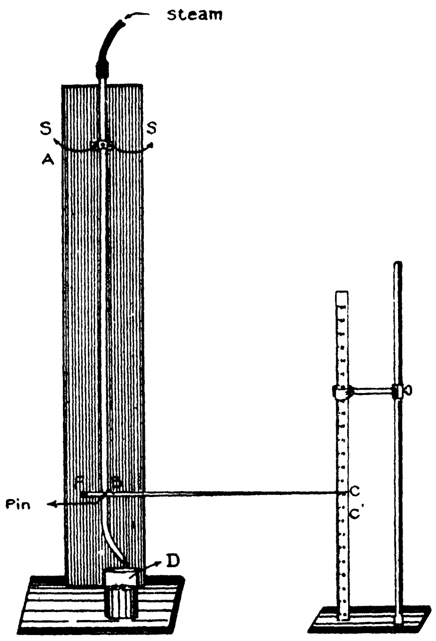

FIG. 41.

The apparatus (fig. 41) consists of a metal tube AB fixed on to a vertical wooden board, standing on a horizontal base board. The tube is open at both ends and is provided with two thick pins, A and B, soldered to it. The top pin A fits into a hole at the centre of a brass plate fixed to the plank by means of two screws SS. This prevents the tube moving upwards relatively to the plank. At the bottom of the board a lever (a wooden lath) FBC is provided and moves round the fulcrum F. The lower pin B of the tube just fits in a hole in the lever. The position of the end C of the lever is read on a vertical half metre scale CC′. A vessel D receives the condensed steam.

Note the distance AB with a metre scale, and read the temperature of the tube by inserting the thermometer into it. Find the length of the longer arm of the lever FC with the metre scale and that of the shorter arm FB, carefully with dividers and scale. Read the position C on the scale. Arrange the apparatus as shown in the figure. Connect the steam generator to the apparatus before reading the position C on the scale and then heat the generator. After a time steam passes through and the pointer FBC is pushed downwards. (Why?) The pointed end creeps down the scale and after a time when the tube expands no more, and the steam issues at the other end of the tube freely, it stands steady at, say, C′. Take this reading. Note the temperature of the steam t°C. (Is this the temperature of the metal tube throughout its length?)

FB′C′ is the position occupied by the lever at the end, B′ (not shown in the figure) being the position of the lower pin after expansion. BB′ is the increase in length of the tube. In the similar triangles, FBB′ and FCC′, $\frac{BB'}{CC'} = \frac{FB}{FC}$ and $BB' = \frac{CC' \times FB}{FC}$.

The increase in length BB′ divided by the rise in temperature of the tube gives the increase in length per degree centigrade rise of temperature. The coefficient of linear expansion of the material of the tube is the ratio of the increase in length per degree centigrade rise of temperature to the length at 0°C (l_0). But the length l_0 does not appreciably differ from l_t, the length at the laboratory temperature, as the coefficient of linear expansion of solids is quite small. Hence, $\alpha = \frac{CC' \times FB}{FC \times AB(t - t_1)}$ where t_1 is the room temperature. All the quantities on the right-hand side of the equation are observed and α is calculated. Note that the expansion of a unit length of the tube for a unit rise of temperature is numerically

equal to α. How is the value of α altered if the unit degree in the above definition is on the Fahrenheit scale?

If the solid is in the form of a rod or wire, the apparatus is essentially the same, except for an outer glass jacket with an inlet and outlet for steam. In the case of a rod or tube the increase in length can be measured with advantage, directly with a screw, a spherometer being used in the vertical position and a screw-gauge in the horizontal position.

Practical example.—

FB = 2·0 cm.; FC = 102 cm.; AB = 100 cm.; $t = 99°C$; $t_1 = 25°C$; CC′ = 91·7 − 85·2 cm. = 6·5 cm.

The tube cooled after the experiment and its temperature was found to be 28°C and the position of C on the scale was 85·6 cm.

The material of the tube used was copper and the coefficient of linear expansion of copper was found to be

$$\frac{6{\cdot}5 \times 2}{102 \times 100 \times 74} = 0{\cdot}0000172(2)$$

At this rate of increase in length with temperature, the length of AB at 0°C would be

$$100 - \frac{6{\cdot}5 \times 2}{102} \times \frac{25}{74} = 99{\cdot}957 \text{ cm.}$$

∴ According to the strict definition, we have

$$\alpha = \frac{6{\cdot}5 \times 2}{102 \times 74 \times 99{\cdot}957} = 0{\cdot}0000172(3)$$

The difference in the value thus obtained is only in the fourth significant figure and the observations made cannot be depended upon to that accuracy. This is why l_t is taken to be equal to l_0 in practice.

Using the reading 85·6 cm. for the position of C at the lower temperature 28°C as noted at the end of the experiment, α is found to be 0·0000168(3). The mean coefficient of linear expansion of copper is thus equal to 0·000017.

COMPENSATION FOR EXPANSION IN CLOCKS AND WATCHES.

The pendulum of a clock generally consists of a metal rod, to the lower end of which is attached a metal bob. In summer

the length of the pendulum increases and the time of oscillation also increases and therefore the clock loses. (Why ?) Similarly it gains in cold weather. Accurate time measurement requires accurate clocks. So the variation in the length of the pendulum must be compensated, for all temperatures. For this purpose the pendulum is made of two different materials—iron and brass or iron and mercury, as is generally the case. One of them expands upwards and the other downwards so that the centre of gravity of the bob remains at the same distance from the point of suspension. Again, in the case of a watch, its rate is controlled by the time of oscillation of the balance wheel, which depends on the radius of the wheel and the radius increases with temperature. The rim of the wheel is made of strips of two metals, into two sectors, the more expansible metal strip being on the outer edge. As temperature rises, the curvature of the wheel increases due to the closing in of the more expanding outer strip and this compensates for the expansion of the radial metal spokes of the wheel.

CORRECTION OF THE OBSERVED BAROMETRIC READING FOR EXPANSION.

A scale is correct at a particular temperature, say t°C (generally 16·6°C or 62°F). At a higher temperature t_1°C, each centimetre of the scale will expand to $1+a(t_1-t)$ cm. where a is the coefficient of expansion of the material of the scale. This is the distance between consecutive marks and is considered as a centimetre in taking the observed height. If h cm. is the height observed, the height H, corrected for the linear expansion of the scale, will be $H = h[1+a(t_1-t)]$ cm. If the scale is engraved on a metal or alloy whose coefficient of expansion is very small, unlike that of brass on which the scale is generally engraved, this correction is unnecessary.

This H cm. is the height of the mercurial column at t_1°C. As already explained in the exercise on the barometer, the

height of the mercurial column at a standard temperature, usually 0°C, is wanted. The pressure of the column remaining the same, the heights are proportional inversely to the densities. If H_0 is the height at 0°C and d and d_0 are the densities at t_1°C and 0°C respectively, we have

$$\frac{H_0}{H} = \frac{d}{d_0}$$

Consider m gm. of mercury at 0°C and at t_1°C. Then $m = Vd = V_0\ d_0$, if V is the volume of mercury at t_1°C and V_0 at 0°C. If b is the coefficient of absolute expansion of the liquid, we have

$$\frac{d}{d_0} = \frac{V_0}{V} = \frac{V_0}{V_0(1+bt_1)} = 1-bt_1 \text{ approximately.}$$

$$\therefore\ \frac{H_0}{H} = 1-bt_1.$$

$$\therefore\ H_0 = h[1+a(t_1-t)]\ (1-bt_1) = h[1-(b-a)t_1-at] \text{ approximately.}$$

$$\therefore\ H_0 = h\ [1-(b-a)t_1], \text{ if } t = 0°\text{C}.$$

The working formula for mercury-brass is therefore

$$H_0 = h(1-0{\cdot}000162\ t_1).$$

An approximate formula for pressures near 760 mm. is

$$H_0 = h-0{\cdot}123\ t_1.$$

EXPANSION OF A LIQUID.

The cubical expansion of liquids is complicated on account of the expansion of the containing vessel which cannot be dispensed with. Suppose the level of a liquid in a graduated glass jar is at a to start with (fig. 42a). Now at a higher temperature, let the level of the liquid be at b. The levels can be measured on an outside vertical scale which does not undergo the change in temperature and then the rise of the liquid will be measured to be ab. But suppose we read the difference in level on the graduated jar. The jar also expands and the level

of the point a on the jar would now rise to, say, a' and the liquid level b would appear to rise only through $a'b$ on the jar though it really rises through ab. Hence the increase in volume or the cubical expansion of the liquid, measured with reference to the containing vessel, is less than that measured without any reference to the vessel. The former is called the apparent or the relative cubical expansion of the liquid and the latter, the absolute or the real expansion. It is thus clear from the above figure that the absolute expansion is the sum of the relative expansion of the liquid and the cubical expansion of the vessel containing the liquid.

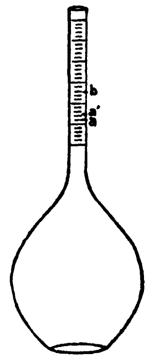

FIG. 42*a*.

The coefficient of apparent cubical expansion of a liquid may be determined by the following method:

The *apparatus required* are a specific gravity bottle, a boiling water-bath, a centigrade thermometer, a beaker containing the liquid, pipette, test-tube holder, filter-paper cuttings, etc.

Weigh the bottle empty, correct to a centigram (w_1). Fill the specific gravity bottle with the given liquid taking care that no air bubbles remain in it, insert the glass stopper carefully and wipe off the liquid on the outside of the bottle with a clean cloth or filter paper. Weigh the bottle to the nearest centigram (w_2). Note the temperature of the remaining liquid in the beaker (t_1°C). Tie up a thread to the neck of the bottle and hang it up from a Bunsen stand (fig. 42*b*), into the boiling water bath, immersing it to the neck with the test-tube holder, if necessary. Wipe off the escaping liquid with filter paper cuttings and wait until no more of the liquid escapes out of the bottle. Note the temperature of the bath t_2°C and remove the bottle from it. Dry the outside, remove the thread, allow it to cool and then

weigh it to a centigram (w_3). Let V c.c. be the inside volume of the bottle at t_1°C and w_2-w_1 gm. of the liquid occupy V c.c. at this temperature. At the higher temperature t_2°C, the inside volume will increase to, say, V_1 c.c. and $V_1 = V(1+3\,\alpha\,\overline{t_2-t_1})$ where α is the coefficient of linear expansion of the glass of the bottle. w_3-w_1 gm. of the liquid occupy V_1 c.c. at t_2°C and contract on cooling to v c.c. at t_1°C or, v c.c. at t_1°C would expand to V_1 c.c. at t_2°C. Let d gm. per c.c. be the density of the liquid at t_1°C. Then $v = \dfrac{w_3-w_1}{d}$. If the increase in volume V_1-V of the bottle with rise of temperature is neglected, as being very small, then its volume at the higher temperature is also V and it is the volume of w_2-w_1 gm. at t_1°C which is equal to $\dfrac{w_2-w_1}{d}$ c.c.

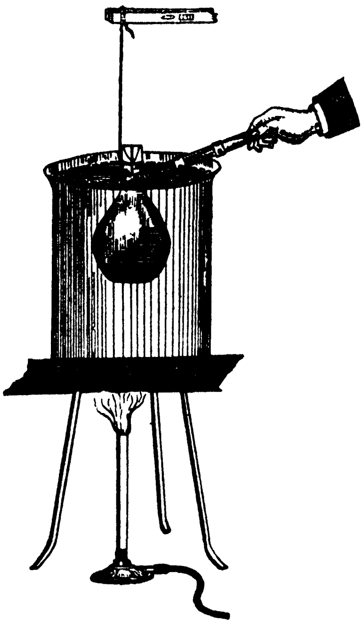

FIG. 42*b*.

$$\therefore \quad V = V_1 = \frac{w_2-w_1}{d} \text{ c.c.}$$

and $v = \dfrac{w_3-w_1}{d}$ c.c. and V_1 c.c. at t_2°C contract on cooling to v c.c. at t_1°C or v c.c. at t_1°C would, if heated to t_2°C, expand to V_1 c.c.

The increase in volume per 1°C rise $= \dfrac{V_1-v}{t_2-t_1} = \dfrac{w_2-w_3}{d(t_2-t_1)}$ c.c.

But the coefficient of apparent or relative cubical expansion γ of a liquid is the ratio of the apparent increase in volume of

the liquid for 1°C rise in temperature to the volume at 0°C. Since the coefficient in the case of liquids also is small, the volume v at t_1°C may be approximately taken to be equal to the volume at 0°C.

$$\therefore\ \gamma = \frac{w_2-w_3}{d(t_2-t_1)} \times \frac{d}{(w_3-w_1)} = \frac{w_2-w_3}{(w_3-w_1)(t_2-t_1)}.$$

The coefficient of absolute expansion of a liquid may be determined in the following manner. This is the ratio of the real increase in volume per 1°C to its volume at 0°C. In the above consideration the quantity (V_1-V) c.c. has been neglected and the increase in volume of the liquid relative to that of the containing vessel has been considered. But let us consider now the actual increase in volume (V_1-v) c.c.

$$\text{Total actual increase in vol.} = V_1-v$$
$$= V(1+3\alpha\,\overline{t_2-t_1})-v$$

$$\text{The increase in volume per 1°C rise} = \frac{V-v}{t_2-t_1}+3V\alpha$$

$$\therefore\ \frac{\text{actual increase per 1°C rise}}{\text{vol. at the lower temperature}} = \frac{V-v}{(t_2-t_1)v}+3\alpha\,\frac{V}{v}$$

$$\therefore\ \text{Absolute coefficient} = \text{relative coefficient} + 3\alpha\,\frac{V}{v}$$

$\frac{V}{v}$ can be nearly taken to be unity and we can write

Coefficient of absolute cubical expansion of liquid	=	Coefficient of relative cubical expansion of liquid	+	Coefficient of cubical expansion of the containing vessel.

Practical example.—

The liquid used was kerosene oil.

$w_1 = 15{\cdot}18$ gm.; $w_2 = 55{\cdot}57$ gm.; $w_3 = 53{\cdot}125$ gm.
$t_1 = 28$°C.; $t_2 = 98$°C.

$$\therefore\ \gamma \text{ for kerosene oil} = \frac{2{\cdot}445}{37{\cdot}945\times 70} = 0{\cdot}00092.$$

The increase in vol. from 28° to 98°C = the volume of 2·445 gm. of the liquid. The rate of increase of volume being uniform, the volume of the liquid at 0°C would be less than that at 28°C by the volume of a mass of liquid $= \frac{28}{70} \times 2{\cdot}445$ gm. or 0·978 gm. Therefore, if, according to the strict definition of the coefficient, the volume at 0°C is to be employed in the denominator, we have the following for the relative coefficient γ :

$$\gamma = \frac{2{\cdot}445}{70 \times (37{\cdot}945 - 0{\cdot}978)} = 0{\cdot}000945$$

The linear coefficient of expansion of glass is 0·000009 and the coefficient of cubical expansion is nearly 0·000027; the coefficient of absolute expansion of kerosene oil is, therefore, approximately equal to 0·000945+0·000027 = 0·00097.

The coefficient of decrease of density with rise of temperature of a body may be defined as the ratio of the decrease in density for 1°C rise in temperature to the density at 0°C. In the above example, supposing that the volume of the inside of the bottle is constant throughout, the densities of kerosene at 28°C and 98°C, are $\frac{40{\cdot}39}{v}$ and $\frac{37{\cdot}945}{v}$ gm. per c.c. respectively. The change in density for a range of 70°C is $\frac{2{\cdot}445}{v}$, and if the density at 28°C is approximately taken to be equal to that at 0°C, the coefficient of apparent decrease of density with rise of temperature $= \frac{2{\cdot}445}{v \times 70} \times \frac{v}{40{\cdot}39} = \frac{37{\cdot}945}{40{\cdot}39} \times 0{\cdot}00092$, which is roughly the same as *the coefficient of cubical expansion* of kerosene relative to glass.*

---

* Mass $= V_0 \rho_0 = V\rho$, where V and ρ are the volume and density at a temperature t.

$$\therefore \quad \frac{V}{V_0} = \frac{\rho_0}{\rho} = 1 + \gamma t, \ \gamma = \text{coefficient of cubical expansion.}$$

$$\therefore \ \frac{\rho_0}{\rho} - 1 = \gamma t, \text{ i.e., } \frac{\rho_0 - \rho}{\rho t} = \gamma.$$

EXPANSION OF A GAS.

When a confined mass of a gas is heated its volume increases appreciably. The volume of the gas is also controlled by the pressure to which it is subject and any slight change in the pressure will appreciably alter the volume and introduces an error in the observation of the thermal expansion of the gas. In the case of solids and liquids the compressibility is negligibly small and so this consideration did not arise. Hence is the necessity to keep the pressure of the gas constant while studying the thermal expansion of a gas.

Again, the effect of heat on the pressure of a confined mass of gas may as well be studied, the volume of the gas being kept constant, for otherwise the change in volume would cause a change in pressure. This effect is studied in the next exercise.

The coefficient of dilatation of a gas at constant pressure is determined in the following manner :

A narrow glass tube of uniform bore, dry mercury, half metre scale, centigrade thermometer, a glass jacket tube, steam generator, etc., are *the apparatus required.*

Take a long glass tube nearly 2 mm. in diameter. Wash it well with (i) caustic soda solution and (ii) nitric acid. Wash it again with water. Connect it to the bellows and pass a stream of air through, while the tube is gently moved through a Bunsen flame. The tube will get dry. Cool it and suck up a small pellet of mercury, about a centimetre long, into

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Again, $\rho = \dfrac{\rho_0}{1+\gamma t}$ from the above.

$= \rho_0 (1 - \gamma t)$ approximately, since γ is very small.

$\therefore \dfrac{\rho_0 - \rho}{\rho_0 \, t} = \gamma$, approximately.

$= \gamma'$, say.

This is the definition given above.

Hence $\dfrac{\gamma'}{\gamma} = \dfrac{\rho}{\rho_0}$ or $\gamma' = \gamma . \dfrac{\rho}{\rho_0}$ as found above.

it and bring it somewhere near the middle of the tube. Close one end of the tube in the flame. Now a mass of air is confined, at atmospheric pressure, in the tube. Place it along the half metre scale so that the inside of the closed end is against the zero of the scale. Tie the tube in this position to the scale, with the thermometer alongside, as shown in figure 43.

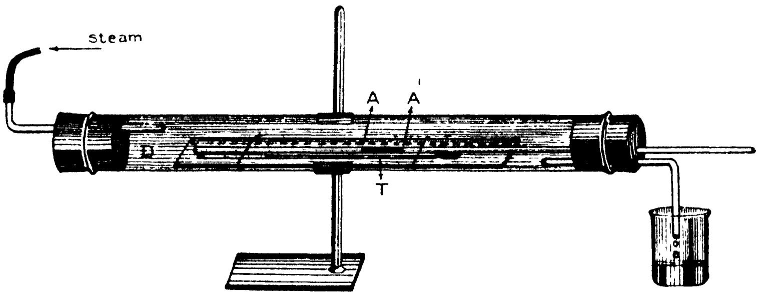

FIG. 43.

As the tube is practically uniform in section, the volume of the air inside the tube is proportional to the length. Note the length of the air column DA and its temperature t°C registered by the thermometer T. Insert the half metre scale with the tube and thermometer tied to it into the jacket tube and arrange as shown in the figure. Pass steam. Note the steady temperature t_1°C and the steady position of the left end A′ of the mercury plug. Stop the passing of steam. The pressure of the confined air is practically (why ?) constant throughout the experiment and is equal to the atmospheric pressure. The increase in volume is $(\mathrm{DA}'-\mathrm{DA})\times a = \mathrm{AA}'\times a$ c.c., where a is the mean sectional area of the tube: the rise in temperature is (t_1-t)°C.

The rate of increase in volume is $\dfrac{\mathrm{AA}'\times a}{t_1-t}$ c.c. per 1°C. According to this rate, the volume of the confined air at 0°C would be $\dfrac{\mathrm{AA}'\times a\times t}{t_1-t}$ c.c. less than that at t°C.

$\therefore$ the calculated volume at 0°C $= \left(\mathrm{DA} - \dfrac{\mathrm{AA}' \times t}{t_1 - t}\right) a$ c.c.

The coefficient of dilatation of a gas at constant pressure is the ratio of the increase in volume of the gas per 1°C to its volume at 0°C.

$$\therefore \quad \alpha = \frac{\mathrm{AA}'}{(t_1 - t)\left[\mathrm{DA} - \dfrac{\mathrm{AA}' \times t}{t_1 - t}\right]}$$

Practical example.—

Length of air column cm.		Temp.	Pressure.
To start with ..	24·75	30·05°C	Time of experiment being 20 minutes, the atmospheric pressure would not have appreciably changed and hence was taken as constant.
After steam was passed ..	30·50	100·0°C	
Some time after ..	30·55	,,	
After some more time ..	30·55	,,	

Increase in volume .. $(30{\cdot}55 - 24{\cdot}75) \times a$ c.c.

Increase in temperature .. $100 - 30{\cdot}05 = 69{\cdot}95$°C.

Rate of increase of vol. .. $\dfrac{5{\cdot}8 \times a}{69{\cdot}95}$ c.c., per 1°C.

Volume of confined air at 0°C $= \dfrac{5{\cdot}8 \times a}{69{\cdot}95} \times 30{\cdot}05$ c.c.,
less than $24{\cdot}75 \times a$, c.c.
$= 22{\cdot}26 \times a$ c.c.

$\therefore$ the coefficient of dilatation of air $= \dfrac{5{\cdot}8 \times a}{69{\cdot}95} \times \dfrac{1}{22{\cdot}26 \times a} = 0{\cdot}00372(4)$

In the above experiment the volume of the confined air changes, according to Charles' law,* regularly with temperature, the position of the mercury plug being constant for a

* The volume of a gas increases for each 1°C rise of temperature by a constant fraction of its volume at 0°C, if the mass and pressure of the gas are kept constant.

constant temperature. Here, air takes the place of mercury in the mercurial thermometer. The length of the fundamental interval in the above experiment was $30{\cdot}55-22{\cdot}26 = 8{\cdot}29$ cm. Therefore, the length on the tube that would correspond to a degree centigrade is 0·0829 cm. The length of the stem below the position corresponding to 0°C is 22·26 cm. and so $\frac{22{\cdot}26}{0{\cdot}0829} = 268{\cdot}5$ degrees can be marked, below 0°C, on the tube. If the tube were placed at a temperature corresponding to 268·5°C below 0°C, or at −268·5C, the volume of the air confined would be zero. Hence such a temperature is called the absolute zero temperature and such a scale of temperature, with this for its zero or the starting point, is called the absolute scale of temperature and the reading on such a scale corresponding to any known temperature on the centigrade scale, say t°C, is $(268{\cdot}5+t)^\circ$ absolute*.

Does the value obtained above refer to the coefficient of apparent dilatation or absolute dilatation of the gas?

Why is it not generally stated as such?

Show how it follows from the above experiment, that the volume of a given mass of gas at constant pressure is proportional to its absolute temperature.

What is the value of the coefficient per 1° Fahrenheit? What is the reading for the absolute zero on the Fahrenheit scale?

The coefficient of increase of pressure of a gas at constant volume may be determined in the following manner.

The constant volume air thermometer, a centigrade thermometer, water bath, retort stand, etc., are the *apparatus required*.

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* The number 268·5 is obtained by taking α to be 0·00372 as obtained above; but if the accepted value, viz. 0·00366 is followed, the absolute zero would be −273°C and the temperature on the absolute scale corresponding to t°C $= (273+t)^\circ$ absolute.

The constant volume air thermometer consists of a capillary glass tube CA (fig. 44) bent twice at right angles. On one of its ends is blown a bulb A. The other end is connected by a length of pressure tubing to an open glass tube B. These two tubes are mounted on a vertical plank of wood which is fixed on to a heavy cast-iron base. The tube B can slide on a vertical iron rod D and can be clamped at any point by means of a thumb screw. The rubber tubing and part of the glass tubes are filled with dry, clean mercury. A metre scale, divided on either edge into millimetres, is fixed vertically along the middle of the plank, and the mercury levels in the tubes are read on it.

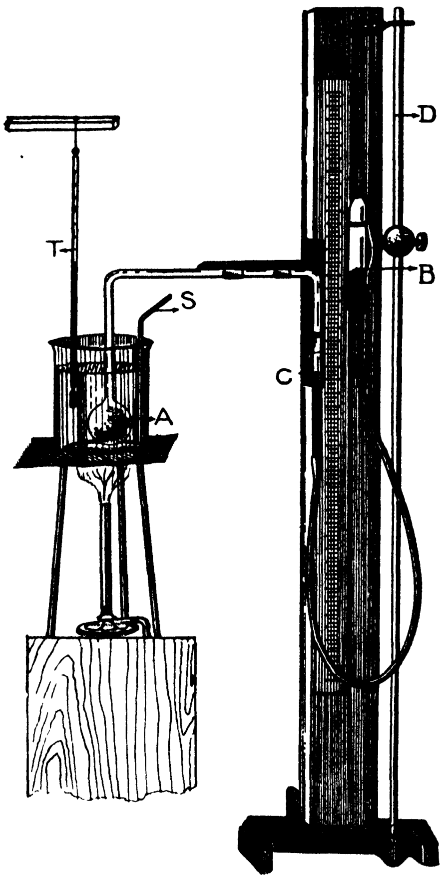

Fig. 44.

Read the barometer in the beginning. Arrange the apparatus as shown in the figure. Take the initial temperature (t_1°C) of the cold water, on the thermometer T. Raise the tube B such that the level of mercury C in the left-hand tube is at a convenient mark on the scale. It is better to have it near the bend so that as little of the air column as possible is exposed to the atmosphere. Take the reading of the level

B on the scale. Raise the temperature through nearly 10°C, lower the flame, stir the water bath well, and maintain the temperature steady for a few minutes; (the temperature perhaps might change through a fraction of a degree). As the temperature is raised the volume of the confined gas increases and the level of the mercury in the left-hand tube, therefore, goes down. Raise the tube B, so that the level of mercury C, is the same as before, thus keeping the volume of the confined air constant. Note the mercury level in the open tube B and the reading of the thermometer (t_2°C). Raise the temperature again and proceeding as before take the readings in the tube B and on the thermometer T. Repeat the process and take a number of readings until the temperature of the bath rises to about 90°C. Repeat the experiment as the bath cools. Read the barometer at the end of the experiment. The mean observed height represents the mean atmospheric pressure during the experiment. Tabulate the readings. Plot the pressures against the corresponding temperatures of the confined air. Draw a smooth curve and, producing it backwards, find the value of the pressure of the air at 0°C. Calculate within the observed limits the mean increase of pressure of the air for 1°C rise. The ratio of the increase of pressure for a rise of 1°C to the pressure at 0°C, gives the coefficient of increase of pressure of the confined mass of air at constant volume.

Practical example.—

Obs. barometric ht. in the beginning	= 73·61 cm.
Obs. barometric ht. at the end (nearly 2 hrs. after).	= 73·44 ,,
Mean ht.	= 73·5 (25) cm.

No.	Temperature of bath. °C.	Const. Mercury level cm.	Mercury level in open tube cm.	Difference in levels cm.	Pressure of confined air cm.
1	25·7	26·0	25·4	−0·6	72·9
2	42·6	,,	29·4	3·4	76·9
3	56·2	,,	32·8	6·8	80·3
4	67·0	,,	35·5	9·5	83·0
5	77·0	,,	38·0	12·0	85·5
6	83·0	,,	39·4	13·4	86·9
7	71·7	,,	36·9	10·9	84·4
8	63·0	,,	34·7	8·7	82·2
9	48·2	,,	30·6	4·6	78·1
10	34·0	,,	27·4	1·4	74·9
11	30·5	,,	26·7	0·7	74·2

Rate of increase of pressure $= \dfrac{86{\cdot}9-72{\cdot}9}{83{\cdot}0-25{\cdot}7} = \dfrac{14}{57{\cdot}3}$ cm. per 1°C.

Pressure at 0°C read from the graph .. 66·7 cm.

$$\therefore \quad \alpha = \frac{14}{57{\cdot}3}\times\frac{1}{66{\cdot}7} = 0{\cdot}00366(3).$$

The pressure of air at constant volume changes regularly with temperature and this property of the gas can be utilized in measuring temperature as in the previous exercise.

At 83°C the level of mercury in B was 39·4 cm. If the temperature were 100°C the level would have been $\left(39{\cdot}4+\dfrac{14}{57{\cdot}3}\times 17\right)= 43{\cdot}55$ cm. If the temperature were 0°C, the level would have been $\left(25{\cdot}4-\dfrac{14}{57{\cdot}3}\times 25{\cdot}7\right) = 19{\cdot}1$ cm. The length of the fundamental interval is therefore (43·55−19·1) = 24·45 cm. The length of a degree = 0·2445 cm. The pressure at 0°C being 66·7 cm. the pressure of the confined gas would be zero if the right-hand tube is lowered 66·7 cm. below the mark 19·1 cm. or if the tube is moved through a

length equivalent to $\frac{66 \cdot 7}{0 \cdot 2445} = 273$ degrees on the scale. Therefore, the temperature at which the pressure of the air at constant volume would be zero is 273° C below 0°C. This is the zero on the absolute scale of temperature.

How do you use the constant volume air thermometer to read the temperature of a bath?

Chapter VI

CALORIMETRY AND RADIATION

UNIT QUANTITY OF HEAT.

Bodies absorb heat when raised in temperature. Equal masses of different bodies require different quantities of heat to be raised through the same range of temperature. The quantity of heat required to raise a given body through a degree of temperature is called *the capacity for heat of the body.* The heat capacity of water is large enough compared with that of many other substances and, therefore, it is chosen as the standard of comparison. The ratio of the heat capacity of a body to that of an equal mass of water is called *the specific heat of the body.* The heat required to raise a gram of water through 1 °C (14·5—15·5°C) is taken as a unit of heat quantity in the C.G.S. system of units and is called a calorie. If a gram of water is heated through 1 °C at any other range, say from 30°C to 31 °C, the heat required is slightly different but may be taken to be nearly one calorie for all our purposes.

To determine the heat capacity of a given mass of a body, it is heated to a known temperature and is transferred to a known mass of water at a given temperature. The two are mixed together by a stirrer, so as to accelerate the equalization of temperature. The resulting final temperature is measured. The rise in temperature of the water is noted, the amount of heat absorbed by the water is calculated, and this heat must have been supplied by the body in cooling through the given range. The amount C given up in cooling through 1 °C is calculated. But the heat required to raise the temperature of a body through 1°C is equal to the heat given out by the same body when its temperature falls

through the same 1°C. So, the amount C represents the capacity for heat of the body. This is the plan generally adopted in calorimetry and is called *the method of mixtures.* Here, it has been assumed that the heat gained is equal to the heat lost, there being no loss of heat due to any cause. But in practice, heat is lost by radiation, etc., and the errors are to be suitably eliminated or corrected for.

Again, the heat given out by the hot body is also shared by the vessel containing the water and this heat absorbed by the vessel must be calculated and added to that absorbed by the water. For this purpose, the heat capacity of the vessel is determined by a separate experiment. This vessel which contains a liquid at a lower temperature always occurs in calorimetry, and is called a calorimeter.

WATER EQUIVALENT OF A CALORIMETER.

A calorimeter, sensitive thermometer, ordinary centigrade thermometer, measuring jar reading to a c.c., beakers with hot and cold water, etc., are the *apparatus required.*

The calorimeter consists of a small copper vessel put inside a bigger one, with cotton stuffed between the two. (Why?) A copper stirrer is also provided. Measure out w_1 c.c. of cold water into the calorimeter. Note the temperature t_1°C with the sensitive thermometer. Heat water in a beaker to about 50°C, as ascertained with the ordinary thermometer and when it is within the range of the sensitive thermometer, note the temperature of the hot water t_2°C, with it. Quickly pour some hot water into the calorimeter, stir briskly and note the resulting temperature, t_3°C, of the mixture with the sensitive thermometer. Measure out the water in the calorimeter, w_2 c.c.

The hot water added is (w_2-w_1) gm. and the fall in temperature is (t_2-t_3)°C. The heat lost is $(w_2-w_1)(t_2-t_3)$ calories. The heat gained by the cold water is w_1 (t_3-t_1) calories. The balance, $[(w_2-w_1)$ $(t_2-t_3)-w_1$ $(t_3-t_1)]$ calories, must

have been taken up by the calorimeter in rising from $t_1°$ to $t_3°C$, assuming that none others have shared the heat given out by the hot water. The capacity for heat of the calorimeter or the heat absorbed by the calorimeter to be raised through 1°C, is

$$\frac{(w_2-w_1)\ (t_2-t_3)-w_1\ (t_3-t_1)}{t_3-t_1} = w \text{ calories.}$$

But these w calories of heat can raise also w gm. of water through 1°C. The number of grams of water that can be raised through 1°C by the heat required to raise the calorimeter through 1°C is called the water equivalent of the calorimeter and is equal to w gm. This is obviously numerically equal to the heat capacity of the calorimeter.

Repeat the experiment 3 or 4 times and take the mean value for the water equivalent.

Which is the chief source of error and how does it affect the value ?

Are we justified in assuming that w c.c. weigh w gm. in the above experiment ?

Can an ordinary thermometer be used instead of a sensitive one ?

Practical example.—

Wt. of calorimeter = 49·0 gm.

The sensitive thermometer used was graduated to 0·1°C.

The ratio of the capacity for heat of m gm. of a body to that of the same mass of water is called the specific heat (s) of the body. Therefore, the water equivalent of m gm. of copper is numerically equal to the product of the specific heat of copper and the number of grams of mass of the copper or $w = m \times s$.

∴ The water equivalent (calculated) for the calorimeter = $49 \times 0·1$ = 4·9 gm.

This value is nearly half the observed value.

	I	II	III	IV
Vol. of cold water w_1	99 c.c.	90 c.c.	80 c.c.	93·5 c.c.
Initial temperature t_1°C ..	28·3°C	29·5°C	28·6°C	29·6°C
Temp. of hot water t_2	47·0 ,,	41·5 ,,	41·5 ,,	37·2 ,,
,, mixture t_3	32·6 ,,	32·5 ,,	32·6 ,,	31·8 ,,
Vol. of mixture w_2 ..	129·5 c.c.	125·0 c.c.	120·0 c.c.	137·0 c.c.
Vol. of hot water $(w_2 - w_1)$ ..	30·5 ,,	35·0 ,,	40·0 ,,	43·5 ,,
Heat lost by hot water	439·2 cal.	315 cal.	356 cal.	234·9 cal.
Heat gained by cold water ..	425·7 ,,	270 ,,	320 ,,	205·7 ,,
Heat gained by calorimeter ..	13·5 ,,	45 ,,	36 ,,	29·2 ,,
Rise in temperature of calorimeter ..	4·3°C	3·0°C	4·0°C	2·2°C
Water equivalent—w	3·1 gm.	15 gm.	9 gm.	13·3 gm.

Mean value = 10·1 gm.

It is obvious that the reading for the temperature of the hot water, noted above, should be higher than what it would be when the hot water is just mixing with the cold water, as some heat is lost by radiation while the water is being poured through cold air. This loss in temperature may be anything like 0·5°C to 1°C. So, if in one of the sets of observations taken above (3 for example), the temperature of hot water were 41°C and not 41·5°C, the heat lost by hot water would be $(40 \times 0{\cdot}5)$ 20 cal. less than before and so the water equivalent would be 4 gm. and not 9 gm. Each 0·1°C fall will lower the value of the water equivalent by unity. Here lies the chief source of error in this experiment.

If cold water is poured into hot water contained in the calorimeter, better results may be obtained. (Why ?)

SPECIFIC HEAT OF A SOLID.

The capacity for heat of equal masses of different substances is different. Water has a large heat capacity and hence the capacity of any body is compared with that of an equal mass of water and expressed as a fraction. This fraction is called the specific heat of the body.

Lead shot, sensitive thermometer, ordinary thermometer, calorimeter, test tube with a perforated cork, boiling water bath, balance, etc., are the *apparatus required* for determining the specific heat of lead shot.

Arrange the apparatus as shown in figure 45. Fill the test tube to nearly a third of its height with the shot and insert the ordinary thermometer into it. Weigh the calorimeter with the stirrer (w_1). Pour sufficient cold water into it. Find its weight again (w_2). Place it in the outer vessel with the sensitive thermometer in it. When after a time the temperature of the shot in the tube gets steady, note the temperature of the calorimeter t_1°C. Take the test tube out, remove the cork, note the temperature of the shot t_2°C and empty it quickly into the calorimeter. Stir and note the resulting temperature t_3°C. Weigh the calorimeter again (w_3). $w_1 \times C$ gm. is the water equivalent of the calorimeter where C is the specific heat of its material. Adding this to the mass of cold water ($w_2 - w_1$) gm. avoids a separate consideration of the calorimeter. This total mass of cold water gains $(w_2 - w_1 + w_1C)(t_3 - t_1)$ calories of heat, and this must have been supplied by $(w_3 - w_2)$ gm. of shot in falling through $(t_2 - t_3)$°C. Hence the heat capacity of the shot is

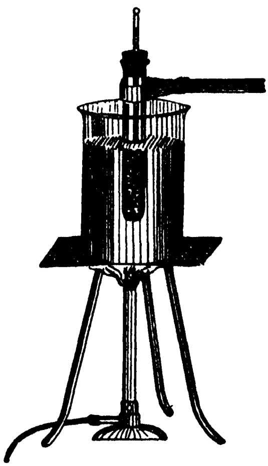

FIG. 45.

$$\frac{(w_2 - w_1 + w_1C)(t_3 - t_1)}{(t_2 - t_3)} \text{ cal.}$$

The capacity of an equal mass of water is $(w_3 - w_2)$ cal.

$$\therefore \text{ specific heat of the shot, } S = \frac{(w_2 - w_1 + w_1C)(t_3 - t_1)}{(t_2 - t_3)(w_3 - w_2)}$$

Take at least two sets of observations. On account of the

fall in the temperature of the shot, while being transferred into the calorimeter, the value obtained will be too low.

Practical example.—

The calorimeter is made of copper.
Sp. ht. of copper .. .. .. 0·095

	I	II
Wt. of calorimeter and stirrer w_1 ..	49·1 gm.	49·1 gm.
,, ,, with water w_2 ..	145·7 ,,	134·9 ,,
t_1 ..	29·0°C	28·9°C
t_2 ..	95·0 ,,	96·0 ,,
t_3 ..	30·0 ,,	29·9 ,,
w_3 ..	199·3 gm.	184·7 gm.
$w_3 - w_2$ ..	53·6 ,,	49·8 ,,
$w_1 \times C$ ..	4·7 ,,	4·7 ,,
Total water ..	101·3 ,,	90·5 ,,
Heat gained by the water and calorimeter	101·3 Cal.	90·5 Cal.
Capacity of shot	$\frac{101 \cdot 3}{65}$,,	$\frac{90 \cdot 5}{66 \cdot 1}$,,
Capacity of $w_3 - w_2$ gm. of water ..	53·6 ,,	49·8 ,,
Specific heat of lead shot	0·029	0·0275
Mean specific heat	0·028	

If the temperature of the shot is lowered by 5°C during its transference in II above, calculate the value of the specific heat of the shot (0·030).

The specific heat of liquids can be determined by this method, by using a solid of known specific heat.

CHANGE OF STATE.

Matter exists in three physical states, solid, liquid and gas. All bodies are conceived to be composed of minute particles of matter and are called molecules. These molecules have the same properties as the body and any further subdivision of these destroys their characteristic physical properties. These molecules are conceived to be at small distances apart and intermolecular forces are supposed to exist between them. This distance is quite small in the case of gases, more small in liquids and still smaller in solids.

Whenever a body passes from one state into the other, say from a liquid to a gaseous state, work is done in widening the distance between the particles. The energy necessary for this work is supplied to the body in the form of heat and the change of state is brought about. Again, if the process is reversed, *i.e.*, if a gas is condensed into the liquid state, heat energy is given out instead of being absorbed and as much is absorbed during one change as is given out in the other. This heat, which is absorbed or given out in the change of state, is not detected by the thermometer and is hence called *latent heat* of change of state. This quantity of heat is proportional to the mass of the body undergoing the change and so *the quantity of heat required to change the state of one gram of a body is defined as the latent heat of change of state of the body.*

This can be determined by the method of mixtures.

Calorimeter, sensitive and ordinary thermometers, balance, a watch provided with a seconds hand, steam generator and trap to catch condensed steam, etc., are the *apparatus required* to find the latent heat of condensation of steam.

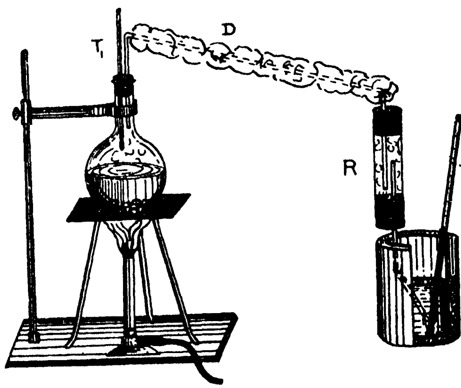

FIG. 46.

The steam generator consists of a flask (fig. 46), its mouth being fitted with a cork, through which pass the ordinary centigrade thermometer T_1 and the steam delivery tube D. This tube is bent into the shape shown in the figure and is surrounded by cotton wool (?). The other end passes into the top cork of the glass trap R and more or less dry steam escapes out of the jet tube at the bottom.

Weigh the inner vessel of the calorimeter and stirrer to a centigram (w_1) by the method of oscillations. Pour water and weigh again (w_2). Read the sensitive thermometer T_2 (t_1°C). When steam is freely escaping out of the jet tube, note the temperature t_2°C on T_1. Introduce the nozzle into the calorimeter and let steam condense, noting the time of condensation, until the temperature rises by nearly 10°C. Be stirring the contents of the calorimeter as the steam condenses. Remove the calorimeter and note the temperature t_3°C of the mixture. Cool the calorimeter for the same interval of time—the time of condensation—and note the temperature t_4°C on T_2. Add half the difference (t_3-t_4) to the observed maximum temperature t_3°C. This is the final temperature of the mixture corrected for cooling.* Cool the mixture and weigh again (w_3). w_3-w_2 gm. of steam condense into water at t_2°C and this water cools to the corrected maximum temperature of the mixture $\left(t_3+\frac{t_3-t_4}{2}\right)$°C. The heat gained by the calorimeter and cold water minus the heat given out by the condensed water in cooling from t_2°C to that of the mixture is the heat given out by w_3-w_2 gm. of steam while condensing into water. The heat given out by one gram of steam condensing into water without change of temperature can therefore be calculated and the latent heat of condensation of steam at t_2°C can thus be obtained.

$$L=\frac{(w_2-w_1+w_1\times C)(t_3+\frac{t_3-t_4}{2}-t_1)-(w_3-w_2)\left[t_2-\left(t_3+\frac{t_3-t_4}{2}\right)\right]}{w_3-w_2}$$

Take two sets of observations and note the mean calculated value of L. In spite of the trap R, the steam carries condensed

* The reasoning underlying the correction is explained in a subsequent exercise on Newton's Law of Cooling.

water with it and hence the value obtained will be low. (Why ?) Why are the masses in the exercise weighed correct to 0·01 gm. ?

Practical example.—

	I	II
Wt. of calorimeter empty, w_1 ..	49·05 gm.	49·05 gm.
,, ,, with cold water, w_2	151·10 ,,	147·24 ,,
Initial temperature, t_1	27·6°C	28·6°C
Time of passage of steam ..	2 min.	2 min.
Obs. maximum temperature, t_3 ..	46·0°C	45·8°C
Temp. after cooling for 2′ more, t_4 ..	45·0 ,,	45·0 ,,
Corrected maximum temperature ..	46·5 ,,	46·2 ,,
Temp. of steam, t_2	99·0 ,,	99·0 ,,
Final mass, w_3	154·41 gm.	150·11 gm.
Mass of condensed steam ..	3·31 ,,	2·87 ,,
,, cold water	102·05 ,,	98·19 ,,
Water equivalent of calorimeter ..	49·05 × 0·095 = 4·66 gm.	4·66 ,,
Total mass of cold water ..	106·71 ,,	102·85 ,,
Heat gained by cold water ..	106·71 × 18·9 = 2017 cal.	1810·16 cal.
Heat given out by condensed steam to cool to the temp. of mixture	3·31 × (99−46·5) = 173·8 cal.	151·54 ,,
Latent heat at 99°C	$\frac{2017-173\cdot8}{3\cdot31}$ = 556·8 cal.	577·9 ,,

Mean value for L = 567 calories.

The steam issued vigorously and when the nozzle was introduced into and taken out of the calorimeter, very small quantities of water were splashed out and this water together with the small quantities removed on the jet tube and the thermometer go to compensate for the error introduced by the passing of slightly wet steam into the calorimeter.

What do you notice in the reading of the thermometer T_1 when the water in the flask is briskly boiling ?

Is the boiling point the same for all pressures and is the latent heat of steam the same at all boiling points ?

NEWTON'S LAW OF COOLING.

Heat is always transferred from hot to cold bodies. Conduction, convection and radiation are the three principal modes of transference of heat. The hot body is then said to cool and the cooling is affected generally by all the three modes operating at the same time. If the first two causes of cooling are eliminated as far as possible, the rate of cooling, due to radiation alone, can be studied. The quantity of heat radiated by a hot body in a second is called the rate of cooling of the body in that second. This rate depends, among other things, on the extent of the radiating surface. The quantity of heat radiated per square centimetre of the surface is called the coefficient of emission of the body. This coefficient is experimentally found to depend upon (1) the nature of the radiating surface, (2) the difference in the temperatures of the radiating body and the surrounding atmosphere. If the extent and nature of surface of a hot body and the temperature of the surrounding atmosphere are kept constant, the rate of cooling is found to be, more or less, directly proportional to the excess of temperature of the body over that of the surrounding atmosphere. This was first stated by Newton, and hence it is known as Newton's Law of Cooling.

A copper calorimeter fitted with a wooden cover provided with two holes, one for the thermometer and the other for the stirrer, another thermometer, hot water bath, asbestos pad, stop-watch, etc., are the *apparatus required* to verify Newton's Law of Cooling.

Pour water at about 90°C into the calorimeter nearly to the top and place the wooden cover in position, with the thermometer and stirrer slipped into the holes. Place it on the asbestos pad (?). It saves time if the outer surface of the calorimeter is coated black (?). Put up the other thermometer in the air very nearby. Stir the water well and note the temperature at the end of every minute and

when the temperature falls considerably, at the end of every two minutes.

If the temperature falls from t_1 to t_2°C in a minute, the mean rate of fall of temperature of the body is $\frac{t_1 - t_2}{60}$ degrees per second at the mean temperature $\frac{t_1 + t_2}{2}$. If t°C be the temperature of the atmosphere, the excess of this mean temperature over t is $\left(\frac{t_1 + t_2}{2}\right) - t$ and $\frac{t_1 - t_2}{60} \times \frac{1}{\left(\frac{t_1 + t_2}{2}\right) - t}$ is the ratio of the rate of fall of temperature to the excess. Calculate such ratios at different temperatures of the body. These ratios will be found to be nearly constant. It follows, therefore, that the rate of fall of temperature is proportional to the excess of temperature. But, if θ be the rate of fall of temperature and m be the mass of water including the water equivalent of the calorimeter, then the rate of cooling of the calorimeter (R) is $m \times \theta$ calories per sec. and so the rate of cooling is directly proportional to the rate of fall of temperature of the body. Therefore, it follows from the results of the experiment that the rate of cooling (R) is directly proportional to the excess of temperature of the radiating body over that of the surrounding atmosphere. This verifies the law. You find that the ratios agree better for smaller differences in temperature between the hot body and the surrounding space.

Practical example.—

Time.	Temperature.	Rate of fall of temp. °C per sec.	Mean temp. °C.	Temp. of atmosphere.	Excess °C.	$\frac{\text{Rate}}{\text{Excess}} \times 10^6$
3 hr. 6′	85					
7	84	$\frac{2}{60}$	83		48·0	694
8	82					
9	80·3	$\frac{3·2}{120}$	80·4		45·4	587
10	78·8					
11	77·4	$\frac{2·6}{120}$	77·5		42·5	510
12	76·2					
13	75·0	$\frac{2·4}{120}$	75·0		40·0	500
14	73·80					
15	72·7	$\frac{2·1}{120}$	72·75		37·75	464
16	71·70			35°C		
17	70·7	$\frac{1·9}{120}$	70·75		35·75	443
18	69·80					
19	68·9	$\frac{1·8}{120}$	68·9		33·9	442
20	68·0					
22	66·1	$\frac{1·7}{120}$	65·25		30·25	468
24	64·4					
26	63·0	$\frac{1·5}{120}$	62·25		27·25	459
28	61·5					
30	60·2	$\frac{1·3}{120}$	59·55		24·55	441
32	58·9					
34	57·7	$\frac{1·2}{120}$	57·1		22·1	452
36	56·5					

The law may also be verified graphically.

A METHOD OF APPLYING COOLING CORRECTION.

One of the initial temperatures of the calorimeter used in the determination of the latent heat of steam was 27·6°C and was very nearly the same as that of the outside air. The tempera-

ture gradually rose to 46°C in two minutes. During this time the calorimeter had been cooling at a gradually increasing rate, which reached the maximum at 46°C. The initial rate of cooling was zero and the maximum was proportional to the maximum difference of temperature, 18·4°C. Therefore, the mean rate of cooling, as well as the mean rate of fall of temperature, correspond to the rate at the mean temperature $\left(27{\cdot}6+\frac{18{\cdot}4}{2}\right)=36{\cdot}8$°C. Hence the total fall in temperature during the two minutes of the passing of the steam is the fall that would occur if the calorimeter were kept steady at 36·8°C during that time. But the rate of fall at the observed maximum temperature 46°C is twice the rate at the mean temperature according to Newton's Law of Cooling. Hence the fall in temperature of the calorimeter during the two minutes of the passing of steam is half the observed fall for the same 2′ at or near the maximum observed temperature.

The cooling correction for the calorimeter can be eliminated by starting with an initial temperature of the calorimeter which is as much below that of the surrounding atmosphere, as the final temperature is above. (Why ?)

SPECIFIC HEAT OF A LIQUID BY THE METHOD OF COOLING.

Apparatus required.—All the apparatus of the previous exercise and a beaker containing the liquid.

Weigh the calorimeter empty (W_1 gm.) without the wooden cover. Take observations as before with water in the vessel and obtain the cooling curve of water and calculate the rates of fall of temperature (θ_1) at some definite points. Weigh the calorimeter (W_2 gm.) at the end with the water, removing the cover.

Empty the vessel, dry it, and fill it with the hot liquid to very nearly the same height. (?). Let the initial temperature

of the liquid be nearly the same as that of water. Proceed as before and plot the cooling curve of the liquid. Remove the wooden cover and thermometer and weigh the calorimeter (W_3) with the liquid and stirrer, after cooling the liquid sufficiently. Calculate from the curve the rates of fall of temperature (θ_2) at the same points. The water equivalent of the calorimeter and the stirrer is $W_1 \times$ sp. ht. of the material of the vessel $= W$ gm.

The heat lost by cooling per sec. by the calorimeter containing water $= [(W_2-W_1)+W] \times \theta_1$ cals., θ_1 being the fall in temperature per sec. at any definite temperature of the calorimeter. Similarly, the rate of cooling of the calorimeter, containing the liquid at the same temperature, is $[(W_3-W_1) \times S+W] \times \theta_2$; S is the sp.ht. of the liquid, θ_2 is the rate of fall of temperature.

The nature and extent of the radiating surface and the temperatures of the calorimeter and the surrounding atmosphere are the same in both cases. The rate of cooling of the calorimeter under similar conditions must therefore be the same, whatever be the liquid with which it is filled.

$$\therefore \; [(W_2-W_1)+W] \times \theta_1 = [(W_3-W_1)S+W] \times \theta_2$$

$$\therefore \; [(W_2-W_1+W)] \times \theta_1 = (W_3-W_1)S \times \theta_2 + W.\theta_2$$

$$\therefore \; S = \frac{(W_2-W_1+W)\theta_1 - W.\theta_2}{(W_3-W_1)\theta_2}$$

Calculate the value of the specific heat at each of the definite temperatures chosen and take the mean value.

Practical example.—

The liquid used was cocoanut oil.

Temperature of the outside atmosphere = 35°C.

$W_1 =$ 45·6 gm.	Wt. of water $W_2 - W_1$		= 119·8 gm.
$W_2 =$ 165·4 ,,	,,	oil $W_3 - W_1$	= 106·9 ,,
$W_3 =$ 152·5 ,,	Water equivalent		
	$W = 45{\cdot}6 \times 0{\cdot}093$		= 4·3 gm.

Calorimeter with oil inside.

Time.	Temperature.
3 hr. 42′	85° C
43	82·3
44	79·6
45	77·3
46	75·4
47	73·3
48	71·4
49	69·9
50	68·3
51	67·0
52	65·7
54	63·3
56	61·0

Values for θ_1 were taken from the curve of the previous exercise. θ_2 is obtained from the curve given in fig. 47.

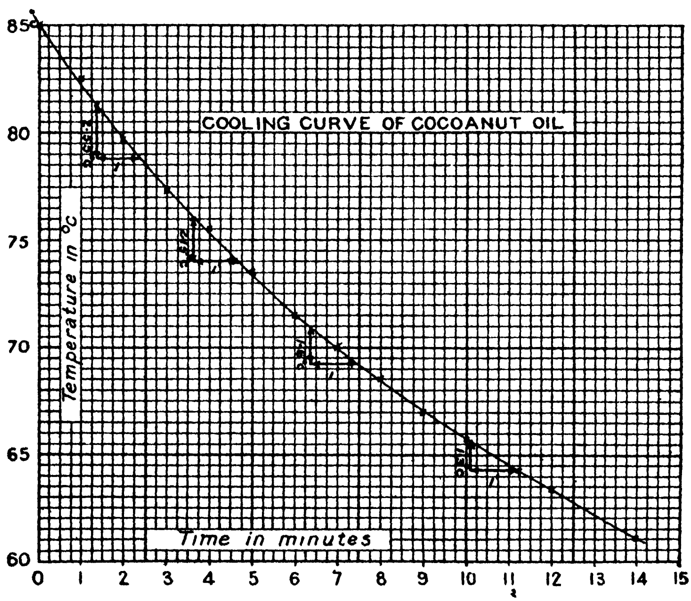

FIG. 47.

Temperature of calorimeter °C.	Rate of fall of temperature in °C per 2 minutes.		Sp. ht. of cocoanut oil S
	Oil θ_2	Water θ_1	
80	5·1	3·25	$\dfrac{124{\cdot}1\times3{\cdot}25-4{\cdot}3\times5{\cdot}1}{106{\cdot}9\times5{\cdot}1}=0{\cdot}70$
75	4·3	2·6	$\dfrac{124{\cdot}1\times2{\cdot}6-4{\cdot}3\times4{\cdot}3}{106{\cdot}9\times4{\cdot}3}=0{\cdot}66$
70	3·2	2·0	$\dfrac{124{\cdot}1\times2{\cdot}0-4{\cdot}3\times3{\cdot}2}{106{\cdot}9\times3{\cdot}2}=0{\cdot}69$
65	2·6	1·7	$\dfrac{124{\cdot}1\times1{\cdot}7-4{\cdot}3\times2{\cdot}6}{106{\cdot}9\times2{\cdot}6}=0{\cdot}72$

Mean value of S = 0·69

EMISSIVE POWER.

The coefficient of emission of a body depends upon two conditions, (1) the nature of the radiating surface and (2) the difference in the temperatures of the radiating body and the surrounding atmosphere. The effect of the second condition when the first is kept constant has been studied already under Newton's Law of Cooling. The effect of the nature of surface when the second condition is kept constant, *i.e.*, *how the rate of cooling varies with the nature of the radiating surface can be now studied in the following manner :*

Two calorimeters of the same size, shape and water equivalent, provided with wooden covers, thermometers, stirrers, asbestos pads, hot water bath, watch, etc., are the *apparatus required.*

Let one of the calorimeters be blackened outside and the other well polished. Pour water at a high temperature into the calorimeters to the same height, and fit the wooden tops. Stir and note separately the intervals of time taken by the calorimeters to fall through the same ranges of temperature, say of 5°C. Take 3 or 4 sets of observations.

If the mass of water poured (m) and the water equivalent (W) of the calorimeters are equal, the quantities of heat (q)

given out by the calorimeters must be equal, if the range of fall of temperature (t) for both is the same.

$$q_1 = (m_1+W_1)t_1 \text{ cal.}$$
$$q_2 = (m_2+W_2)t_2 \text{ cal.}$$
$$\text{if } m_1 = m_2,\ W_1 = W_2 \text{ and } t_1 = t_2;$$
$$q_1 = q_2 = q = (m+W)\,t.$$

If the times taken (T_1 and T_2 sec.) by the calorimeters to cool through the same range of temperature are different, the rates (R_1, R_2) of cooling would also be different and they are inversely proportional to the times.

The rate of cooling $R_1 = \frac{q}{T_1}$ cal. per sec. for the 1st calorimeter.

The rate of cooling $R_2 = \frac{q}{T_2}$ cal. per sec. for the 2nd calorimeter

$$\therefore \quad \frac{R_1}{R_2} = \frac{T_2}{T_1}$$

Calculate from the observations made the ratio of the rate of cooling of the polished surface to that of the black surface for each set of observations and you get nearly the same fraction. You find that the rate of cooling of the black surface is greater than that of the polished surface. This constant ratio or fraction obtained measures *the emissive power of the polished surface.*

How is the ratio altered if the polished surface is tarnished ?

Why is a black surface chosen as a standard of comparison ?

How is it that the surfaces of the thermo-flask are very finely polished and the inside is mirrored ?

Which do you prefer, a white or a black dress, while out on the field in the summer sun ?

Practical example.—

Bright surface.		Black surface.	
Temp.	Time.	Temp.	Time.
77°C	3 hr. 52′ 0″	77°C	3 hr. 51′ 0″
72 „	„ 57′ 50″	72 „	„ 55′ 10″
67 „	4 „ 4′ 50″	67 „	4 „ 0′ 35″
62 „	„ 14′ 0″	62 „	„ 7′ 10″

Range of fall in temp.	Time of cooling.		Coefficient of emission of bright surface *divided by* Coefficient of emission of black surface.
	Bright.	Black.	
77 to 72°C	350 sec.	250 sec.	$\frac{250}{350} = 0{\cdot}714$
72 to 67 „	420 „	325 „	$\frac{325}{420} = 0{\cdot}774$
67 to 62 „	550 „	395 „	$\frac{395}{550} = 0{\cdot}718$

Mean emissive power of the polished surface = 0·74.

MECHANICAL EQUIVALENT OF HEAT.

All bodies when left to themselves remain in the same state, the state being either of rest or of uniform motion in a straight line. This is the principle of *inertia*. A force when applied to a body changes that state and causes uniformly accelerated motion along the line of application of the force. If the point of application of a force is displaced through a length, work is said to be done and energy is the capacity to do work. The bob of an oscillating pendulum has energy. At the middle point of its path the energy of the bob is all kinetic and at the extremities it is all potential. At other points it is partly kinetic and partly potential; in varying proportions. The total energy, however, remains constant. Similarly in the case of a falling body, the sum of the two forms of energy at any point of its fall is constant. It is from such

facts as these that *the principle of the conservation of energy* has been enunciated. It states that *the total amount of the Energy in the Universe is a constant* or *Energy can neither be created nor destroyed.* When a falling body reaches the ground it comes to rest and hence has no kinetic energy and being on the ground, it has no potential energy also. What then has happened to the energy it possessed before reaching the ground? Strike flint against steel. A spark is produced and the energy of the stroke has disappeared. What has happened to it? The iron hoof of a running horse produces a spark against the metal road. Where has this spark of fire come from? Whenever we feel cold we rub our hands and feel warm. What has rubbing to do with the warmth produced? In all these cases, energy or work done must have taken up another form according to the principle of the conservation of energy. Energy of a completely different type is produced. In all the above illustrations mechanical energy has been transformed into the energy of heat. Not only can heat be produced by the expenditure of mechanical work but the reverse process, namely, the production of mechanical work by the expenditure of heat energy is also common experience. What is it that causes the mechanical motion of an express passenger train? You immediately think of the engine. But what is the motive power of the engine due to? Steam. How is steam produced? It is thus clear that heat is the cause of the mechanical energy of motion of the train. Again, what do you find when you enter a workshop or a factory? Mechanical work in different forms is turned out by the employment of engines. So you find around you a regular and constant transformation of heat into mechanical energy and *vice versa.*

Whenever this transformation occurs, a relation exists between the work done (W) and the heat produced (H). The ratio $\frac{W}{H}$ is found to be a constant and this constant

is known as the mechanical equivalent of heat and is represented by the letter J. In the C.G.S. system of units, J ergs of work has to be done to produce a calorie of heat.

Apparatus required to find the value of J.—Searle's apparatus, a sensitive thermometer, a watch, a few hectogram masses, retort stand, foot rule, balance, etc.

Searle's apparatus (fig. 48*a*) contains two truncated hollow

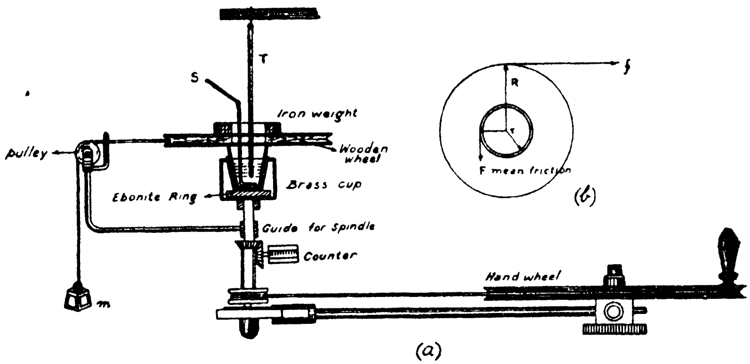

Fig. 48.

brass cups one fitting into the other. From the bottom of the outer cup project two metal pins which fit tightly into an ebonite ring which is screwed to the bottom of a surrounding brass cup. This brass cup is rigidly fixed to the vertical spindle of the apparatus. A leather band works round a hand-wheel and a brass wheel on the spindle. The revolutions of the spindle are registered on a counter attached as shown in the figure.

From the top of the inner conical brass cup project two steel pins which fit into holes in a wooden wheel. This wheel rests on the inner cup, and an iron ring-weight placed on the wheel produces the required pressure between the two cups. To a point on the edge of the wooden wheel is attached a fine string which passes over a guide-pulley and carries a few hundred grams mass (*m*). This pulley is supported on

a bent steel rod fixed to the outer framework of the apparatus. The tension of the string prevents the wooden wheel and therefore the inner brass cup, from revolving round the vertical axis, when the hand-wheel is worked. Thus, friction is caused between the inner surface of the outer cup and the outer surface of the inner cup. A drop or two of oil between the surfaces reduces the friction to a convenient value. The moments of the tension (f) of the string and of the mean friction (F) between the cups, about the vertical axis of the apparatus, are opposed to each other as shown in fig. 48*b*. These opposing moments would be equal in magnitude when the speed of the hand-wheel is so regulated that the hanging mass is kept at a mean constant height from the ground level.

The frictional force F dynes is overcome through a distance of $2\pi r$ cm. for every revolution of the spindle, where r is the mean radius of the conical vessels at the surfaces of contact. If n is the total number of revolutions made, the work (W) done against friction is $2\pi r \times n \times \mathrm{F}$ ergs. Since the moments of the two opposed forces are equal in magnitude, under the condition stated above, $\mathrm{F} \times r = f \times \mathrm{R}$ where R is the radius of the wooden wheel. Therefore, the work done W against friction is $2\pi n \times f \times \mathrm{R}$, ergs. Of these quantities, n is registered by the counter, $f = m.g.$ dynes, and R can be measured with a scale.

The mechanical energy thus spent (W) is completely converted (?) into heat energy which raises the temperature of the brass cups and the contents. The thermometer T registers the rise in temperature. The ebonite ring at the bottom of the outer conical brass cup prevents heat from being conducted away. Thus the equivalent heat (Q) can be determined calorimetrically, and equated to the energy spent. The ratio $\frac{\mathrm{W}}{\mathrm{Q}}$ gives the number of units of mechanical energy required to produce a unit of heat energy and is called

the mechanical equivalent of heat (J). In the C.G.S. system, it is expressed in ergs per calorie.

Practical example.—

$2R = 25$ cm., $m = 100$ gm., $n = 673$, sp. ht. of brass $s = 0{\cdot}092$.

$g = 980$ cm. per sec. per sec.

(M) $= 203{\cdot}4$ gm. = mass of the brass cups with stirrer.

$(M_1) = 18{\cdot}0$ c.c. = Vol. of water in the inner cup.

30·1°C = Initial temp. of the water.

33·0 „ = Final temp. observed.

10 min. = Time of experiment.

32·0°C = Temp. 10′ after the final temperature was observed.

33·5 „ = Corrected maximum temp.

Heat gained by the calorimeter, $H = \theta\,(M_1 + Ms)$ cal., θ = rise in temp.

Work done against friction, $W = 2\pi n \times f \times R$ ergs,

$= 2\pi n\ mg.\ R$ ergs.

$$\therefore\ J = \frac{W}{H} = \frac{2\pi n\ mg.\ R}{\theta\,(M_1 + Ms)} \text{ ergs per calorie.}$$

$$= \frac{2\pi \times 673 \times 100 \times 980 \times 12{\cdot}5}{(33{\cdot}5 - 30{\cdot}1)\,(18 + 203{\cdot}4 \times 0{\cdot}092)} = 4{\cdot}15 \times 10^7 \text{ ergs per calorie.}$$

An erg is found to be too small a unit of work in practice and 10^7 ergs is found to be a convenient practical unit of work. This practical unit is called a Joule, after James Prescott Joule, who was the pioneer in conducting classical experiments to determine the value of J by mechanical methods. The apparatus now used for this determination in laboratories is a modification of his original design.

Chapter VII

HUMIDITY AND THERMAL CONDUCTIVITY

SATURATION PRESSURE.

The atmosphere round us is never absolutely dry, but contains varying quantities of water vapour. This vapour exerts a partial pressure which, together with that of the air, is equal to the atmospheric pressure.

In an enclosed volume of air in which a quantity of water is present the space takes up as much water vapour as it can hold at that temperature, and it is said to be saturated with the vapour. The space holds the maximum quantity of water vapour it can hold at that temperature, and hence the partial pressure of the vapour is called its maximum tension or its saturation pressure at that temperature. If the temperature of the space is raised, the capacity to hold aqueous vapour increases and some more of the water vaporises and the pressure of the vapour reaches the maximum at the new temperature. If there is no water present and the temperature is raised, the space cannot hold all the water vapour it can at that temperature and the space is said to be unsaturated. The degree of saturation decreases with the rise of temperature. If, on the other hand, we start with a volume of air unsaturated with aqueous vapour and cool it gradually, a temperature will be reached at which the pressure exerted by the water vapour, which remains nearly the same throughout, becomes the maximum pressure for that temperature and any further cooling causes the water vapour to condense and dew appears. The remaining quantity of the vapour is just enough to saturate the space at the lowered temperature. If the temperature is still further

reduced, more of the vapour condenses into dew, because the quantity of water vapour required to saturate the same volume decreases with temperature. The pressure of the water vapour therefore falls, but always continues to be the maximum for the temperature. It is obvious that the space remains in an unsaturated condition till the dew first appears; only, the degree of saturation gradually increases to a maximum. The temperature at which dew appears is called the *dew point*. The saturation pressure of the aqueous vapour at the dew point is the initial pressure of the vapour present in the air.

RELATIVE HUMIDITY.

The state of the atmosphere in regard to the water vapour it contains (the degree of saturation) is called the hygrometric state or the relative humidity and is measured by the ratio of the quantity of aqueous vapour (q) which a given volume of air contains at a given temperature to the quantity (Q) that would saturate it at the same temperature. Air containing aqueous vapour approximately obeys Boyle's Law so long as the air is not saturated with the water vapour and the quantities of water vapour in a given volume of air at the same temperature are proportional to the pressures they exert until the air just becomes saturated. Therefore the ratio q/Q or the relative humidity can also be measured by the ratio of the pressure (p) of the aqueous vapour actually present in the air at any temperature to the maximum pressure (P) at the same temperature.

The second method is the one usually employed. The pressure P is known from the table of maximum tension of aqueous vapour as the temperature of the atmosphere is known and the value p is to be determined experimentally. This is done in two ways. (I) The dew point is first determined and the value of the maximum tension at the temperature of the dew point is found from the tables and this

is p. (II) The actual quantity of water vapour present in a litre of air in the atmosphere is determined by drawing the air through drying tubes. The difference in the masses of the tubes before and after the experiment is known, and the value of p is worked out from the mass of aqueous vapour present in a litre of air thus found, by the application of the gas laws.

The instruments employed in determining the humidity or the hygrometric state of the atmosphere are called hygrometers and those employed in the first method are called the *dew point hygrometers* and those used in the second are called the *chemical hygrometers* as the aqueous vapour is absorbed by a chemical process. *The method of determining the humidity of the atmosphere with a Regnault's hygrometer will now be described.*

The *apparatus required* are a Regnault's hygrometer and an aspirator.

This consists of two glass tubes (fig. 49), the lower portions of which S, S, are of thin silver plate. The test-tubes are secured in position to the two arms of the tube L by thick rubber tubing. The right-hand test-tube is closed with a rubber cork, through which pass a thermometer T_1 and a bent tube D open at both ends. The lower end of this tube and the bulb of the thermometer dip into ether at the bottom of the test-tube. A thermometer T_2 is held in the other test-tube which is empty. The silver thimbles are well polished. L is a tube fixed to the stand P and is connected by a rubber tube to the aspirator A, which is provided with two stop cocks C_1, C_2. A is filled with water and when the two cocks are open, water runs down and the outside air rushes through the open end of D, bubbles through ether, passes through L and fills the top of the aspirator. The hollow head of the tube L is closed from the left-hand arm as indicated at B. (Why?) As the air bubbles through ether the liquid evaporates and the heat required for the change

of state is taken from the liquid and the silver bottom. The temperature of the liquid falls rapidly, and silver being a

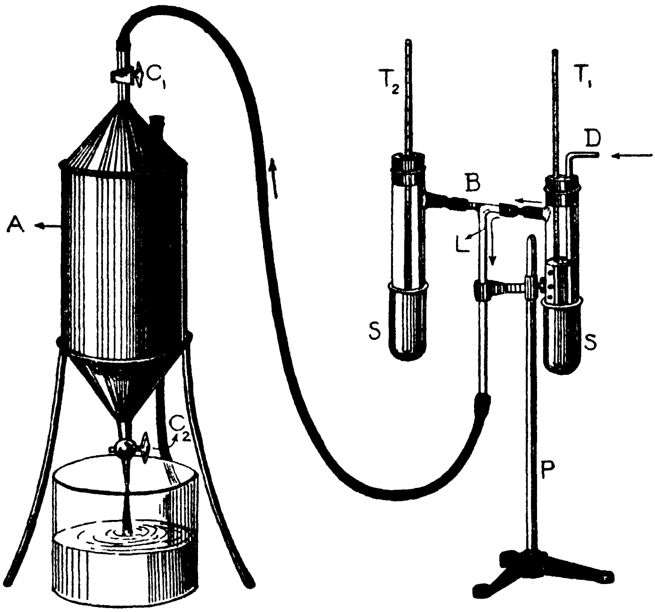

Fig. 49.

good conductor of heat readily takes up the temperature of the liquid. The air round the silver thimble also cools and a temperature will soon be reached when the air just becomes saturated with the water vapour it contains. Dew just begins to appear and the bright silver surface gets dim. The change in brightness is easily judged * in comparison with the other silver surface which is always bright. When the dew just appears read T_1, turning the stop cock C_2. The temperature of the cooled air along with that of the other gradually rises by drawing in heat from the neighbouring

* The formation of dew can be easily detected with a small magnifying lens.

hot air and after a time the dew on the silver surface just completely disappears. Read the thermometer T_1 once again and take the mean. Repeat a number of times. The mean of all these readings can be taken as the dew point. The reading on T_2 gives the temperature of the atmosphere. Read the pressures p and P at the two temperatures from the table giving the maximum tension at different temperatures and, if necessary, calculate them by the method of interpolation. The ratio gives the humidity.

An alternative method is to employ a wet and dry bulb thermometer.

Apparatus required is a wet and dry bulb thermometer or Mason's hygrometer.

This consists of two thermometers hung side by side as shown in fig. 50 with their bulbs near each other. Round the bulb of one of them is tied a washed and wet clean muslin piece, the lower portion of which dips into a bottle of water and thus the bulb is always kept wet. Water will be evaporating always from the wet bulb and the rate of evaporation depends obviously on the hygrometric state of the atmosphere, increasing with the dryness of the weather. The reading on the wet bulb thermometer is therefore lower than that on the dry bulb thermometer and this difference in the readings is great in dry weather and small in wet weather.

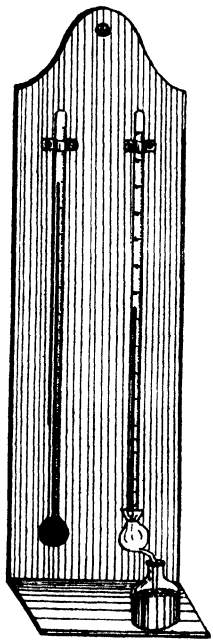

FIG. 50.

Take two sets of readings on the thermometers at an interval of five minutes. Take the mean reading on each thermometer. The dew point can be calculated from these mean readings with the help of an empirical table of factors

necessary to multiply the excess of the reading of the dry bulb over that of the wet, to give the excess of the temperature of the atmosphere over that of the dew point for the temperatures that ordinarily occur. Such a table is given below.

Glaisher's Factors.

Dry bulb temp.	0	1	2	3	4	5	6	7	8	9
0° C	3·32	2·81	2·54	2·39	2·31	2·26	2·21	2·17	2·13	2·10
10 „	2·06	2·02	1·99	1·95	1·92	1·89	1·87	1·85	1·83	1·81
20 „	1·79	1·77	1·75	1·74	1·72	1·70	1·69	1·68	1·67	1·66
30 „	1·65	1·64	1·63	1·62	1·61	1·60	1·59	1·58	1·57	1·56
70°F	1·77	1·76	1·75	1·74	1·73	1·72	1·71	1·70	1·69	1·69
80 „	1·68	1·68	1·67	1·67	1·66	1·65	1·65	1·64	1·64	1·63
90 „	1·63	1·62	1·62	1·61	1·60	1·60	1·59	1·59	1·58	1·58
100 „	1·57									

These are called Glaisher's Factors. Mr. Glaisher took many thousands of observations with the wet and dry bulb hygrometer in Greenwich, India and Toronto, and from simultaneous readings of a dew point hygrometer (Daniell's) drew up the table of factors.

Saturated vapour pressure of water is expressed in mm. of mercury at 0°C; $g = 980{\cdot}67$ cm. per sec. per sec.

Temp.	0	1	2	3	4	5	6	7	8	9
0°C	4·58	4·92	5·29	5·68	6·10	6·54	7·01	7·51	8·04	8·61
10 „	9·21	9·84	10·51	11·23	11·98	12·78	13·62	14·52	15·46	16·46
20 „	17·51	18·62	19·79	21·02	22·32	23·69	25·13	26·65	28·25	29·94
30 „	31·71	33·57	35·53	37·59	39·75	42·02	44·40	46·90	49·51	52·26
	0	2	4	6	8	10	12	14	16	18
40°C	55·13	61·30	68·05	75·43	83·50	92·30	101·9	112·3	123·6	135·9
60 „	149·2	163·6	179·1	195·9	214·0	233·5	254·5	277·1	301·3	327·2
80 „	355·1	384·9	416·7	450·8	487·1	525·8	567·1	611·0	657·7	707·3

These figures are taken from Kaye and Laby's Physical and Chemical constants.

Practical example.—

Regnault's Hygrometer :—

Dew appearing.	Dew disappearing.	Mean.
10·8°C	11·6°C	11·2°C
10·4 ,,	11·2 ,,	10·8 ,,
10·3 ,,	11·2 ,,	10·8 ,,
10·4 ,,	11·4 ,,	10·9 ,,
11·0 ,,	11·2 ,,	11·1 ,,
	Mean dew point	11°C

Reading of the thermometer in the empty tube 33·2°C

$$\text{Relative humidity} = \frac{\text{saturation vapour pressure of water at } 11°C}{\text{saturation vapour pressure of water at } 33{\cdot}2°C}$$

$$= \frac{p}{P}$$

$p = 9{\cdot}84$ mm. from the Tables.

The value of P is obtained by interpolation in the following manner.

t°C	Pressure	Difference per 1°C	Difference for 0·2°C	At 33·2°C
33	37·59 mm.	2·16 mm.	0·43 mm.	38·02 mm.
34	39·75 ,,			

$$\text{Relative humidity} = \frac{9{\cdot}84}{38{\cdot}02} = 0{\cdot}259$$

Percentage humidity $= 25{\cdot}9$

Wet and dry bulb hygrometer :—

Readings were taken just before and after the above observati of the dew point.

	Wet bulb.	Dry bulb.
	69° F	94°F
	69·5 ,,	94 ,,
Mean	69·25°F	94°F

Factor for 94°F = 1·60

Excess of the dry bulb reading over that of the wet = 24·75°F

Excess of the dry bulb reading over that of the dew point = 1·6 × 24·75
= 39·6°F

Dew point $= 94-39{\cdot}6 = 54{\cdot}4°F = (54{\cdot}4-32)\times\frac{5}{9} = 12{\cdot}4°C$

Dry bulb reading $= (94-32)\times\frac{5}{9} \quad = 34{\cdot}4°C$

Max. aqueous vapour pressure at $12{\cdot}4°C = 10{\cdot}51+0{\cdot}4\times(11{\cdot}23-10{\cdot}51)$
$= 10{\cdot}80$ mm.

" " " " at $34{\cdot}4°C = 39{\cdot}75+0{\cdot}4\times(42{\cdot}02-39{\cdot}75)$
$= 40{\cdot}66$ mm.

Relative humidity $= \frac{10{\cdot}80}{40{\cdot}66} = 0{\cdot}266$

Percentage humidity $= 26{\cdot}6$

The wet and dry bulb hygrometer was carried from inside the laboratory into the outside verandah and hung there for about a quarter of an hour and the readings were again taken and the humidity calculated.

Wet bulb.	Dry bulb.
69°F	96·5°F

Factor for 96·5°F is 1·59

Dew point $= 96{\cdot}5-1{\cdot}59\times(96{\cdot}5-69) = 52{\cdot}78°F = 11{\cdot}5°C$

Dry bulb reads $(96{\cdot}5-32)\times\frac{5}{9} = 35{\cdot}8°C$

Relative humidity $= \frac{10{\cdot}20}{43{\cdot}92} = 0{\cdot}232$

Percentage humidity $= 23{\cdot}2$

Why is the humidity lower outside the room?

Does it mean that the quantity of aqueous vapour per litre of air is proportionately smaller outside the room?

How does the humidity generally vary during the different parts of the day and night and during different seasons of the year?

On dewy mornings and nights, what is the percentage humidity of the atmosphere?

What is the relation between the feeling of sultriness of the weather and the relative humidity?

The method of determining the relative humidity with the chemical hygrometer will be now described.

Apparatus required are a U shaped drying tube, a drying bottle, an aspirator of about 3 litres capacity with a tubulure

and stop-cock at the bottom, a centigrade thermometer, a measuring glass of capacity one litre, a balance, etc.

Pumice stone is broken into small bits, fried and dropped into strong sulphuric acid and is soaked for a day or two, underneath a desiccator. These pieces are afterwards dropped into the U tube (fig. 51) with tongs and the tube is

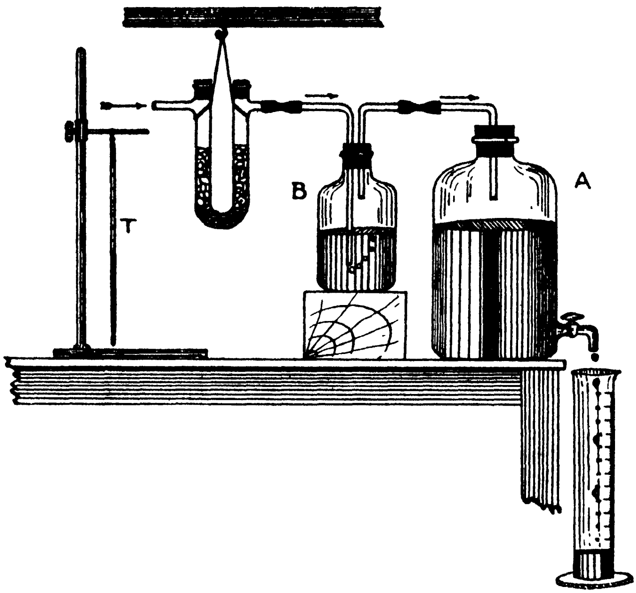

FIG. 51.

stoppered. The drying bottle (B) contains strong sulphuric acid and through the rubber cork closing its mouth, pass two glass tubes, one dipping into the bottom of the liquid and the other standing outside the surface. The aspirator (A) is filled with water and the measuring jar is put below the tap. The thermometer is hung by the side of the apparatus as shown in the figure.

Close the left-hand stop-cock of the U tube opening the other. Open the tap of the aspirator. If water does not continue to run out the apparatus is air-tight. (?). Close

the tap and the cock of the drying tube. Disconnect and weigh the U tube correct to a milligram by the method of interpolation. Connect it back and open the stop-cocks. Partly open the tap of the aspirator so that a slow (?) stream of air bubbles through the apparatus as judged from the bubbling in B.

Draw out 2 or 3 litres of water. Close the tap and the cocks. Weigh the tube once again to a milligram. The increase in the weight (M) is the mass of the water vapour actually present in the volume (V) of the air drawn out. The drying bottle B prevents water vapour from the aspirator from reaching the dry tube. It is assumed that the tube absorbs all the water vapour from the air drawn out. For greater accuracy another drying tube may be used in series. The drying tubes may as well contain calcium chloride in place of pumice stone. The stone offers a large surface and renders the absorption of aqueous vapour complete.

Calculate the mass of aqueous vapour (M/V) contained in a litre of the atmosphere. Let it be q gm. Let the temperature of the atmosphere be t°C. A table taken from Kaye and Laby's Physical and Chemical constants, giving the mass of water vapour in grams contained in a cubic metre of saturated air at 760 mm. total pressure for different temperatures, is given below. Read the value Q for t°C for a litre of the atmosphere. The ratio q/Q, according to the definition, gives the hygrometric state of the atmosphere.

Grams of water vapour that saturate a cubic metre (1000 litres) of air.

Temp.	0	1	2	3	4	5	6	7	8	9
0°C	4·84	5·18	5·54	5·92	6·33	6·76	7·22	7·70	8·21	8·76
10 ,,	9·33	9·93	10·57	11·25	11·96	12·71	13·50	14·34	15·22	16·14
20 ,,	17·12	18·14	19·22	20·35	21·54	22·80	24·11	25·49	26·93	28·45
30 ,,	30·04	31·70	33·45	35·27	37·18	39·18	41·3	43·5	45·8	48·2

Note.—The corresponding ratios q/Q and p/P for any two sets of temperatures can now be calculated and the degree of accuracy of the assumption made, namely $q/Q = p/P$ can be ascertained. After finding the dew point with the Regnault's hygrometer, the quantities q and Q can be looked for from the above table and the relative humidity determined. Do you find good agreement in the values obtained? q is now obtained not directly but from the knowledge of the dew point.

Alternatively, we may proceed in the following manner. Having obtained from the chemical hygrometer the quantity q gm. per litre, we can proceed to calculate the pressure p of the water vapour actually present in the atmosphere and determine the humidity with the help of the tables, used in the previous part of the exercise. p is the partial pressure of the water vapour in the atmosphere. The water vapour contained in a litre of moist air at $(273+t)°$ abs. weighs q gm. The volume of the vapour is the volume of the moist air which is one litre and its pressure is p. q gm. per litre is therefore the density of the aqueous vapour at t°C and p mm. pressure.

A litre of water vapour weighs

$$q \times \frac{(273+t)}{273} \times \frac{760}{p} \text{ gm. at N.T.P.}$$

This is the density of the vapour at N.T.P. Here we assume that the water vapour obeys the gas laws, which state that the density is inversely proportional to the absolute temperature and directly proportional to the pressure of the gas. 1·293 gm. is the mass of a litre of dry air at N.T.P. and 0·622 is the density of water vapour relative to that of air, at the same temperature and pressure.

A litre of water vapour weighs $1{\cdot}293 \times 0{\cdot}622$ gm. at N.T.P.

$$\therefore \quad q \times \frac{273+t}{273} \times \frac{760}{p} = 1{\cdot}293 \times 0{\cdot}622.$$

$$\therefore \quad p = \frac{q \times (273+t) \times 760}{273 \times 1{\cdot}293 \times 0{\cdot}622} \text{ mm. of mercury.}$$

q and t being observed, p is calculated. This is the saturation pressure at the dew point. Hence by a reference to the table giving maximum pressure of water vapour at different temperatures, and interpolating, if necessary, the dew point can be ascertained.

This is generally found to be nearly the same as the temperature at which q gm. of water vapour would saturate a litre of air as read from the last table.

Having found p as shown above, P can be read in the table of aqueous tensions against t°C and the ratio p/P determined. The values obtained by the two different methods will be nearly the same.

If the gas laws hold good in the case of water vapour, to the point of saturation, then at the temperature t°C, we have, as is clear from the above discussion

$$P = \frac{Q \times (273+t) \times 760}{273 \times 1{\cdot}293 \times 0{\cdot}622} \text{ mm. of mercury.}$$

P is the saturation pressure of vapour at t°C and Q is the mass of vapour contained in a litre of air saturated with water vapour at t°C. $\therefore \frac{q}{Q} = \frac{p}{P}$. Differences occur in the ratios, in practice, as the assumption made is not strictly correct. It might be interesting to note that the three instruments described in detail in the above are each an example of the three different classes of hygrometers acting under three different processes, namely, condensation, evaporation and absorption. In the first class of instruments, the temperature of condensation of the water vapour contained in the air is determined and the dew is made to appear on a polished, smooth surface. To this class belong the Daniell's, the Dines' and the Regnault's instruments and these are called the dew point hygrometers. Of these, the

last named is the most reliable and accurate, the cooling caused by the evaporation of the ether being regulated by the aspirator. In the second class, the action of the instrument depends upon the rate of evaporation of the water vapour from the surface of the wet bulb of a thermometer, into the air outside. In the third class, the water vapour in the air is absorbed by a chemical process. This class of instruments is necessarily slow in action and the determination of humidity or the fractional saturation of the air takes considerable time.

THERMAL CONDUCTIVITY.

The thermal conductivity of a substance is a measure of its ability to transmit heat through it. The quantity of heat that flows across the substance from one face of a centimetre cube to the opposite one in one second, when the temperature difference between the two faces is one degree centigrade, is called the thermal conductivity of the substance. When one end of a metal rod is surrounded by a body maintained at a constant high temperature it very soon acquires that temperature. The layers of the rod next to that end receive heat by conduction at a rate, say, Q calories per sec. These layers are surrounded by colder air and so a small fraction of Q is lost to the air through convection and radiation and the remaining goes to raise the temperature of the layers. The temperature of the rod thus gradually rises and with it the rate of loss of heat due to radiation, as this increases with the difference of temperature of the rod and that of the surrounding atmosphere. Thus the fraction of the heat that is conducted through the rod gradually decreases. A stage is reached when all the heat conducted to each layer is just enough to make up for the loss of heat due to cooling. Until this stage is reached, the temperatures at all points on the rod rise, though at gradually decreasing rates, but afterwards they get steady

at each point on the rod, though the temperatures at different points along the length of the rod are different, falling gradually towards the colder end. This end, if sufficiently distant from the source, would be at the temperature of the surrounding air. This stage is called *the stationary or the steady state* of the rod in regard to temperature and the earlier is called *the variable state.* During the steady state, the heat that passes any layer is all radiated along the rod from that point to the colder end.

When the ends of different rods of the same cross-section are maintained at the same high temperature—since the rate of flow of heat, due to conduction, depends upon the material of the rod—the distance along the rods from the hot end to the points where the temperatures are the same, say t°C, would be different, when the rods have reached their steady state. It can be proved that the thermal conductivities of the materials of such rods are proportional to the squares of such distances and hence their conductivities can be compared.

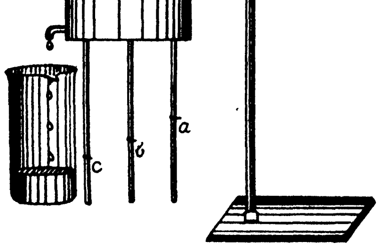

FIG. 52.

Apparatus required for carrying out such an experiment are Edser's apparatus, a steam boiler, millimetre scale, etc.

The apparatus consists of a brass vessel A (fig. 52) closed at both ends except for two small openings, one on the top and the other on one side at the bottom, to let in and let out steam respectively. The vessel is supported on a stand. To the bottom of the vessel are soldered three rods of copper (*c*), brass (*b*) and iron (*a*) of the same cross-section. Indexes made of brass wire are slipped on to them. A thin coating

of melted paraffin wax is given to the rods, which, after solidification, holds the indexes in position. Steam is let into the chamber A. The wax begins to melt and the indexes begin gradually to slide down with the melting wax. The temperature at which the indexes stand on each rod at any time is the melting point of the wax. (?). This point of temperature travels down the rods during the variable state and the indexes stand steady at different points on the rods, soon after the steady state is reached. Wait for a few minutes after the steady state is reached and measure the distances along the rods from the bottom of the vessel to the pointer. These are the distances along which the wax has melted on the rods. If K_c, K_b and K_a represent the thermal conductivities of the metals and l_1, l_2 and l_3 are the corresponding distances measured, then as stated above $K_c : K_b : K_a :: l_1^2 : l_2^2 : l_3^2$.

Practical example.—

$$l_1 = 21{\cdot}0 \text{ cm.};\ l_2 = 12{\cdot}5 \text{ cm.};\ l_3 = 8{\cdot}2 \text{ cm.}$$

$$K_c : K_b : K_a :: 21^2 : 12{\cdot}5^2 : 8{\cdot}2^2$$

$$\therefore \frac{K_c}{K_a} = \left(\frac{21}{8{\cdot}2}\right)^2 = 6{\cdot}5$$

$$\text{and } \frac{K_b}{K_a} = \left(\frac{12{\cdot}5}{8{\cdot}2}\right)^2 = 2{\cdot}3$$

The mean conductivities between temperature 18—100°C for these metals, obtained from standard tables, are

$K_c = 0{\cdot}91$ calorie cm^{-1} sec^{-1} temp^{-1}.
$K_b = 0{\cdot}26$,, ,,
$K_a = 0{\cdot}14$,, ,,

The calculated ratios are $\dfrac{K_c}{K_a} = 6{\cdot}5$ and $\dfrac{K_b}{K_a} = 1{\cdot}9$.

CHAPTER VIII

REFLECTION AND REFRACTION AT PLANE SURFACES

GENERAL.

Certain bodies like the sun, the stars, an electric lamp, and a candle flame emit light produced by them and are said to be *luminous*. Rays of light proceed from a luminous body and travel out in all directions along straight lines. Most bodies are not luminous. The whole landscape around us which is clearly visible during the day but not so in the dark is an example. Darkness is absence of light. A body can be seen only when rays of light proceeding from it fall on the eye. Whenever light from a body, like the sun, falls on ordinary bodies, they send back the light in all directions and become visible. This phenomenon of sending back light is called reflection. Some bodies reflect light *regularly in definite directions* and many others *diffuse* light *irregularly in various directions*. Surfaces which cause good regular reflection are called *mirrors*. Bodies like glass allow light to pass through freely and are said to be *transparent*. Others like stone, wood, and metals do not allow light to pass through and are said to be *opaque*. Substances like oiled paper and ground glass allow some light to pass through though objects cannot be seen through them. Such substances are said to be *translucent*. In all cases of reflection and transmission some fraction of the incident light is *absorbed* by the body.

Incident rays from a point-source after reflection or transmission generally converge to or appear to diverge from another point, which is called the *image*. In the former case the image is said to be *real* and in the latter *virtual*. Real

images can be received on a screen and it is impossible to do so with virtual images.

LAWS OF REFLECTION.

(1) The incident ray, the reflected ray, and the normal to the surface at the point of incidence lie in the same plane.

(2) The angle which the incident ray makes with the normal at the point of incidence is equal to the angle which the reflected ray makes with it.

In the case of reflection at a plane surface, it follows from the second law, that the virtual image formed is as far behind the surface as the source is in front of it. This may be proved in the following manner. ABC (fig. 53) is the reflecting surface. P is the point source at a perpendicular distance of AP from the surface. PB, BR and PC, CS are two sets of incident and reflected rays and BM and CN are the corresponding normals at the points of incidence. R and S meet at Q when produced backwards. Join QA. According to the second law of reflection $\angle PBM = \angle MBR$ and $\angle PCN = \angle NCS$. In the triangles PBC and QBC, $\angle PBC = \angle QBC$ and $\angle PCB = \angle QCB$ and BC is common and therefore the two triangles are similar. So $QB = PB$. Now in the two triangles ABP and ABQ, $\angle PBA = \angle QBA$, $QB = PB$ and AB is common; so the triangles are similar and $AQ = AP$. Since $\angle PAB$ is a right angle, $\angle QAB$ is another and PAQ is a straight line at right angles to AB.

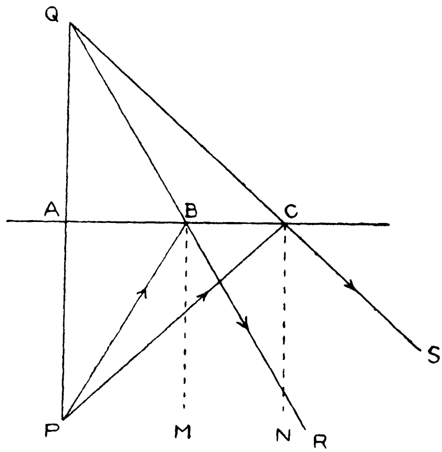

FIG. 53.

VERIFICATION OF THE LAWS OF REFLECTION AT A PLANE SURFACE.

Apparatus required are a plane mirror strip fixed to a vertical block of wood, a drawing board with white paper pinned on to it, four pins, instrument box, and a millimetre scale.

Draw a line MM_1 (fig. 54) on the drawing board. Place

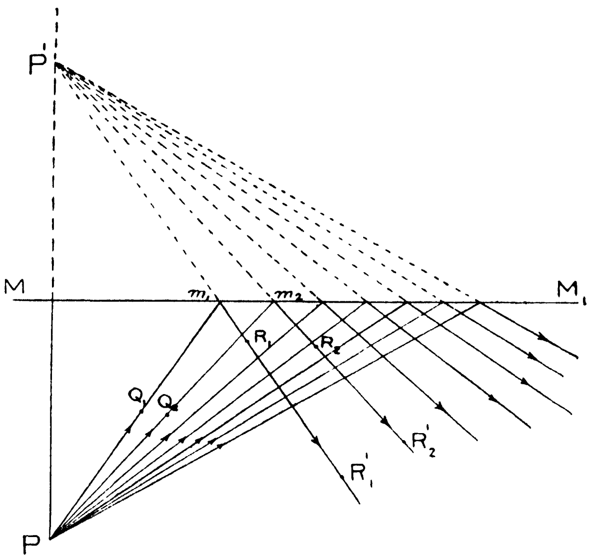

FIG. 54.

the mirror strip so that the edge of the mirrored surface (the back surface) lies on the line. Fix a pin P in front of the mirror towards one side. Fix another pin Q_1* nearer the mirror. Look into the mirror such that the two images appear to be exactly one behind the other. Look at the bottoms of the pins only as they may not be parallel all

* Let the distance between two pins in any one direction be not less than 7 or 8 cm. in all the exercises where the pin method is employed.

through their height. Fix a third pin R_1 in the line of sight such that it hides the two images. Fix also a fourth pin R_1' similarly and this hides the two images and the third pin. P is the object, PQ_1 an incident ray and R_1R_1' is the reflected ray. The position of the mirror is not to be disturbed till the end. Shift the second pin to another position Q_2 and trace R_2R_2' the direction of the corresponding reflected ray. Repeat the same for four or five more incident rays. Remove the mirror. Produce the incident and the reflected rays to meet the line MM_1. The points of intersection m_1, m_2, etc. of each pair of the incident and the corresponding reflected rays lie almost on MM_1. Measure with the protractor the angles of incidence and reflection for each set and tabulate them. You will find that the angle of incidence is nearly equal to the angle of reflection in each case and that the second law of reflection is thus verified. Again, the incident rays, the reflected rays, and the normals at m_1, m_2, etc. (the points of incidence) are all in the plane of the paper and this verifies the first law. The point of the pin P lying in the plane of the paper has been considered as the source of light. Any other point of the pin may be taken as the source of light and the above considerations hold equally well in the plane parallel to that of the paper and passing through the point.

Produce the reflected rays backwards. They intersect nearly at a point P′. Join PP′, cutting the trace of the mirror at M. The point P′ determines the position of the image of the object P. MP and MP′ would by measurement be found to be equal and the angles $P'MM_1$ and PMM_1 will be found to be right angles. This verifies the statement that the reflected image is situated as far behind the plane mirror as the object is in front of it.

It is not uncommon that we enjoy the moving landscape by looking through the window of a passenger train. Objects nearer to us (telegraph posts) appear to move away from

us in a direction opposed to that of the train and those farther off (distant trees) appear to move with us. Therefore the relative directions of apparent motion of the objects give us an idea of their relative positions. This apparent relative motion is called *parallax.* Of any two objects, that which moves with the eye is the farther.

Fix a pin in front of a mirror strip MM_1 at P (fig. 55) at a distance of not less than 7 or 8 cm. Take another pin whose height is greater than that of the strip and fix it behind the mirror at *a*, so that its top portion, the image, and the object pin appear to be in the same line. Move the eye to the right; suppose the image appears to move to the left and the second pin *a* to the right. This means that the image is nearer to the eye than *a*. Now fix the pin at *b*; suppose the image appears to move in the direction of the eye; the image is farther away than *b*. The position of the image is therefore between *a* and *b*.

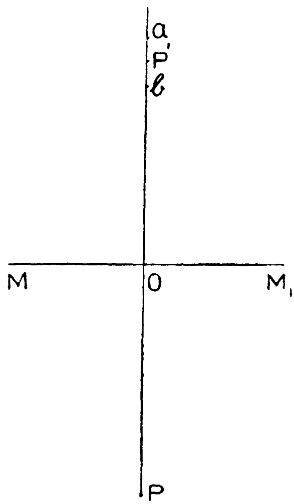

FIG. 55.

Try and fix the second pin between *a* and *b*, say at P′, such that there is no relative motion between this pin and the image of the first pin, when the eye is moved to the right or to the left; P′ coincides with the position of the image. PO produced will be found to pass through P′ and you find that OP is nearly equal to OP′. Care must be taken to erect pins at P and P′ vertically. Due to the imperfectness of this adjustment and the thickness of the pins some error will creep into the observations. Repeat the observations for two or three different distances and tabulate the results.

Practical example.—

Angle of incidence in degrees.	Angle of reflection in degrees.
26·0	27·0
33·0	32·0
37·0	36·0
40·5	40·5
43·5	43·5
45·8	45·5
57·0	56·7

Distance of the object from the mirror .. 7·0 cm.
„ image „ .. 7·0 cm.

In this experiment, the first five reflected rays when produced backwards met at P′ and the other two just a little nearer the mirror. This is due to the greater obliquity of the incident rays and the consequent effect of the thickness of the mirror strip.

The following results are obtained by the parallax method :—

Distances along the normal from the mirror:

Of the object, OP	Of the image, OP′
7·45 cm.	7·40 cm.
11·90 „	12·10 „
17·20 „	17·60 „

1. What happens if the edge of the front surface of the mirror is placed on the line MM_1 ?
2. What do you notice regarding the clearness of the image of the pin when looked at more and more obliquely ?
3. What is the effect of the thickness of the glass used ?
4. Does the position of the image change with that of the eye ?
5. What is the relation between the angle of incidence and the angle of deviation for any ray ?
6. What do you note regarding the size of the image, relative to that of the object ? Is the image formed real or virtual ?

INCLINED MIRRORS.

An object placed between two mirrors inclined at an angle has a number of images, the number depending on the angle between the mirrors.

Take two mirrors in rectangular frames hinged about a vertical axis and fixed to the centre of a semicircular base-board on which the mirrors rest edgewise. The edge of the circle is divided into degrees. Place a small object on the graduated circle between the mirrors. A number of images are seen and all of them appear arranged on the circumference of the full circle completed by reflection. Push the object in and notice that the number of images is the same and that they are arranged on the circumference of a smaller concentric circle. Carefully select the position of the eye so that the maximum possible number of the images is seen for the given angle between the mirrors. The following are some of the observations noted with such an arrangement.

No.	$\theta=(\alpha+\beta)$	N observed	$\frac{360}{\theta}$	α	β	Angle between successive images in degrees
1	90°	3	4	30	60	60, 120—C 120, 60—A
	90°	3	4	45	45	90, 90—C 90, 90—A
2	72	4	5	40	32	80, 64—C 64, 80—A →72
3	60	5	6	40	20	80, 40, 80—C 40, 80, 40—A
4	51·4	6	7	30·4	21	60·8, 42, 60·8—C 42, 60·8, 42—A →51·8
5	45	7	8	10	35	20, 70, 20, 70—C 70, 20, 70, 20—A
6	65	5	$5\frac{7}{13}$	45	20	90, 40—C 40, 90, 40—A R = 60
7	110	3	$3\frac{3}{11}$	70	40	140, 80—C 80—A R = 60
8	160	2	$2\frac{1}{4}$	70	90	140—C 180—A R = 40

The position of the object between the mirrors divides the angle into two parts α and β and these can be given any set of values. The particular values in the experiment tried are given in the above table. The positions of the images

and the object are read on the reflected and thus completed graduated circle. The angle between the object and the nearest and the first image in the clockwise direction and that between the first and the second images, etc. in the same direction are given in the last column of the table and are marked C. The same is done in the anticlockwise direction marked A.

In the first five observations θ is an exact sub-multiple of 360 and the number of images, $N = \frac{360}{\theta} - 1$, as could be seen from the table. If the number of images seen is odd, the number of angular distances in the last column is even and they are alternately 2α and 2β. In all these cases the reflected graduated circle appears to be equally divided into N sectors. If the number of images is even, the number in the last column is odd, and all of them except one are alternately 2α and 2β. The remaining angular distance is that of the remotest image from the object and is equal to θ. The total of the angular distances in each case is 360°.

In the last three cases θ is not an exact sub-multiple of 360 and N = the integral part of the quotient $\frac{360}{\theta}$. The number of angular distances is also equal to N and some portion of the arc (the remotest portion of the completed circle from the object) is left out and is marked R in the table. When θ increases and approaches to be an exact sub-multiple of 360, the two remotest of the images approach each other, R gradually getting smaller and smaller, and they coalesce into a single image. In the last case as θ increases the two images get near and only one image is seen when $\theta = 180$. $N = \frac{360}{180} - 1$. What do you observe if θ decreases and becomes an exact sub-multiple of 360, say from 65° to 60°?

What is the number of images that must be formed when $\theta = 36°$. Test the number by actual observation. Do you see all the images?

What do you notice regarding the brightness of the image as the image is more and more remote from the object?

How many images should be seen between two parallel mirrors, and how many do you practically see?

The images may be constructed graphically in the following manner.

In fig. 56, OM_1 and OM_2 are the traces of the mirrors,

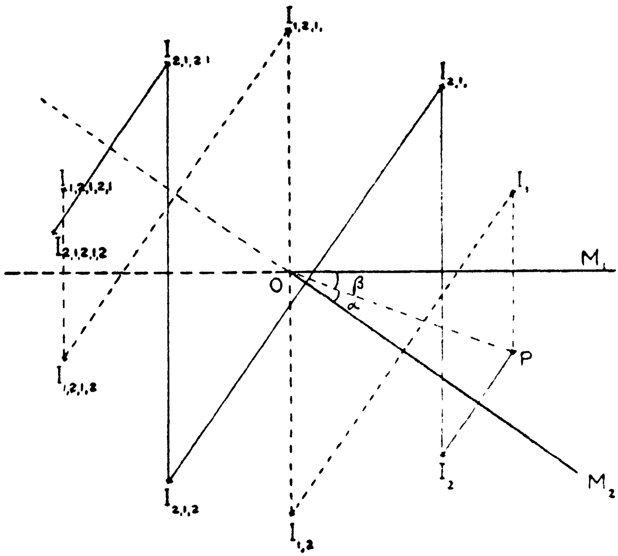

FIG. 56

and P is the object between them. $\theta = 35°$, $\alpha = 15°$, and $\beta = 20°$. Draw PI_1 perpendicular to OM_1, and I_1 be such a point that it is as far behind OM_1 as P is in front of it. Therefore I_1 is the image of P in the mirror OM_1. This construction is made with a set square and dividers. Regard I_1 as a source of light in front of the mirror OM_2 and draw

by a similar construction the image $I_{1,2}$, formed by reflection first at the mirror OM_1 and then at OM_2; hence the image is marked $I_{1,2}$. Similarly, $I_{1,2,1}$ is the image of $I_{1,2}$, formed in the mirror OM_1, $I_{1,2,1,2}$ the image of $I_{1,2,1}$ in the mirror OM_2, and finally $I_{1,2,1,2,1}$. The last image is behind the two mirrors and cannot be further reflected. Similarly starting with the image I_2 of P, formed first at the mirror OM_2, a series of images are obtained, viz. I_2, $I_{2,1}$, $I_{2,1,2}$, $I_{2,1,2,1}$, and $I_{2,1,2,1,2}$ and this last is again behind both the mirrors. $N = 10 =$ integral part of $\frac{360}{35} = 10\frac{2}{7}$. When $\theta = 36°$, the last two images merge into one image and $N = 9 = \frac{360}{36} - 1$. All these images are found to lie on the circle with OP as the radius. $\angle POI_1 = 2\beta$, $\angle I_1OI_{2,1} = 2\alpha$ etc. alternately and $\angle POI_2 = 2\alpha$, $\angle I_2OI_{1,2} = 2\beta$, etc. alternately. The lengths of the arcs PI_1, $I_1I_{2,1}$, etc. may be measured instead of the angles.

REFRACTION AT A PLANE SURFACE.

Apparatus required for verifying the laws of refraction are a rectangular block of glass, three pins, drawing board, paper, and an instrument box.

Draw a line AB (fig. 57) on the paper fixed to the board. Place the block ABCD flat on the board with one of its longer edges coinciding with AB as shown in the figure. Fix a pin P near one corner away from you, touching the block. Look through the slab of glass at the image in any direction and fix a second pin Q_1 in that direction. Fix another pin R_1 such that it is in a line with Q_1 and the image of P; R_1 hides both of them. Mark the points Q_1 and R_1. Fix the second pin at Q_2 and the third at R_2 viewing the image of P in another direction. Repeat the same for four or five more directions.

AB is the surface of separation between glass and air. Since P is touching the block, the rays proceeding from P

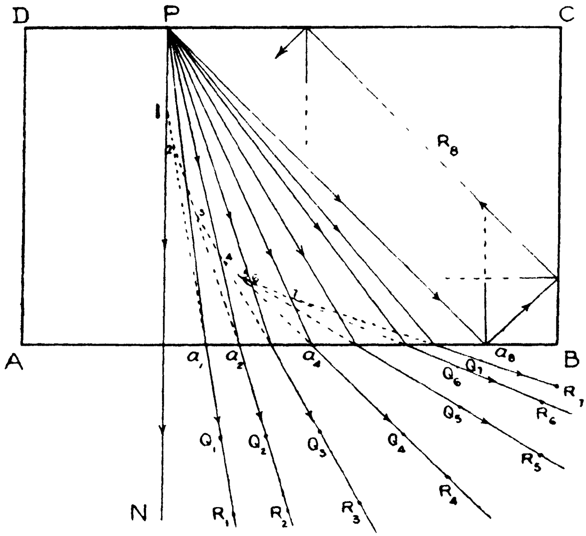

FIG. 57.

in different directions, and incident on AB, have been travelling in glass and Q_1R_1, Q_2R_2, etc. are the directions along which these incident rays emerge out into air. Produce Q_1R_1, Q_2R_2, etc. backwards to meet the line of separation AB at a_1, a_2, etc. These are the points of emergence; join these points to the source P and you get the incident rays. Thus Pa_1 is an incident ray and $a_1Q_1R_1$ is the corresponding emergent ray. Note that the emergent ray bends away from the normal and the light passes from glass to air. Measure the angles of incidence (i) and refraction (r) with the protractor for each set of rays and calculate the ratio $\frac{\sin i}{\sin r}$. This ratio will be found to be nearly a constant. This verifies the second law of refraction. Again, since the incident ray, the normal at the point of incidence or

emergence (?), and the refracted ray are in the plane of the board, the first law is also verified. In this exercise light is refracted at the bounding surface while it passes from the thicker to the rarer medium, i.e. from glass to air and therefore the constant obtained is called the *refractive index of the material* of the slab when light travels *from glass to air* (${}_g\mu_a$). But what is ordinarily called the refractive index of glass (μ) is the ratio of the sine of the angle of incidence to that of the angle of refraction, when light travels *from air into glass* and is strictly expressed as ${}_a\mu_g$. Therefore the refractive index of the slab is the reciprocal of the mean ratio obtained above.

TOTAL INTERNAL REFLECTION AND CRITICAL ANGLE.

When light travels from glass to air, the maximum possible angle of refraction is 90° and the corresponding angle of incidence α at the bounding surface inside glass is such that $\frac{\sin \alpha}{\sin 90} = {}_g\mu_a = \frac{1}{\mu}$. Light incident at an angle greater than α has been found not to emerge out into the thinner medium, air, but is found to be *totally reflected back into the thicker medium.* This phenomenon is called *total internal reflection.*

The limiting angle of incidence α for which a refracted ray is just possible is called the *critical angle.* For all angles of incidence less than α part of the light is refracted and part is reflected back into the slab; but for angles greater than α no light is refracted and all of it is reflected back into the medium. Hence the name *total internal* reflection. Calculate the critical angle from the constant obtained above and verify the value by tracing the extreme possible ray Q_7R_7. To do this move the eye towards B until the image seen through just disappears. Fix two pins in this line of sight as already explained. The image would not be very clear at this stage (?) and some difficulty

would be felt in fixing the pins accurately. Draw the refracted and the incident rays and compare the angle of incidence thus observed with the critical angle calculated. Note that the ray Q_7R_7 is not the theoretically possible grazing emergent ray but the nearest approach to it.

Practical example.—

No.	i	r	log. sin i	log. sin r	$\frac{\sin i}{\sin r}$
1	4·5	8·0	$\bar{2}$·8436	$\bar{1}$·1436	0·5012
2	11·0	17·5	$\bar{1}$·2806	$\bar{1}$·4781	0·6346
3	16·7	25·0	$\bar{1}$·4634	$\bar{1}$·6259	0·6879
4	21·5	34·0	$\bar{1}$·5641	$\bar{1}$·7476	0·6554
5	26·2	41·7	$\bar{1}$·6449	$\bar{1}$·8230	0·6636
6	33·5	58·0	$\bar{1}$·7419	$\bar{1}$·9284	0·6509
7	43·0	79·0	At nearly grazing emergence.		

Mean refractive index $({}_g\mu_a) = 0{\cdot}66$

In the first set of observations noted above, the values for i and r are too small for accurate measurement, so that the observational error is a considerable fraction of the reading taken. Hence such observations should be avoided.

$$ {}_a\mu_g = \frac{1}{{}_g\mu_a} $$

$$ \therefore\ \mu = \frac{1}{0{\cdot}66} = 1{\cdot}51 $$

but $\sin\alpha = {}_g\mu_a = 0{\cdot}66$

$\therefore$ critical angle $= 41^\circ\ 18'$.

The value obtained by direct observation is 43°, as shown in the last column above. This indicates that the pin method is not very accurate.

The position of the image for different positions of the eye is different. Draw PN at right angles to AB in fig. 57. Produce the refracted rays backwards. Mark the points of intersection of each set of consecutive rays, PN and Q_1R_1, Q_1R_1 and Q_2R_2, etc. Points 1, 2 . . . 7 in the figure are thus obtained. All these lie on a smooth curve and all the possible virtual images of P seen through the slab lie on it.* A similar curve would be obtained if the incident rays from P on the left side of the normal were traced and the emergent rays produced back similarly. These curves are symmetrical about the normal PN and are of the form ⅄. This double curve is called the *Caustic Curve.* In the case of reflection at a concave surface, a similar curve is obtained and the images formed on this curve are real.† A broad beam of parallel light from a distant luminous source like the sun is, after reflection by a concave mirror, concentrated on this curve and bodies like cotton and wool can be burnt along the curve. Hence the name caustic (burning). Whenever a similar curve is obtained by refraction or reflection, though the images formed are not real, the curve is, by similarity, called a caustic curve or simply a caustic.

Note that the number of rays that go to form an image decreases as we pass down the caustic, i.e. as we look more and more obliquely into the slab. How does this affect the intensity or clearness of the image seen ?

REFRACTION THROUGH A PRISM.

We shall now *trace the path of a number of rays passing through a prism by the pin method.*

Apparatus required are an equilateral glass prism, four pins, drawing board and paper, and an instrument box.

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* This is strictly true when the points a_1, a_2, etc. are sufficiently near each other.

† This curve is often observed on the surface of a cup of milk or coffee and is due to reflection from the inner surface of the cup.

Trace the boundary of the prism ABC on the board (fig. 58).

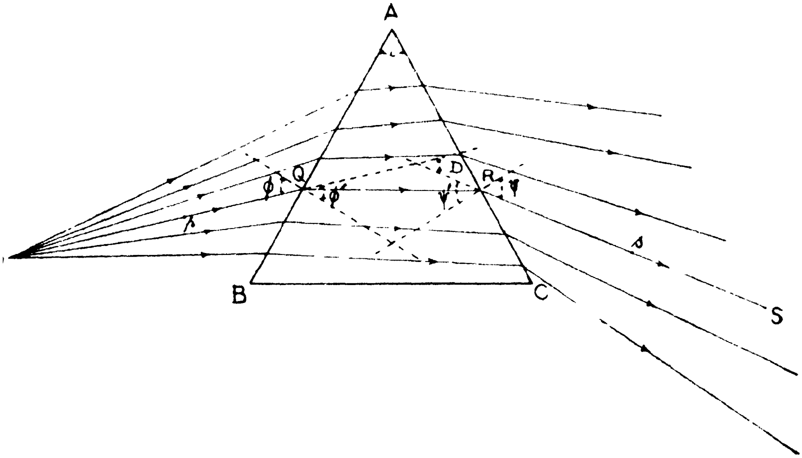


FIG. 58.

Fix a pin P near one side. Put the prism aside and with the protractor and set square draw six lines from the point P such that they are incident on AB at angles greater than 30°. Replace the prism. Fix the second pin at *p* on one of these lines. Look at the images from the other side AC of the prism and fix the pins *s* and S such that these two simultaneously hide the images of the other two. Let the line *s*S produced backwards meet the face AC at R. PQ is an incident ray and RS is the corresponding emergent ray. The line QR is the path of the ray inside the glass. Trace the path of the other rays similarly. Remove the prism and measure the angles of incidence and refraction at the two bounding surfaces AB and AC. Note that all the incident rays bend towards the thicker part of the prism. Produce the emergent ray backwards to meet the direction of the incident ray in each case and measure the exterior angle D thus formed and this is the *total deviation* for the ray. Measure the angle of the

prism i with the protractor directly. Tabulate the observations and verify the relations

$$\text{(i) } D+i = \phi+\psi, \text{ and (ii) } i = \phi'+\psi'.$$

Practical example.—

No.	ϕ	ϕ'	ψ'	ψ	D	i	D+i	$\phi+\psi$	$\phi'+\psi'$
1	30·5	20·5	38·0	73·5	45·0	. .	104·5	104·0	58·5
2	34·5	23·0	35·5	63·7	39·5	. .	99·0	98·2	58·5
3	39·0	25·5	33·2	57·5	38·2	. .	97·7	96·5	58·7
4	45·0	27·0	31·5	51·0	38·0	59·5	97·5	96·0	58·5
5	50·0	29·5	29·3	46·0	37·5	. .	97·0	96·0	58·8
6	55·0	33·0	26·7	41·5	38·0	. .	97·5	96·5	59·7
7	27·0	18·5	40·0	85·0	at grazing emergence.				

An examination of the observations shows that the deviation of a ray decreases as the angle of incidence on the first face increases and has a minimum value of about 37°·5 and then increases with the angle of incidence. Note how the change in the angle of deviation D for an increase of one degree in the angle of incidence ϕ reaches a minimum value. Note also how as the angle of incidence ϕ increases, the difference $\psi'-\phi'$ gradually decreases to a minimum and how after the minimum deviation position is passed ϕ' becomes bigger than ψ' and the difference changes sign.

We shall next determine *the minimum angle of incidence on the first face of the prism, for which an emergent ray from the second face is possible.*

In the above experiment lower the position of the second pin p towards B until the images just cease to appear in the same line of sight. Trace this emergent ray by the pin method. Measure ϕ, ϕ', ψ' and ψ. ψ will be nearly 90° and ψ' is the critical angle for the given glass. The observations made in such an experiment are given in the last column of the above table.

We can calculate the minimum angle of incidence, assuming that $\mu = 1{\cdot}5$ and $i = 60°$ for the prism. This is a case of

grazing emergence at the second face and so the angle of incidence at the second face in glass $\psi' = \sin^{-1}\left(\frac{1}{1 \cdot 5}\right)$

$$\therefore\ \psi' = 41^\circ\ 48'$$

$$\therefore\ \phi' = i - \psi' = 18^\circ\ 12'$$

$$\text{but } \frac{\sin\phi}{\sin\phi'} = 1 \cdot 5$$

$\therefore\ \phi = 27^\circ\ 56'$ and this is nearly equal to the observed value, viz. 27°. An experimental method of determining accurately the angle of minimum deviation for a prism is given later on (Chapter X).

(1) Plot the values of ϕ and D and determine graphically the value of ϕ, for which D is a minimum.

(2) What would be the relative values of ϕ, ψ, ϕ' and ψ' in the minimum deviation position for the prism ?

(3) If $\mu = 1 \cdot 5$ for the material of the prism, calculate the value of ϕ in the minimum deviation position and compare this value with that obtained in (1).

(4) When would a ray emerge out of the third face ? (BC in fig. 58.)

(5) Sketch the path of a ray (i) incident normally, and (ii) at an angle less than 27° on the first face of the equiangular prism. Find the angle of emergence at the third face and compare it with the angle of incidence on the first face.

(6) Note that the position and clearness of the virtual images of the pins change with the position of the eye when looked through the second face. In which position of the prism are the images best defined ?

We shall now find *the critical angle for the material of the prism.* Trace the boundary of the prism, ABC (fig. 59), on the drawing board. Fix a pin P touching the face AB. Let the diffused light from a door or a window fall directly on P. Light rays proceed from P into the glass prism in all directions.

Let us consider some rays incident on the face AC. Let POP′ be the normal to the face AC. P′ is the reflected image

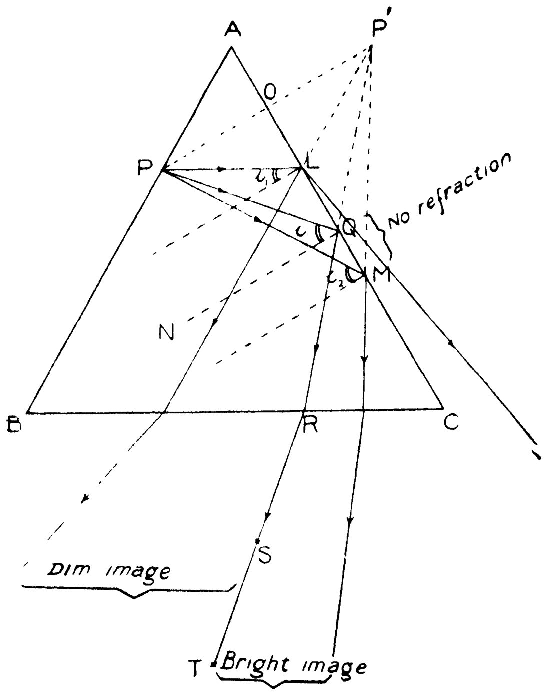


FIG. 59.

of P such that OP = OP′, the face AC acting as a mirror. A ray of light incident at L at an angle i_1, less than the critical angle would be partly refracted out into the rarer medium air and partly reflected into the prism and this reflected ray would emerge out of the face BC. Light incident at a point like Q which is nearer to C than L will have the angle of incidence i greater than i_1 and if i is just equal to the critical angle all the light incident at Q will be reflected back into the prism along QR and partly emerges out along RST. Light incident at points between Q and C will have their angles of incidence i_2 greater than i and will be totally reflected and

emerge out of the third face BC. Thus if the eye is passed from C towards B, while looking into the face BC, the image of the pin P appears very bright and when the eye passes across the direction RT the image suddenly gets dim and continues to remain so as the eye moves towards B.

In order to trace the ray PQRST, look into the face BC moving the eye from C towards B. Note where the image suddenly gets dim. Fix two pins S and T in that direction. Remove the prism, join TS and produce it to meet the trace of the face BC at R. As P′ is the image of the pin P′R is the direction of the reflected ray incident at Q, the point of intersection of the line P′R with the face AC. Measure the angle PQN with the protractor and this is the critical angle.

Repeat the experiment by fixing the pin to touch the side AC of the prism. Look in the face BC and, proceeding as before, find again the critical angle i and take the mean.

Practical example.—

Critical angle observed	=	41°
,, ,, ,,	=	42°
Mean	=	41°·5

We shall now find *the angle of the prism by the pin method.*

Apparatus required are a drawing board with a white paper pinned on, five pins, protractor, set square, and the glass prism.

Place the prism on the board and trace its boundary ABC (fig. 60). Fix a pin P at a distance of 10 or 15 cm. from the apex A. Fix two pins P_1 and P_3 as shown in the figure. Look into the side AB for the reflected images of P and P_1 and by the pin method fix the direction of the reflected ray QP_2S. Similarly trace the direction RP_4T from the face AC. Draw QN and RM normals to the surface at Q and R. i_1, r_1 and i_2, r_2 are the respective angles of incidence and reflection.

Produce the directions RT and QS backwards to meet at a point O. The line PA divides the angle A into two parts A_1

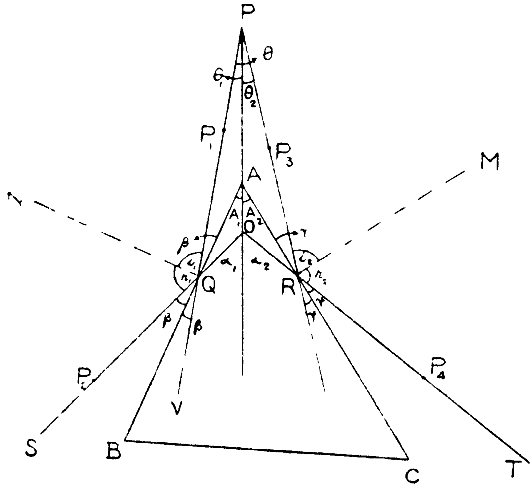


FIG. 60.

and A_2 and the angle SOT into α_1 and α_2. Let $\alpha_1+\alpha_2=\alpha$ and $\theta_1+\theta_2=\theta$. Measure α and θ with a protractor. Put P at different distances from A and repeat the observations and you find that $\alpha+\theta$ is nearly constant. From the fig. 60 it can be shown that $\alpha+\theta=2A$, in the following manner.

In the Δ PQO the exterior angle $\alpha_1 = \angle PQO+\theta_1$

$$= \angle SQV+\theta_1$$

$$= 2\beta+\theta_1$$

similarly in the Δ PRO, $\alpha_2 = 2\gamma+\theta_2$

$$\therefore \quad \alpha=\alpha_1+\alpha_2=2\beta+2\gamma+\theta$$

again in the Δ QPA, $A_1=\beta+\theta_1$

and in the Δ RPA, $A_2=\gamma+\theta_2$

$$\therefore \quad A_1+A_2=A=\beta+\gamma+\theta$$

$$\therefore \quad 2A=2\beta+2\gamma+2\theta$$

$$\therefore \quad 2A=\alpha+\theta$$

If P is at a distance more than 25 or 30 cm. and if the points selected Q and R are very near the apex A, the angle θ will be negligible, compared with the experimental errors and then $\frac{1}{2}\alpha = A$. P_1 and P_3 approach to meet the line PA and Q and R can be considered to be just on either side of A.

Note that the case where O, A and P are concurrent is considered in fig. 60. The relation $2A = \theta + \alpha$ can be verified whether O, A and P are concurrent or not. For the simplicity of the proof of the relation $2A = \alpha + \theta$ the particular case is taken. If they are not concurrent, a line parallel to PA is to be drawn through O and the same relation proved.

REFRACTION THROUGH A GLASS SLAB.

In the exercise on refraction at a plane surface it has been pointed out (page 183) that the position of the image depends on the position of the observer's eye. The image appears nearest the object when looked at directly and recedes from it as the line of sight is more and more oblique. Consider a small pencil of rays emanating from P (fig. 61) and directly incident on AB, the bounding surface between glass and air. P is the object and PO, the axis of the pencil, cuts the surface normally. The bounding incident rays PA and PB emerge out along AS and BR after refraction and when produced backwards cut PO at Q. At B draw NBN′ normal to AB.

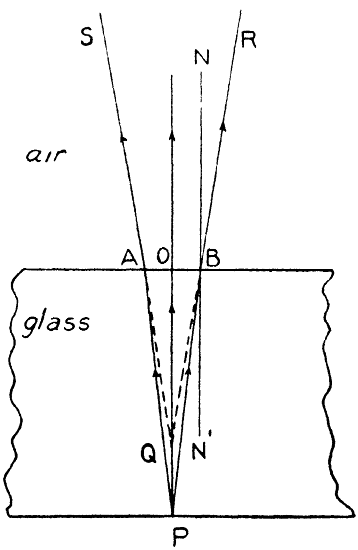


FIG. 61.

$$\mu = \frac{\sin \angle NBR}{\sin \angle N'BP}$$

but $\angle NBR = \angle N'BQ = \angle OQB$ and $\angle N'BP = \angle OPB$

$$\therefore \quad \mu = \frac{\sin OQB}{\sin OPB} = \frac{BP}{BQ}$$

When the pencil of rays is very small, B and A would be very near O and BQ may, to the first approximation, be taken to be equal to OQ and BP to OP. Therefore, under these limitations,

$$\mu = \frac{OP}{OQ} = \frac{\text{Distance of the object from the bounding surface}}{\text{Distance of the image from the same surface}}$$

$$= \frac{\text{Real thickness of glass slab}}{\text{Apparent thickness of the slab}} \quad \cdots \quad \cdots \quad (1)$$

Let us consider an alternative case in which AB and CD (fig. 62) are the traces of the parallel sides of the slab. P is the position of the object. Let a small divergent pencil of rays, PAB, be incident on the surface AB, PO being the axis of the pencil. The rays PA, PB bend towards the normals at A and B after refraction and proceed along AC and BD respectively. The image after this refraction at the surface AB appears to be at Q′. The rays undergo another refraction at the surface CD before they emerge out into the air and bend away from the normal. These directions produced backwards go and meet O′OP, at Q. Q is the position of the image of P after refraction through the slab.

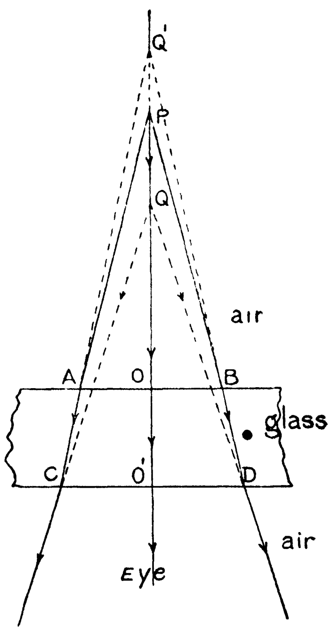


FIG. 62.

It follows from the above consideration of direct refraction at a plane surface that $\frac{OP}{OQ'} = \frac{1}{\mu}$ for the refraction at the surface AB and that $\frac{O'Q}{O'Q'} = \frac{1}{\mu}$ for that at CD, considering Q′ as the object from which the incident rays AC, OO′ and BD all proceed. OO′ is the thickness of the slab, say d.

$$\frac{OP}{OQ'} = \frac{1}{\mu} = \frac{d+OQ}{d+OQ'}$$

$$\therefore \quad OQ' = \mu.OP$$

$$\text{and } d+OQ' = \mu.d+\mu.OQ$$

$$\therefore \quad d+\mu.OP = \mu d+\mu.OQ$$

$$\therefore \quad (\mu-1)d = \mu(OP-OQ)$$

$$= \mu.PQ$$

$\therefore \quad PQ = \frac{\mu-1}{\mu} d$, which is a constant for the slab and is independent of the distance OP.

Again

$$\frac{PQ}{d} = 1 - \frac{1}{\mu}$$

$$\therefore \quad \frac{1}{\mu} = 1 - \frac{PQ}{d} = \frac{d-PQ}{d} \text{ and } \mu = \frac{d}{d-PQ} \quad \cdots \quad \cdots \quad (2)$$

If, therefore, d and PQ are measured the refractive index of the material of the slab is determined.

The two expressions obtained above can be used to find the refractive index of glass by the following methods.

The *apparatus required* in *the parallax method* are a rectangular block of glass, an optical stand, two pins, a small piece of cork, drawing board and millimetre scale.

Fix a pin P (fig. 63) vertically on the drawing board touching the glass block. Pass a second pin through a cork piece and hold it vertically in the optical stand S. Adjust the height of this pin to just touch the top surface of the slab. Look through from E into the block and adjust the stand S such that the image of the pin P and the second pin Q appear to be in the same vertical line. Move the eye about E sideways and observe if there is any relative motion between them. If there is any, move S along POE as explained in a previous exercise until you obtain the position Q where the relative motion ceases. Measure the lengths OP and OQ with the millimetre scale. Disturb the arrangement and repeat it two or three times. The mean ratio $\frac{OP}{OQ}$ gives the refractive index of the glass. Water or alcohol, contained in a thin-walled rectangular glass vessel, may be substituted for the slab of glass and μ for the liquid may be determined.

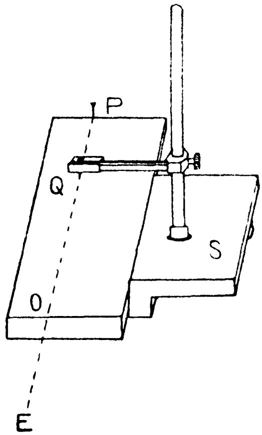


FIG. 63.

Practical example.—

	Glass slab.		*Water*	
	OP	OQ	OP	OQ
	10·15 cm.	6·55 cm.	5·5 cm.	4·2 cm.
	10·15 „	6·65 „	5·5 „	4·0 „
	10·15 „	6·75 „	5·5 „	4·0 „
	10·10 „	6·75 „		
Mean..	10·14 cm.	6·67 cm.	5·5 cm.	4·07 cm.

Mean value of μ for glass $\frac{10\cdot14}{6\cdot67} = 1\cdot52$

„ „ μ for water $\frac{5\cdot5}{4\cdot07} = 1\cdot36$

The *apparatus required* in *the microscope method* is a microscope with an arrangement to move it vertically on a fine graduated scale provided with a vernier. Place the block of glass used in the above experiment on the base board of the microscope. Keep the axis of the microscope vertical. Focus the instrument on the top surface of the block O (fig. 64). Note the position M_1 of the microscope on the vertical scale. Lower the microscope by working the screw and focus the top surface of the base board as seen through the block at Q. Note the position M_2. Remove the slab and focus the base board P directly and note the third position M_3. Whenever an object is seen clearly through a microscope, the distance between the instrument and the object is a constant for the instrument and so $M_1O = M_2Q = M_3P$. Therefore the difference between the readings M_1 and M_2 gives the apparent thickness of the slab OQ and that between M_1 and M_3 gives the actual thickness OP. Repeat each of the observations twice or thrice and take the mean ratio $\frac{OP}{OQ}$.

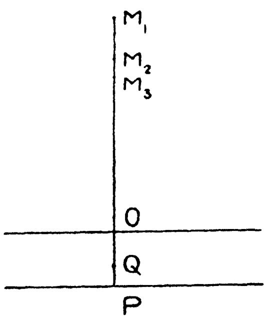


FIG. 64.

Practical example.—

A travelling microscope was used. The vertical scale is graduated to 0·05 cm.; 2·45 cm. on the vernier are divided into fifty equal parts. So the vernier reads to 0·001 cm.

	M_1	M_2	M_3
	12·829 cm.	11·108 cm.	10·200 cm.
	12·826 ,,	11·105 ,,	10·201 ,,
	12·822 ,,	11·107 ,,	10·200 ,,
Mean ..	12·826 cm.	11·107 cm.	10·200 cm.

Mean OP .. 2·626 cm.
,, OQ .. 1·719 ,,

$$\mu = \frac{2{\cdot}626}{1{\cdot}719} = 1{\cdot}528$$

This method is very useful in determining the refractive index of a liquid, say water. Fine sand may be thrown to the bottom of water contained in a glass beaker and position M_2 determined easily. By sprinkling fine cork dust on the surface of water, position M_1 can be noted.

	M_1	M_2	M_3
	cm.	cm.	cm.
	9·520	8·014	7·478
	9·565	8·011	7·473
	9·530	8·014	7·495
	9·535	8·016	7·488
Mean ..	9·538	8·014	7·484

Mean depth of water .. 2·054 cm.

,, apparent depth .. 1·524 ,,

Refractive index of water $\mu = \dfrac{2{\cdot}054}{1{\cdot}524} = 1{\cdot}349$

Parallax method may also be used in the following manner, and the second relation $\mu = \dfrac{d}{d-PQ}$ *applied.*

Apparatus required are a rectangular block of glass, a drawing board with white paper pinned on to it, two pins taller than the thickness of the slab, calipers, millimetre scale, dividers and a set square.

Draw a line AB (fig. 62) on the board and place the block with one of its sides on it. Fix a pin P and look through the slab from the other side, at the image. Fix another pin on the same side as P so that it is in a line with the object and that the upper portion of the second pin and the lower portion of the image appear to be in the same vertical. Let the image of the second pin seen through cause no confusion. Fix the second pin at Q by the method of parallax, and it gives the position of the image of P. Put P at two or more different distances from the block and obtain in each case the position of the image. The image always appears nearer the slab than the object. Measure in each case the apparent shift PQ in the position of the object with dividers and scale.

You find that it is nearly constant. Measure the width of the slab d with a calipers and find the value of μ for the material of the slab, using the relation $\mu = \dfrac{d}{d-\mathrm{PQ}}$.

Practical example.—

A green coloured slab of glass was used.

$d = 4{\cdot}83$ cm.

OP		PQ
cm.		cm.
3·20		1·65
6·30		1·45
8·50		1·50
	Mean ..	1·53

$$\mu = \frac{4{\cdot}83}{4{\cdot}83-1{\cdot}53} = \frac{4{\cdot}83}{3{\cdot}30} = 1{\cdot}46$$

Chapter IX

REFLECTION AND REFRACTION AT SPHERICAL SURFACES

SPHERICAL MIRRORS.

When a ray of light is incident at any surface, it will be reflected according to the two laws stated before. In the case of a plane surface, the normals to the surface at various points are parallel. In the case of a spherical surface, the directions of the various normals are inclined to each other and all of them converge to a point and this point is the *centre of the sphere* of which the surface is a part. This point is called the *centre of curvature of the surface.* A spherical surface is either convex or concave. The edge of the spherical surface in the case of mirrors and lenses is a circle and the diameter of the circle is called the *aperture*.

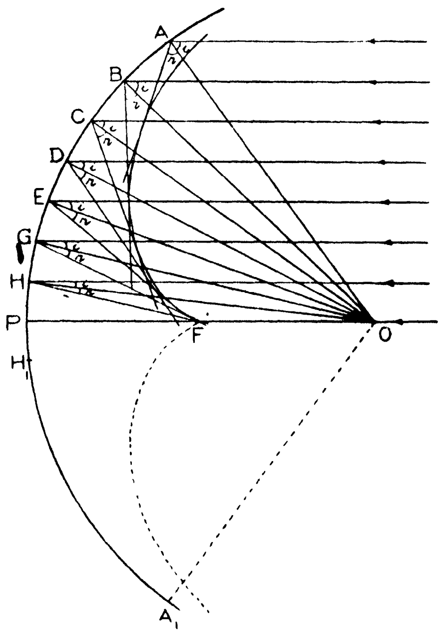


Fig. 65.

The central point of the surface is called the *pole* and the line joining the pole to the centre of curvature is called the *axis*.

Let APA′ (fig. 65) be the principal section of the mirror containing the axis OP; P is the pole of the mirror and O is the centre of

curvature and OP is the radius of curvature. Let a point source of light be situated at a great distance on the axis of the mirror. Rays of light proceeding from the source meet the mirror in a pencil parallel to the axis at different points, A, B, C, etc. Join AO, BO, etc., normals to the surface at the points of incidence. Draw the reflected rays with the help of a protractor so that $i = r$, as shown in the figure. The ray incident along OP will be reflected back along the direction PO. The points of intersection of the consecutive reflected rays will, provided that the points A, B, C, etc. are sufficiently near each other, lie on a curve, the curve of images due to reflection from the upper part of the mirror. Note that the reflected rays are tangential to the curve. A similar curve is obtained in the lower portion of the mirror. The two curves together form the full curve which is called the *caustic curve.* If the whole aperture of the mirror AA is exposed to light from a source, it is evident that a number of images at different distances from the mirror are obtained. It is also clear therefore that in order to study experimentally the reflection at a concave mirror, the aperture of the mirror exposed to light should be small, to avoid confusion of the images obtained. Again since a number of successive images are obtained one next to the other, the image of an extended source like a flame will not be clear on account of overlapping, if a large area round the pole is exposed. Test these facts by observing the image of a flame with the whole aperture exposed and also by shutting out most of the surface and leaving a small aperture of one centimetre radius, round the pole, by means of a card-board disc with a central hole cut into it and fitted to the mirror APA_1.

Let us consider a small portion HPH_1 (fig. 65) of the principal section. HO is the normal and radius. The rays incident at H and P being parallel the angle of incidence at H $= \angle HOP$ and $i = r = \angle OHF$. In the $\triangle OHF$. $\angle OHF = \angle FOH$ and

the triangle is therefore isosceles. In the triangle HFP the angle HFP depends upon the aperture HP and if HP is small enough, HF = PF = FO. In practice it is found that the above relation approximately holds good if the aperture is small and in such a case the various images on the caustic are formed so near each other that they crowd in about F, the beginning of the caustic curve, and the image is nearly clear. The rays involved in this consideration have their angles of incidence within 5° and may be considered to be directly incident on the mirror. The theory of reflection of a pencil of rays incident nearly normally is simple and the following considerations and relations hold good only in such cases of *direct incidence.*

When a small pencil of parallel rays, coming from a distant source of light situated on the axis of a spherical mirror, is incident directly on the mirror, the rays after reflection actually converge to or appear to diverge from a point on the axis of the mirror and this point is called the *principal focus* of the mirror. The distance of the point from the pole of the mirror is called the *focal length*, f, and $f = \frac{r}{2}$, where r is the radius of curvature of the surface of the mirror.

We shall now deal with the case of a convave mirror in some detail. AR is the trace of the mirror (fig. 66). PQ is

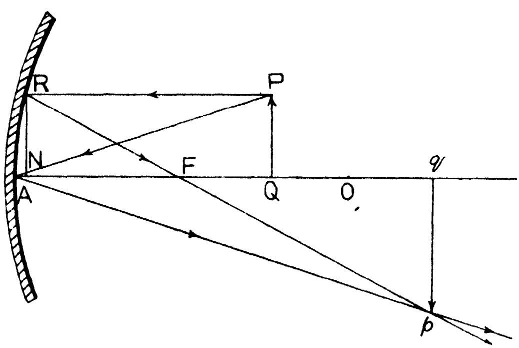


FIG. 66.

the object, O is the centre of curvature and F is the focus. AO $= r = 2$AF $= 2f$. PR is an incident ray parallel to the axis and the reflected ray Rp passes very nearly through F. PA is a ray incident from the same point P on the pole A of the mirror and Ap is the reflected ray so that $\angle$PAO $= \angle p$AO. The two rays incident from the point P meet after reflection at p and therefore p is the image of P. q is the image of Q and the image of any point of the object between P and Q will be found, by a similar geometrical construction, to form at a corresponding point in the line pq. AQ is the distance of the object from the pole and is represented by u and the distance Aq of the image is represented by v. *All the distances are measured from the pole A towards the point concerned. If a distance is opposed in direction to that of the incident rays, the convention is to consider the distance as positive; if a distance is measured in the direction of the incident light, it is taken as negative.* In the particular case u, v, r and f are all positive.

Draw RN perpendicular to the axis cutting it at N a little away from A.

$\dfrac{\text{RN}}{pq} = \dfrac{\text{NF}}{\text{F}q}$ in the two similar Δs, RNF and Fpq.

RN $=$ PQ and FN is very nearly equal to FA $= f$ for direct incidence.

$$\therefore \quad \frac{\text{AF}}{\text{F}q} = \frac{\text{PQ}}{pq}$$

The two Δs PAQ and pAq are similar, and $\dfrac{\text{PQ}}{pq} = \dfrac{\text{AQ}}{\text{A}q} = \dfrac{u}{v}$

$$\therefore \quad \frac{\text{AF}}{\text{F}q} = \frac{u}{v}, \text{ but AF} = f, \text{ and F}q = \text{A}q - \text{AF} = v - f$$

$$\therefore \quad \frac{f}{v-f} = \frac{u}{v}$$

$\therefore \quad uv = uf + vf$ and dividing throughout by uvf we get

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} = \frac{2}{r}.$$

$\frac{pq}{PQ}$ = ratio of the length of the image to that of the object,

and is called the *magnification*, *m*, and $m = \frac{v}{u}$.

When the object is between the pole and the principal focus, a virtual image occurs. The construction for the image (fig. 67)

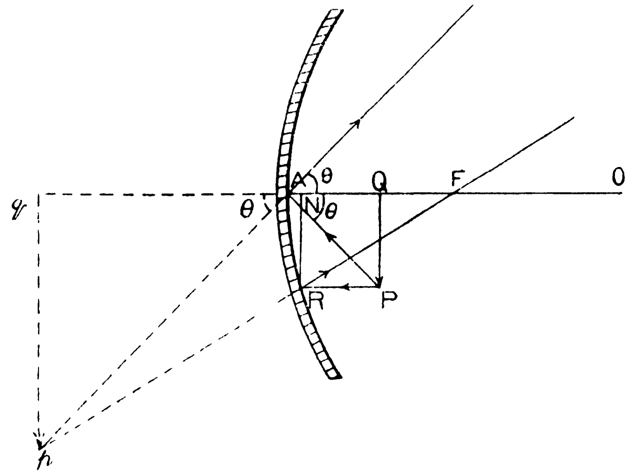


FIG. 67.

is the same as before, and again,

$$\frac{RN}{pq} = \frac{NF}{pF}, \quad \text{i.e.,} \quad \frac{PQ}{pq} = \frac{AF}{qF} = \frac{f}{f+v}$$

$$\therefore \quad \frac{pq}{PQ} = \frac{f+v}{f}.$$

In the two $\triangle$s, Apq and APQ which are similar $\frac{pq}{PQ} = \frac{v}{u} = m$, as before.

$$\therefore \quad \frac{v}{u} = \frac{f+v}{f}$$

$$\therefore \quad uv = vf - uf$$

$$\therefore \quad \frac{1}{f} = \frac{1}{u} - \frac{1}{v} = \frac{2}{r}.$$

In this relation regard is not paid to the signs of the distances and they are considered only numerically. Aq is v and is negative in sign. (?). So the above relation can be written thus $\frac{1}{f}=\frac{1}{u}+\frac{1}{v}=\frac{2}{r}$ and the form is the same as that obtained above.

The case of a convex mirror is dealt with now.—

The same construction as in the previous case is again made (fig. 68).

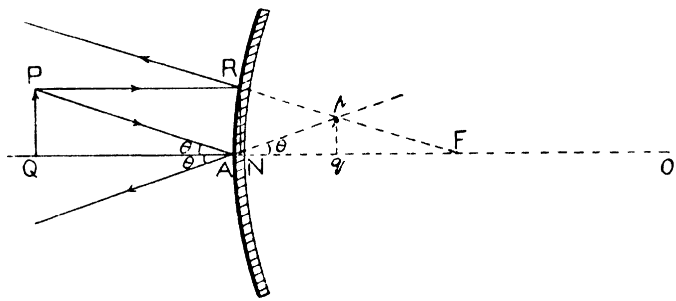


FIG. 68.

$$\frac{RN}{pq}=\frac{NF}{qF}, \quad \therefore \quad \frac{pq}{PQ}=\frac{f-v}{f}$$

In the Δs, APQ and Apq $\frac{pq}{PQ}=\frac{v}{u}=m$

$$\therefore \quad \frac{v}{u}=\frac{f-v}{f}$$

$$\therefore \quad uv=uf-vf \quad \text{and} \quad \frac{1}{f}=\frac{1}{v}-\frac{1}{u} \text{ (numerically).}$$

f and v, for a convex mirror, are always negative and u is positive, hence $-\frac{1}{f}=-\frac{1}{v}-\frac{1}{u}$ or $\frac{1}{f}=\frac{1}{v}+\frac{1}{u}=\frac{2}{r}$. The equation is of the same form for all cases of reflection at a spherical mirror when proper signs are assigned to the distances.*

* The observed numerical values in an experiment can be directly substituted in the numerical formula and if the general formula is to be used, proper signs have to be assigned to the numbers before substitution.

Notice the following points in the case of images formed after reflection at spherical mirrors.

In the case of a concave mirror, as the object approaches the mirror from a great distance, the image recedes from the mirror starting from the focus and meets the object at the centre of curvature and as the object is brought to the focus the image continues to move away from the mirror in the same direction to a great distance from the mirror. If the object is continued to be moved towards the mirror and when it just passes the focus the image suddenly appears at a great distance on the other side of the mirror and is virtual. As the object continues to move towards the pole the image also appears to move very rapidly towards the pole and finally meets it.

In the case of a convex mirror, when the object is brought from infinity to the pole, the image is always virtual and moves in the opposite direction from the focus to the pole. The range of the positions of the image is thus confined to that between the focus and the pole in the case of a convex mirror, whereas for a concave mirror this is the only distance within which images are not at all formed. Thus the gap in the range of the image in the concave mirror is filled up by the range in the convex mirror.

Practical example.—

The *apparatus required* for *verifying the relation* $\frac{1}{v}+\frac{1}{u}=\frac{2}{r}$ *for a concave mirror* are two optical stands, a ground glass screen with holder, concave mirror, a metre scale, a glass jet held in an adjustable stand.

Mount the mirror, the screen and the source of light on stands and arrange them one behind the other at the *same height* on a table. Bring the source of light from a distance, gradually, near the mirror and determine the position of the screen in each case until a clearly defined image is obtained. Measure the distance from the centre of the mirror to the screen (v) and that between the centre of the mirror and the source of light (u) in each case. Start with $u = v$ and take readings on either side. Take a number of sets of readings and tabulate. Note the nature and size of the image in each case.

When the whole face of the mirror is exposed, two or three images of the jet flame are formed on the screen (why ?). Take into consideration the brightest among them and proceed. To avoid this difficulty cover up the reflecting surface with a card board disc cut to just fit in and provided with a central circular hole of about 1·5 cm. in diameter. If there is a difficulty in getting the position of the screen with the card board disc on, adjust roughly with the whole mirror exposed and then accurately with the central portion only exposed. A better way is to have a small wire gauze piece soldered to a metal rod, next to the source, towards the mirror and to consider the illuminated gauze as the source of light. When the source of light is between the mirror and the screen let not the source illuminate the screen directly (why ?).

Work out in each case the sum of the reciprocals of u and v and note that it is very nearly constant throughout and calculate the radius of curvature of the mirror from the mean value. Plot the u and v curve and read the value of r from it and compare.

u cm.	v cm.	Nature of image.	Size of image.	$\frac{1}{v}$	$\frac{1}{u}$	$\frac{1}{v}+\frac{1}{u}=\frac{2}{r}$
143·5	24·6	inverted	diminished	0·0406	0·00697	0·0476
112·5	26·8, 26·4	,,	,,	0·0377	0·0089	0·0466
79·0	30·0, 29·6	,,	,,	0·0335	0·0127	0·0462
57·0	34·3, 33·8	,,	,,	0·0294	0·0175	0·0469
45·5	39·8, 39·3	,,	,,	0·0254	0·0220	0·0474
43·2	43·2	,,	same size	0·0231	0·0231	0·0462
37·5	48·6, 49·0	,,	magnified	0·0205	0·0267	0·0472
35·5	54·8, 54·4	,,	,,	0·0183	0·0282	0·0465
30·0	72·6, 73·0	,,	,,	0·0137	0·0333	0·0470
21·8	on a wall nearly 40 ft. away.	,,	,,	0·0008	0·0459	0·0467

Mean value of $\frac{2}{r}=0{\cdot}0468$; $r=42{\cdot}6$ cm.

r (direct observation) = 43·2 cm.; r (from graph) = 42·8 cm. The results are shown graphically in fig. 69.

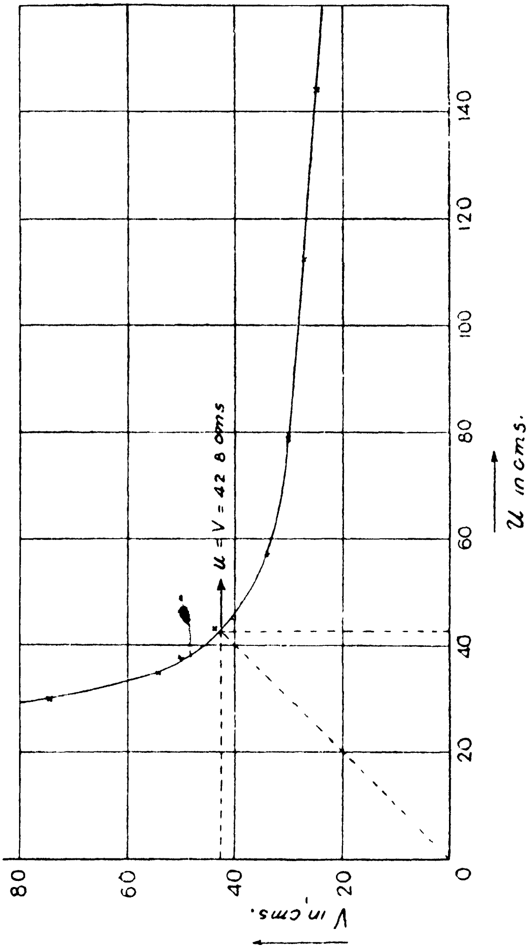


FIG. 69.

1. Can an image be caught on the screen for all values of *u* ? What is the least possible value for *u* and where will the image be found in such a case ?

2. How do you calculate the magnification in any case ? Measure the length of a convenient number of meshes on the gauze in the above exercise and also the corresponding length on the image and see if the ratio gives the magnification worked out from theory.

3. What change does the intensity of illumination of the image undergo with v?

4. Obtain an image of a distant object and compare the distance of the image from the pole of the mirror with the constant obtained.

DIFFERENT KINDS OF LENSES.

We shall now consider refraction of light through a transparent material bounded by two spherical surfaces called a lens. The two surfaces may or may not have the same curvature. The surfaces may be *convex* or *concave*. Generally the two surfaces are both convex or both concave and the radii of curvature of the two surfaces are nearly equal. These are called *double convex* or *double concave* lenses. If one of the surfaces is a plane, they are called *plano-convex* or *plano-concave* lenses. Convex lenses are thickest at the centre and gradually get thinner towards the edge whereas concave lenses are thinnest at the centre and grow thicker towards the edge.

Lenses are generally made of glass and are circular in shape. The line passing through the centres of curvature of the two spherical bounding surfaces is called the *axis of the lens.* The distance between the central points or the poles of the two surfaces is called the *thickness of the lens.* The point midway between the poles is called the *centre of the lens.*

We are here concerned with refraction through *thin lenses* only. The theory of thick lenses is complicated and is studied in an advanced course.

In treating of thin lenses we neglect the thickness and a ray of light incident towards the centre of the lens is considered to pass out without refraction. As a matter of fact the incident and refracted rays are parallel to each other, there being a very small lateral displacement due to refraction as in the case of a parallel slab of glass of very small thickness.

Again we are concerned with the refraction of an incident pencil of rays whose axis is parallel to that of the lens. This is the case of *direct incidence.* The case where the two axes are inclined is one of *oblique incidence*; its treatment is also complicated and is studied in an advanced course. Further we confine our attention to the pencil of rays starting from a source on the axis of the lens and passing through a small portion round the centre of the lens.

If a pencil of rays from a distant object, say the sun, fall on a *convex lens* the axis of the pencil being parallel to the axis of the lens an image is formed on the other side of the lens, at a distance from the centre of the lens. This distance is called the *focal length.* The rays after refraction cut each other at a point on the axis and this point is called the *principal focus* of the lens. The image is in this case formed at the principal focus. The rays after refraction through a convex lens converge towards a point and hence it is called a *convergent lens.* Its focal length is measured *from the centre of the lens* along the direction of the incident light and hence the focal length is negative in sign. A convex lens is therefore called a *negative lens.*

A pencil of parallel rays incident similarly on a *concave lens* diverges after refraction and appears to come from a point on the same side of the lens as the object. This point is called the principal focus of the lens and the distance from the centre of the lens to the principal focus is called the focal length. This is *positive* in sign. (?). The concave lens is therefore called a *diverging lens* and a *positive lens.*

In the case of thin lenses the following two statements are of importance and are useful in tracing the path of rays after refraction.

1. A ray of light passing through the centre of the lens passes out without change of direction.

2. A ray of light parallel to the axis of the lens converges to or appears to diverge from the principal focus.

CONVEX LENSES.

The following is the method of constructing an image of an object situated on the axis of a thin lens.

AB (fig. 70) is a section of a thin lens in the plane of the

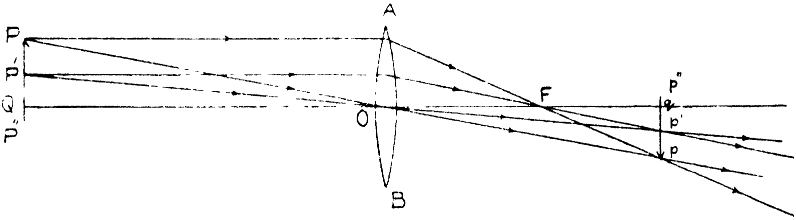


FIG. 70.

paper and O is the centre of the lens. PQ is the object and F the principal focus. QOF is the axis of the lens. Every point of the object sends out a pencil of rays directed to every point in space. Let us consider the point P and the ray PA parallel to the axis. This ray would pass through the focus F after refraction. Consider another ray PO passing through the centre of the lens and this would pass out as PO*p*. The two incident rays PA and PO meet at the point *p* after refraction. Any other ray incident from P on any point of the lens other than A and O also passes after refraction through *p* so that all the rays within the cone BPA converge after refraction to *p*. All the rays cross each other at *p* and diverge out from the point. This point is the image of P. The intensities of light coming along the different rays from P concentrate at the point *p* and contribute to the brightness of the image. The position of the image *p* is completely determined by considering the two rays PA and PO. Consider another point P′ on the object. Similar considerations show that *p′* is the image of P′ and it is easily seen that any other point between P and P′ has a *corresponding* point image between *p* and *p′*. Thus it is clear that the line *pq* is the image of the linear source PQ. Q is shown in the figures

to be on the axis of the lens. If the object extends downwards below the axis say to P″, the image extends along *pq* above the axis to *p*″. It is convenient to consider a linear object lying on one side of the axis.

In fig. 70 $OQ = u$, $Oq = v$, and $OF = f$ and these three distances are measured *from the centre of the lens* and according to the convention of signs, *u is positive*, and *v and f are negative.*

If the object PQ is situated at a great distance from the lens, the pencil of rays, from any point of the object, that catches the lens is practically a parallel pencil because the angle of the cone of the pencil of rays is negligibly small. Therefore the image of every point of PQ is formed practically at F. If u is very large, v is very small and the image formed is almost a point. As PQ comes near the lens, v increases gradually and the size of the image also increases. When PQ gets near enough the image is very large, and is at a great distance. These statements can be verified by, tracing the rays in a number of typical cases or better still experimentally. After tracing the rays for a given value of u_1, if another trace is drawn for a value of u_2 equal to that of v_1 obtained in the first case, it will be found that v_2 in the second case will have exactly the same value as u_1. The positions of the object and the image are interchangeable in the two cases, and are called *conjugated positions.* If the distance between the object and the image $(u_1+v_1 = u_2+v_2)$ is fixed there would be two positions of the lens in which a real image is formed on the screen and the distance between the positions will be equal to u_1-u_2 or v_2-v_1.

If the object is brought very near the lens, the rays after refraction do not converge but when looked from the other side of the lens appear to diverge from an image situated on the same side of the lens as the object. Such an image is called a virtual image in contrast to the real image considered already.

In all these cases, where the image formed is either real or virtual, the distances u, v and f have a particular and constant relation for a given lens.

The relasion may be stated thus: $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$, where v, u and f are given their proper conventional signs of direction.

When a real image is formed, this law may be deduced thus.

Trace the rays as shown in fig. 71. u is positive and v and f are negative in sign.

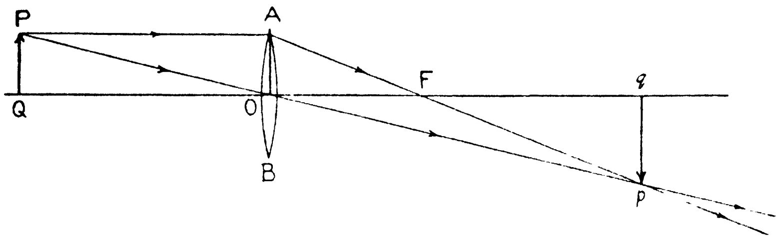


FIG. 71.

In the similar triangles POQ and pOq

$$\frac{qp}{PQ}=\frac{Oq}{OQ}=\frac{-v}{u}.$$

Again PQ = AO and in the similar triangles AFO and Fqp

$$\frac{qp}{AO}=\frac{Fq}{OF}=\frac{Oq-OF}{OF}=\frac{-(v-f)}{-f}=\frac{v-f}{f}$$

but $\frac{pq}{AO}=\frac{qp}{PQ}$

$$\therefore \quad \frac{v-f}{f}=\frac{-v}{u}$$

$$\therefore \quad \frac{v}{f}-1=-\frac{v}{u}.$$

Dividing throughout by v, we have $\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$.

$$\text{Again } \frac{pq}{\text{PQ}} = \frac{\text{Linear dimension of image}}{\text{Corresponding linear dimension of object}}.$$

$$= \text{magnification} = m = -\frac{v}{u}.$$

In all real images, magnification is negative since v is negative. This negative sign indicates that the image is head downwards or inverted.

When a virtual image is formed, the same law holds good and it may be deduced thus.

Consider the two rays PA parallel to the axis and PO passing through the centre of the lens (fig. 72). These two

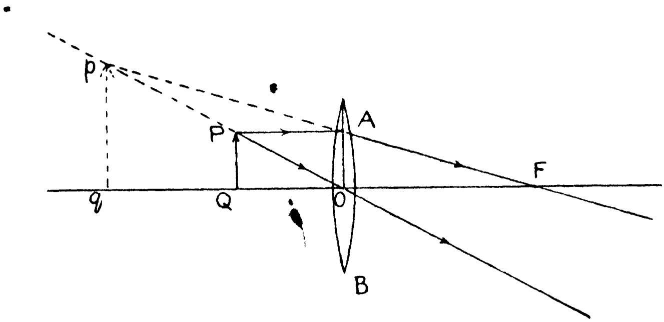


FIG. 72.

diverge after refraction and appear to do so from a point p, on the same side as the object. pq is the erect virtual image. u and v are positive and f is negative.

In the similar triangles Opq and OPQ

$$\frac{pq}{\text{PQ}} = \frac{\text{O}q}{\text{OQ}} = \frac{v}{u} = m.$$

The image is erect and magnified.

Again, in the similar triangles FAO and Fpq

$$\frac{pq}{\text{AO}} = \frac{\text{F}q}{\text{FO}} = \frac{\text{FO}+\text{O}q}{\text{FO}} = \frac{-f+v}{-f} = 1 - \frac{v}{f}$$

but $$\frac{pq}{AO}=\frac{pq}{PQ}=\frac{v}{u}$$

$$\therefore 1-\frac{v}{f}=\frac{v}{u}.$$

Dividing throughout by v and transposing, we have

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}.$$

CONCAVE LENSES.

AOB is the section of a thin double concave lens (fig. 73). O is the centre of the lens and F is the principal focus. As before consider the point P of the object and the two rays PA and PO. The rays after refraction diverge from a point p. pq is the virtual image formed and u, v and f are all positive. In the similar triangles OPQ and Opq $\frac{pq}{PQ}=\frac{Oq}{OQ}=\frac{v}{u}=m$, (always positive and less than unity).

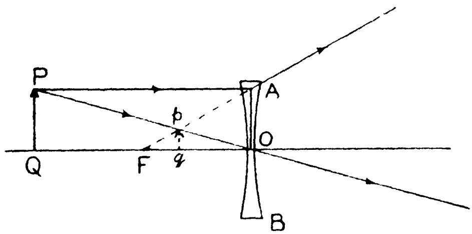


FIG. 73.

Again, in the Δs, Fpq and FAO

$$\frac{pq}{AO}=\frac{Fq}{FO}=\frac{FO-qO}{FO}=\frac{f-v}{f}=1-\frac{v}{f}$$

but $$\frac{pq}{AO}=\frac{pq}{PQ}=\frac{v}{u}$$

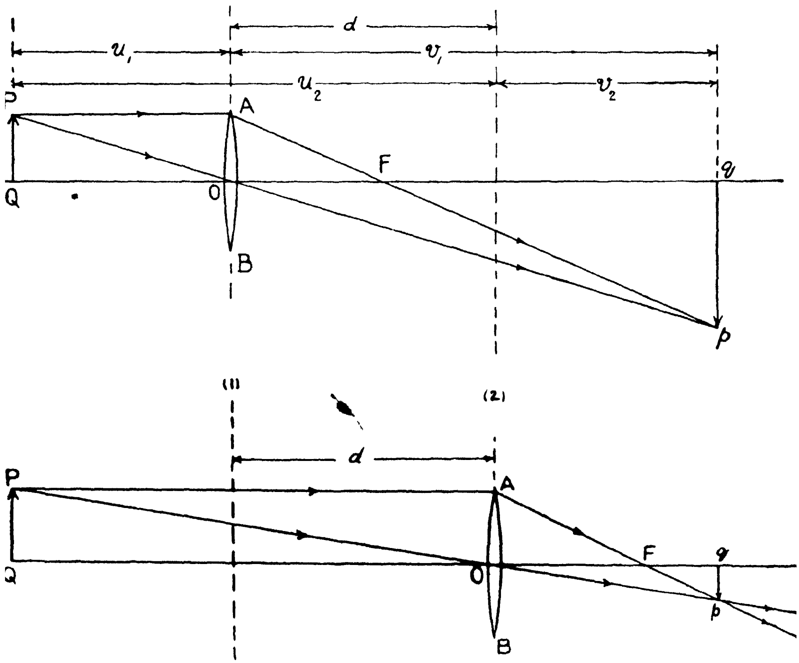
$$\therefore \frac{v}{u}=1-\frac{v}{f}$$

$$\therefore \frac{1}{f}=\frac{1}{v}-\frac{1}{u}$$

Conjugate Positions.

As explained already, there are two positions for a lens in which a real image is obtained for a given position of the object and the image. In the two positions, u and v interchange, $u+v$ being constant.

In figs. 74 and 75 the two positions of the lens are shown.



FIGS. 74 and 75.

The distance between the object and the image is kept constant $u_1+v_1 = u_2+v_2$ and $u_2 = v_1$ and $v_2 = u_1$, in magnitude.

Let us now consider the magnitudes only but not the directions of the distances u, v and f. The relation $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

becomes $-\frac{1}{v}-\frac{1}{u}=-\frac{1}{f}$ since v and f are negative, or $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$. This relation is useful in dealing with definite values for u and v in practice. Let the distance between the object and the image $(u+v)$, be called l and the distance through which the lens is to be shifted into position (2) from position (1) be called d. Then it is clear from the diagram that $d = u-v$, numerically.

$$\frac{1}{f}=\frac{1}{u}+\frac{1}{v}=\frac{u+v}{uv}$$

but
$$uv = \frac{(u+v)^2-(u-v)^2}{4}$$
$$= \frac{l^2-d^2}{4}$$
$$\therefore \quad \frac{1}{f}=\frac{u+v}{(l^2-d^2)/4}=\frac{4l}{l^2-d^2}$$
$$\therefore \quad 4lf = l^2-d^2$$

and
$$f = \frac{l^2-d^2}{4l}.$$

In the case when $u = v$ and $d = 0$, $l = 4f = u+v$. The least distance between an object and its image will be found to be four times the focal length of the lens and in this particular case the magnification is unity. In fig. 74, $m_1 = \frac{v_1}{u_1}$, and in fig. 75, $m_2 = \frac{v_2}{u_2} = \frac{u_1}{v_1} = \frac{1}{m_1}$.

The magnifications are thus reciprocals of each other. If x cm. is the length of the object, then the length of the image in one case is m_1x cm. and the length of the image in the other is m_2x cm. Therefore we find that the product of the lengths of the images in the two cases is $m_1m_2x^2$; but $m_1m_2 = 1$, and hence *the length of the object x is equal to the square root of the product of the lengths of the images in the two conjugate positions of the lens.* This is a useful relation.

Practical example.—

The *apparatus required* to verify the relation $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ for a convex lens are an optical bench carrying three stands for carrying an illuminated piece of wire gauze—the object,—the lens and the screen and an annular piece of card board to cover the lens, and to allow light to pass through a small portion of the lens round the centre.

The heavy bottoms of the stands carry index marks which can be read on the scale engraved along the edge of the optical bench. The vertical plane through the gauze is adjusted to contain the index mark at the bottom of the stand carrying the gauze so that the index reads the position of the gauze. A similar adjustment is made with the stand carrying the screen and its index mark. In the case of the lens stand, this is not possible. The vertical plane passing through the centre of the lens can be adjusted with the eye, to lie at right angles to the graduated scale below, and the distance between the projection of the plane through the centre of the lens and the position of the index of the stand carrying the lens can be once for all determined. This distance is called the index error.

The three stands are so adjusted that the centres of the gauze, the lens and the screen lie in a line parallel to the length of the scale.

Various values are given to u and the corresponding values for v, where a clear image of the wire gauze is formed on the screen, are noted.

No.	u cm.	v cm.	$\frac{1}{u}$	$\frac{1}{v}$	$\frac{1}{u}+\frac{1}{v}$
1	13·0	62·0	0·07692	0·01613	0·09305
2	13·5	50·0	0·07417	0·02000	0·09417
3	14·5	40·2	0·06905	0·02488	0·09393
4	15·5	34·3	0·06458	0·02916	0·09374
5	17·5	27·7	0·05719	0·03611	0·09330
6	19·5	23·7	0·05131	0·04221	0·09352
7	20·0	23·1	0·05000	0·04330	0·09330
8	20·5	22·3	0·04880	0·04485	0·09365
9	21·5	21·9	0·04650	0·04567	0·09217
10	25·5	18·6	0·03920	0·05380	0·09300
11	30·0	16·8	0·03330	0·05995	0·09325
12	34·3	15·9	0·02916	0·06292	0·09208
13	50·0	13·7	0·02000	0·07308	0·09308
14	110·0	11·7	0·00909	0·08561	0·09470

Mean value of $\frac{1}{u}+\frac{1}{v}=0·0933(5)$

This mean constant 0·0933 is $\frac{1}{f}$ and hence $f=10·72$ cm.

Observations 2 and 13 are of conjugate positions. The difference in the values 13·5 and 13·7 is due to observational errors. The mean value is therefore taken into consideration.

$$u+v = 13{\cdot}6+50 = 63{\cdot}6 = l$$

$$u-v = 50-13{\cdot}6 = 36{\cdot}4 = d$$

$$\therefore \quad f = \frac{63{\cdot}6^2-36{\cdot}4^2}{4\times 63{\cdot}6} = 10{\cdot}69 \text{ cm.}$$

Observations 4 and 12 are also of conjugate positions.

$$u+v = 34{\cdot}3+15{\cdot}7 = 50 = l$$

$$u-v = 34{\cdot}3-15{\cdot}7 = 18{\cdot}6 = d$$

$$\therefore \quad f = \frac{50^2-18{\cdot}6^2}{4\times 50} = 10{\cdot}77 \text{ cm.}$$

$$\text{Mean } f = 10{\cdot}73 \text{ cm.}$$

1. Note how the value of v changes for a change of 1 cm. in u as the observations proceed. Observe the significant change at the 9th set of observations in the table.

2. Note that as the lens stand moves along the scale, the stand supporting the screen approaches the lens stand, i.e. the reading of the index mark of the stand gradually decreases. Note that the screen is not moved beyond a particular reading, and that at that position $u+v$ is least. What is the relation between u and v then?

3. Remove the annular card board ring fixed to the lens surface and obseve the increase in the brightness of the image and the decrease in the definition. This loss of definition is due to the formation of a number of images due to the largeness of the aperture and the consequent overlapping.

4. Remove the wire gauze and use the illuminating lamp as the source. Observe that the position of the clearest image on the screen is not so easily judged as with the illuminated wire gauze which has a fine structure.

CHAPTER X

OPTICAL INSTRUMENTS

THE TELESCOPE.

The *apparatus required* to form an astronomical telescope are two lens holders, ground glass screen, two double convex lenses of different focal lengths, dividers and millimetre scale.

The lens holders and screen (fig. 76) slide on a wooden

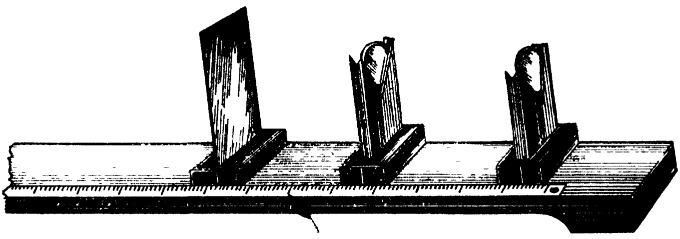


FIG. 76.

base on one side of which is fixed lengthwise a scale divided into millimetres. With the help of index marks the positions of the holders can be accurately read on the scale.

Place the lens of the longer focus on one of the lens holders. Direct the lens, by turning the whole base board, towards a distant tree and move the screen to and fro until a clear image of the distant object is formed on the screen. Note the index readings and the difference gives the focal length of the lens. Disturb the arrangement and take readings again. Note the mean focal length F. Find out the mean focal length f of the other lens similarly.

Mount the two lenses so that the lens of the longer focal length is nearer the object. The line passing through the

centres of the lenses is parallel to the length of the scale. Look through the two lenses at the distant tree and adjust the distance between them until the tree is seen most distinctly. Keep the eye just behind the lens of the shorter focus. Note that the tree appears much clearer than when looked at with the naked eye. It is inverted and magnified and looks as if it were brought near the eye. The lens farther from the eye forms an inverted and very small image of the distant tree at its focus, and the lens next to the eye aids the vision by magnifying the inverted image. The first lens is called the object glass and the second one the eye-piece. Note the distance between the centres of the lenses, i.e. between the two index points on the scale. Repeat the observation two or three times and take the mean distance d, and this would approximately be equal to the sum $F+f$.

Practical example.—

	F cm.	f cm.	d cm.
	16·0	5·1	20·9
	16·0	5·2	20·8
	16·05	5·1	20·5
			20·9
			21·0
Mean	16·0 cm.	5·1 cm.	20·8 cm.

$F+f = 21{\cdot}1$ cm. and this may be compared with the mean value of d which is equal to 20·8 cm. Note that d is slightly less than $F+f$. The *magnifying power* of a lens or combination of lenses as in the above, is the ratio of the apparent size of the image to that of the object when looked at directly. The apparent size of an object is measured by the angle which it subtends at the eye. As the image of the distant object is seen mostly distinctly it must have been at a distance of 25 cm. from the eye, because 25 cm. is the distance of most distinct vision for a normal eye. At distances greater than 25 cm. objects are seen distinctly but the degree of distinctness

gradually decreases. Below 25 cm. distinct vision is not possible and hence this distance of most distinct vision is also called the minimum or least distance of distinct vision.

In figure 77, PQ is the distant object, O and E are the two

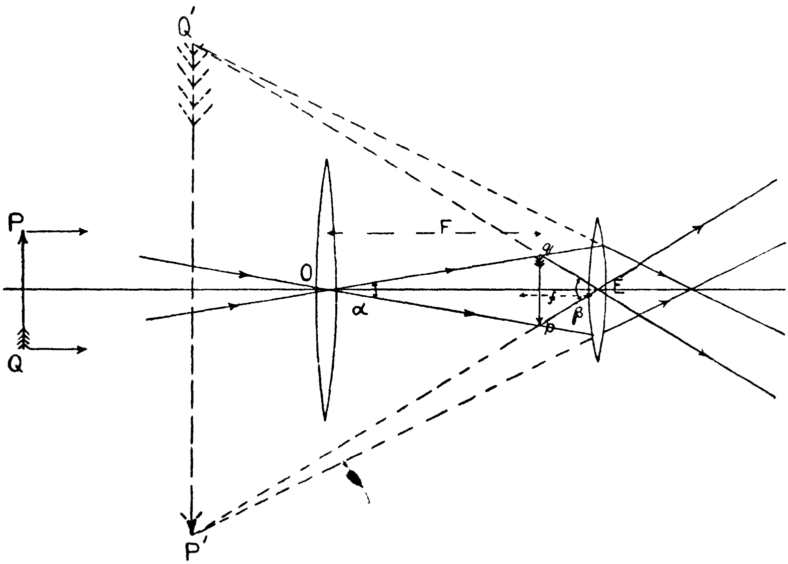


FIG. 77.

lenses of the combination, *qp* is the inverted image formed at the distance F from O. The eye piece E is at a distance f_1, slightly less than f from the image *qp*, such that OE is slightly less than F+f as was observed to be the case already. The magnified image Q′ P′, is most distinctly seen through the eye piece at the distance of most distinct vision from E. The rays after refraction through the eye lens form a divergent pencil as shown in the figure. If the eye piece E is slightly moved away from O such that it is exactly f cm. distant from q the rays after refraction go out in parallel pencils and the image would appear to be formed at a great distance

as shown in figure 78. The image seen in this case would

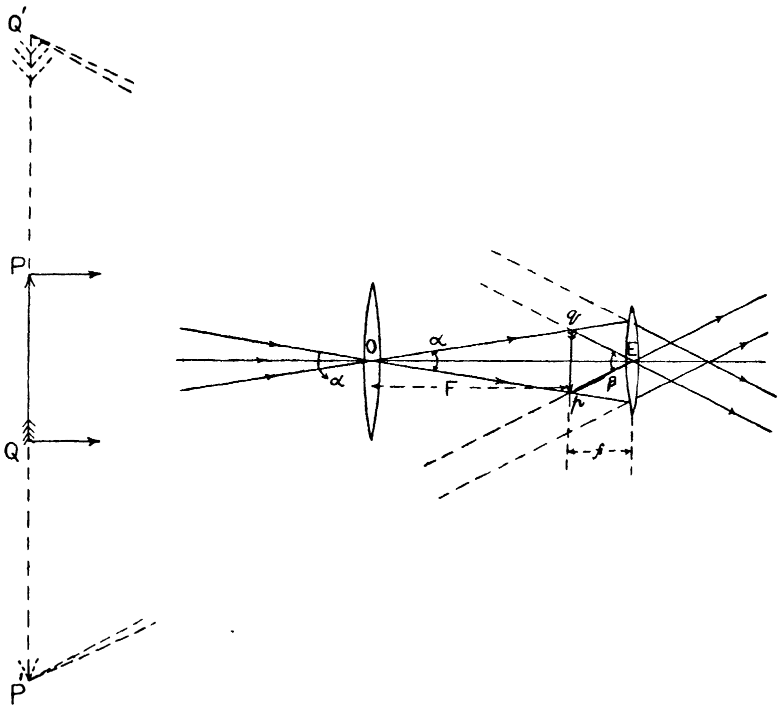


FIG. 78.

not be so very distinct as in the previous case. The first position of the eye piece is therefore the better one. The eye selects this position automatically and the image after refraction appears to be at nearly 25 cm. from the eye.

The apparent size of the image P′Q′ is measured by $\angle Q'EP' = \angle qEp = \beta$ and that of the object by $\angle PEQ = \angle POQ$ to the first approximation as PQ is at a great distance and as OE is small compared with it. The rays PO and QO passing through the centre of the lens O emerge out undeviated and therefore POp and QOq are straight lines.

$$\therefore \quad \angle POQ = \angle qOp = \alpha$$

Therefore by definition the magnification m of the combination is equal to $\frac{\beta}{\alpha}$. qp is really very small but is shown big in the figures for convenience. The angles α and β are very small in magnitude and when expressed in circular measure they are approximately equal to their tangents.

$$\therefore \quad m = \frac{\tan\beta}{\tan\alpha}$$

In the first case (fig. 77) $m_1 = \frac{F}{f_1}$ where f_1 is slightly less than f and in the second case (fig. 78) $m_2 = \frac{F}{f}$ and the image is formed at a great distance from the eye. If f is taken to be 5·1 cm., since the image is formed at 25 cm. from the eye in the better arrangement, and remembering that f is negative we have

$$\frac{1}{v} - \frac{1}{u} = -\frac{1}{f}$$

$$\therefore \quad \frac{1}{25} + \frac{1}{5{\cdot}1} = \frac{1}{u} \quad \left\{\frac{1}{D} + \frac{1}{f} = \frac{1}{u} \quad \therefore \quad \frac{f}{u} = 1 + \frac{f}{D}\right\}$$

$\therefore$ $u = 4{\cdot}2(4)$ cm. $= f_1$, i.e. the image pq is 4·2 cm. from the eye piece but not 5·1 cm. (f).

OE in the best arrangement will be $F + f_1 = 20{\cdot}2$ cm. $= d$. Hence in the readings taken above for d, 20·5 cm. is the nearest approach to the correct position and the image is therefore not formed at 25 cm. from the eye, but much further away. The least distance for the particular eye is therefore much more than that for the normal eye. From the mean value for d, calculate the mean value for v. When the object is at a great distance the value of m depends upon the value of f_1. The limits of magnification are therefore, as shown already (figs. 77 and 78), maximum $\frac{16}{4{\cdot}24} = 3{\cdot}8$ and minimum $\frac{16}{5{\cdot}1} = 3{\cdot}1$.

But $3{\cdot}8 = \frac{16}{5{\cdot}1} \times \frac{5{\cdot}1}{4{\cdot}24} = \frac{F}{f} \times \frac{f}{u} = \frac{F}{f} \times \left(1 + \frac{f}{D}\right)$

$\therefore$ maximum magnification = minimum magnification $\times \left(1 + \frac{f}{D}\right)$, where D = distance of most distinct vision.

A slight or minute variation in the position of the eye piece changes the magnification. The ratio of the focal lengths of the two lenses gives the lowest possible magnification of the telescope.

The magnification of the above combination may be experimentally found in the following manner.

Divide the edge of a wall in the verandah nearly 50 ft. distant from you, into a number of equal parts so that they are clearly visible to the naked eye. Direct the combination of the lenses to the edge. Look at the magnified image with one eye through the lenses and at the object directly with the other eye. Adjust the distance between the lenses by the method of parallax such that an image is formed just by the side of the object. Observe how many divisions of the object are apparently equal in size to one division of the image. This number gives the magnification. Take 3 or 4 sets of observations by looking at different points of the object. Keep the position of the eye the same while taking the readings. Tabulate the observations and the results thus:

Reading on scale directly looked at.	Corresponding reading on image.	Magnification.
x_1 x_2	y_1 y_2	$\frac{x_1 - x_2}{y_1 - y_2}$
x_3 x_4	y_3 y_4	$\frac{x_3 - x_4}{y_3 - y_4}$

Practical example.—

A wooden plank on which inches and feet are marked is placed vertically close to a wall 70 ft. away.

Distance between the lenses = 21·4 cm., which is slightly more than (F + f). (Why ?)

Reading on scale (direct).	Corresponding reading on image.	Magnification.
{ 6 ft. 3 in. 3 ft.	{ 5 ft. 6 ft.	$\frac{3' \, 3''}{1'} = 3{\cdot}25$
{ 8 ft. 6 ft. 4 in.	{ 4 ft. 6 in. 5 ft.	$\frac{1' \, 8''}{6''} = 3{\cdot}33$

Mean magnification = 3·3.

Substitute for the convex eye piece a double concave lens of about the same focal length and form a Galilean telescope. Find the magnification. Draw a neat sketch of the path of the rays coming from a distant object, as they pass out of the arrangement. Why is the image formed here erect ? Why is the Galilean design often used in opera glasses ?

THE SIMPLE MICROSCOPE.

Apparatus required for constructing a simple microscope are three optical stands (Pye & Co.), one provided with a lens holder and the other two with a metre scale holder, a 10 cm. and a 50 cm. boxwood millimetre scale pieces, a double convex lens of a short focal length, a metre scale and dividers.

Mount the lens and the scales on the stands at a suitable height. Place the stand C (fig. 79) carrying the longer scale, at a distance of 25 cm. behind the lens mounted on the stand O. Introduce the other stand L between O and C as in figure.

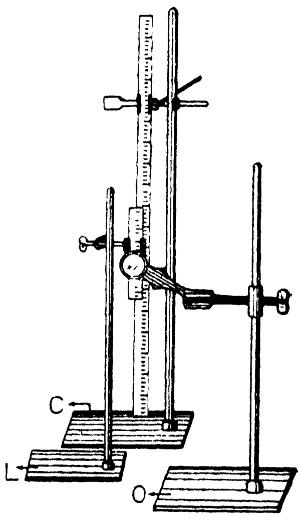


FIG. 79.

Place the eye near the lens and look through at the object scale mounted on L. Perhaps a blurred and magnified image of the scale is seen. Move the stand L to and fro slightly until the magnified image of the scale is clear. Look at this image through the lens with one eye and at the scale on the stand C directly with the other eye. Adjust the distance between the stands L and O by moving L, until the image is formed at the scale on C and does not move relatively to it when the eye is moved laterally. This secures accurate adjustment. A clear magnified image of the scale on L is formed on C which is a copy of the object scale L. Measure the distance (u) between the lens and scale L with the dividers and scale carefully and measure the distance (v) between the lens and the other scale. Disturb the arrangement and determine (u) and (v) again and take the mean values. *When a double convex lens is placed at such a distance from an object that a magnified image is seen distinctly at the least distance of distinct vision for the eye, the lens is said to be used as a simple microscope.* The above arrangement is the best for using the lens as a simple magnifying glass.

The magnifying power of a microscope is defined as the ratio of the angle subtended by the image at the eye to the angle subtended by the object at the eye, if it were placed at the least distance of distinct vision. To determine the magnification arrange the lens, etc. as described above and looking at

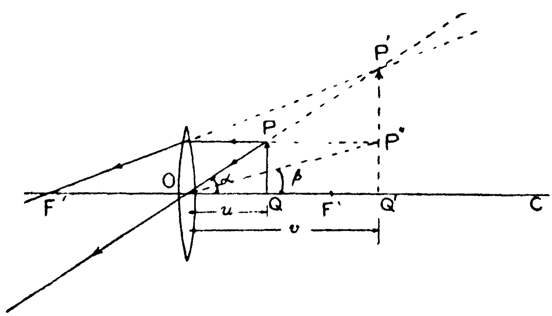


FIG. 80.

the image with one eye and the copy of the object scale with the other, note down sets of corresponding readings, as in the previous exercise. Tabulate and calculate the mean magnification. In figure 80 if PQ the object were placed along with the image at P″Q′, its apparent size is given by the $\angle P''OQ' = \beta$. (The lens O is supposed to be very thin and the eye is next to it.) The apparent size of the image is given by $\angle P'OQ' = \alpha = \angle POQ$.

$$\therefore \quad m, \text{ the magnification, } = \frac{\alpha}{\beta}.$$

Considering very small distances on the object at a time, the angles would be very small and $m = \dfrac{\tan\alpha}{\tan\beta} = \dfrac{P'Q'}{P''Q'} = \dfrac{P'Q'}{PQ}$

$$\text{or } m = \frac{\tan POQ}{\tan P''OQ'} = \frac{OQ'}{OQ} = \frac{v}{u}$$

If $v = 25$ cm. the corresponding value for u is the lowest possible (why ?) and the magnification is therefore the greatest. If the image is formed at a greater distance than 25 cm., u will be slightly greater than before and according to the definition of the magnifying power, as the apparent size of the object when placed at 25 cm. from the eye is always compared with that of the image, the magnification slightly falls. With the particular lens used the maximum magnification is $\dfrac{25{\cdot}5}{4{\cdot}15} = 6{\cdot}15 = \left(1+\dfrac{D}{f}\right)$* and, when the image is at a great distance and $u = 5{\cdot}1$ cm. the minimum magnification is $\dfrac{25{\cdot}5}{5{\cdot}1} = 5$. In the practical determination of the magnification the relation $m = \dfrac{P'Q'}{PQ}$ is utilized thus. In figure 81 the magnified and real scales are shown, E being the position

* $\dfrac{1}{D} + \dfrac{1}{f} = \dfrac{1}{u}$ and multiplying by D throughout, $1 + \dfrac{D}{f} = \dfrac{D}{u}$

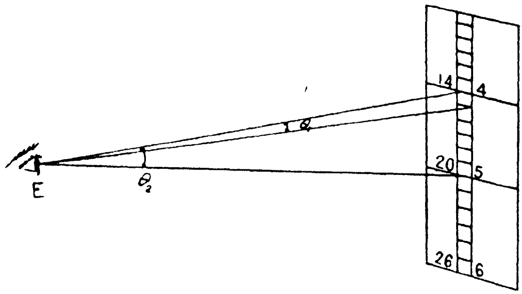


FIG. 81.

of the eye. Six divisions of the copy of the object scale subtend the same angle as one division of the magnified image of the object scale. θ_2 is six times θ_1, and $m = 6$.

Practical example :—

No.	Reading on copy of scale. cm.	Corresponding reading on the image. cm.	Magnification. m	Sets of observations used in calculation.
1	3·8	84·0	6·2	1, 3
2	6·8	84·5	6·3	2, 4
3	10·0	85·0	6·3	3, 5
4	13·1	85·5	6·0	1, 2
5	16·3	86·0	6·4	2, 3
			6·2	3, 4
			6·4	4, 5
			Mean 6·25	

In one of the positions arranged above $u = 4·15$ cm. and $v = 25·5$ cm. Hence applying $\frac{1}{v} - \frac{1}{u} = -\frac{1}{f}$, $f = 4·96$ cm.

This lens is the same as that used in the previous exercise in which f was found to be 5·1 cm. By removing the copy of the scale in the above exercise beyond 25 cm. some more positions might be obtained and the object adjusted so that the image is again formed at the copy of the scale. Get 2

or 3 such positions and from the values of v and u once again calculate f as shown above. This verifies the relation $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ for virtual images obtained with a double convex lens. Where would the image appear to be when the object scale is at the focus of the lens?

How would you practically obtain this focal length by an adjustment like that of the above?

Generally *the minimum or least distance of distinct vision for the eye* varies from 25–30 cm. with different persons and sometimes it is different for the two eyes for the same person. Hence, to have the magnification at its best, it is necessary that this distance should be accurately known for the particular person using the microscope.

To determine this distance for the eye, hold a metre scale horizontally, one end being very near the eye, almost touching it and the scale extending out lengthwise from the eye. Hold a book with the other hand. Look at the bold print and move the book at an arms length along the scale rather quickly, to and fro. The type remains in focus all along, i.e. the accommodation of the eye lens is automatic and quick. Bring the book nearer and repeat. The print appears blurred while in movement. This happens within a particular distance from the eye. Carefully note it on the scale. Take several such readings and the mean gives the least distance of distinct vision. Within this distance the image of the moving print will be blurred because the eye requires more time to thicken itself to focus the print on the retina than when the book was beyond this distance. This distance therefore gives the limit of the natural or automatic accommodation of the eye. If an object is to be seen distinctly within this distance a more or less distinct effort is required.

A normal eye can see objects best at the normal distance of 25 cm. from the eye. At this distance the size of the object is the biggest as the angle subtended at the eye is the

biggest possible, the distance of 25 cm. being the nearest for clear vision. This size is chosen as the standard for comparison.

If it were possible for the eye to see objects nearer than 25 cm. then the size of the object will be the greater, the smaller the distance and this is very desirable in order to examine objects minutely. This is made possible by inserting a convex lens of a small focal length between the object and the eye, the eye being next to the lens. The distance between the lens and the object is so adjusted that the virtual image formed is at the distance of 25 cm. the distance of most distinct vision for the eye. Now the size of the object seen through the lens, i.e. the size of the image, is bigger and is said to be magnified. This size is compared with the standard above mentioned and the ratio of comparison measures the magnification. In figure 80, β is the maximum angular size with the naked eye and α is the effective size brought about by the magnifying lens. The smaller the focal length of the convex lens the greater is the size of the object as seen through the lens and the greater therefore is the magnification.

THE SPECTROMETER.

In figure 82, A is a telescope (astronomical) movable about

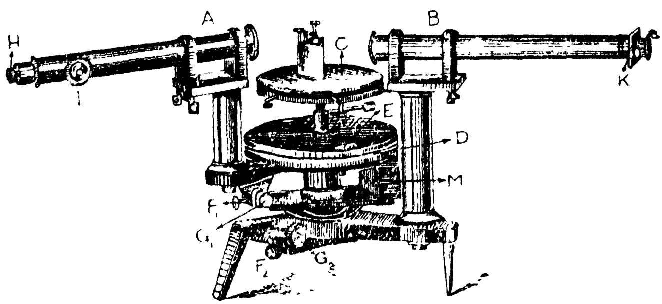


FIG. 82.

the vertical axis passing through the centre of the instrument.

The telescope is counterpoised by the mass M. B is a collimator fixed to one of the three legs on which the instrument is supported. C is the prism table of adjustable height and is movable about the vertical axis through the centre. D is a graduated circle fixed to the telescope and E is the vernier which can be screwed on to the prism table. F_1 and F_2 are two radial screws for clamping the telescope and the prism table respectively, in any desired position. Tangent screws G_1 and G_2 are provided for fine adjustments within a small range.

The spectrometer is adjusted for parallel rays in the following manner. Push in or pull out the eye piece H until the cross wires are distinctly seen.* (Pull out the eye piece completely and notice that inside the tube A are fixed in a vertical plane two very fine wires at right angles to each other. Adjust the eye piece once again as before.)† Direct the telescope towards a distant object and alter the distance between the object glass and the cross wires by turning the side screw I, until there is no parallax between the image of the distant object and that of the cross wires. Now a clear image of the distant object is formed in the plane of the cross wires and the *telescope is adjusted for parallel rays.* Turn the telescope to look into ~~the~~ collimator directly. Perhaps a blurred image of the slit at K is seen. Alter the distance between the slit and the collimating lens fixed to the end of B nearer C, by working the side screw (not shown in the figure) until the slit is clearly defined and until there is no parallax between the image of the slit and that of the cross wires. Now *the rays from the slit emerge out in a parallel pencil* after

* The cross wires are now slightly within the focal distance of the eye piece and the image is formed somewhere near the distance of most distinct vision. The pencil of rays from the cross wires after refraction through the eyelens is not parallel but slightly divergent.

† Are these wires visible to the naked eye at the same distance as before when the eye is aided by the eyelens? What is the least distance, at which these wires are visible to the unaided eye, called?

refraction through the collimating lens. Narrow the width of the slit sufficiently by working the screw K. *The spectrometer is now adjusted for parallel rays.*

The refracting angle of a glass prism is determined in the following manner.

Level the prism table of the spectrometer with a spirit level and mount the prism on it centrally such that the refracting edge is against the axis of the collimator. Reflected images of the slit illuminated by the diffused daylight will be seen when looked into the two polished surfaces of the prism. Direct the telescope to view one of the images, clamp the radial screw of the telescope and work the tangent screw until the cross wire is on the image of the slit. Note down the reading of the zero of the vernier on the scale. Move the telescope to the other side and looking into the other polished surface catch the reflected image on the cross wire as before. Take the reading of the zero of the vernier again. Note down the difference between the readings. Change the relative position of the prism table with respect to the telescope; the position of the vernier E relative to the graduated circle is thus changed. Repeat the observations and note again the difference in the readings of the zero of the vernier. Half the mean difference gives the refracting angle of the prism.

The path of the rays from the slit to the eye is sketched in figure 83.

S Slit.
C Collimating lens.
i The angle of the prism.
O Object lens.
E Eye piece.
P_1 Plane of the cross wires.
P_2 Focal plane of the eye piece.

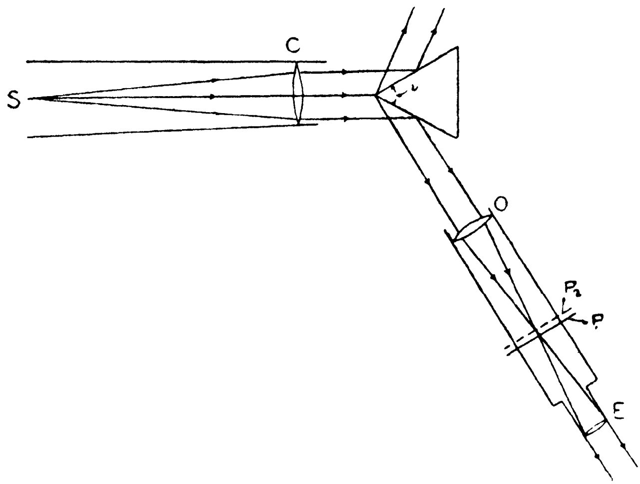


FIG. 83.

Practical example.—

The 'Student' Spectrometer (W. Wilson) was used. The scale was divided into degrees and half degrees. Thirty divisions on the vernier subtended the same angle as 29 half degrees at the centre of the divided circle. The vernier therefore reads to a minute of the arc.

	Scale reading.	Vernier reading.	Position of telescope.	Angle of prism.
I	114° 0′ 233° 30′	6′ 22′	114° 6′ 233° 52′	$\frac{119° 46'}{2} = 59° 53'$
II	81° 30′ 201° 0′	2′ 15′	81° 32′ 201° 15′	$\frac{119° 43'}{2} = 59° 51' 30''$
				Mean .. 59° 52′

The *refractive index of the material of the given prism with sodium light* is found as follows.

The angle of the prism having been determined, the angle of the minimum deviation remains to be found. This is usually done in a dark room. Narrow the slit so that it

appears like a fine line and illuminate it with sodium light.* Mount the prism on the prism table centrally with the refracting edge vertical and turn the table such that the parallel pencil of rays from the collimator is incident at a small angle on the first face of the prism. Turn the telescope and look through the second face at the refracted image of the slit. What is the colour of the image? Is the image single or double? What do you infer regarding the nature of the source of light? In this position the direction of the emergent light, i.e. that of the axis of the telescope nearly grazes the second surface of the prism and hence the deviation is nearly the maximum. Turn the prism table slightly and increase the angle of incidence. Move the telescope towards the apex of the prism and look through for the refracted image of the slit. Continue to increase the angle of incidence, looking at the image through the telescope. As the angle of incidence on the first face is gradually increased the rapidity with which the emergent ray moves towards the apex gradually decreases and at a stage the position of the refracted image gets comparatively steady, i.e. even though the angle of incidence is slightly increased or decreased, the image keeps nearly the same relative position to the cross wires of the telescope. If the angle of incidence on the first face of the prism is further increased the image of the slit, recedes backwards and for all other angles of incidence, the position of the telescope comes nearer the base of the prism. Get back to the position of the prism where the image of the slit appears to move very slowly towards the apex of the prism. Clamp the prism table in this position. Bring the vertical cross wire of the telescope to coincide with the slit. Work the tangent screw of the prism table and find accurately the position of the prism where, when the tangent screw is moved either

* Asbestos cord wound round an iron ring, and soaked in a concentrated solution of sodium chloride is introduced into the Bunsen colourless flame.

way, the refracted image moves away from the apex of the prism, i.e. away from the direction of the incident light. In this position of the prism the emergent refracted pencil of light is nearest the direction of the incident light and hence *the position of the prism is one of minimum deviation.* Clamp the telescope and work the tangent screw so that the vertical cross wire is exactly on the image of the slit. Read the position of the telescope on the scale. Remove the prism from the prism table. Move the telescope and look at the slit through the collimator directly. Work the radial and tangent screws so that the vertical cross wire coincides with the image of the slit. Read the scale and vernier again. This gives *the direct reading of the telescope.* The difference between the minimum deviation reading and the direct reading of the telescope gives the angle of minimum deviation (D). Disturb the position and take another set of observations as before. Take the mean angle of the minimum deviation. The angle of the prism i being already found calculate the refractive index of the prism μ which is equal to $\dfrac{\sin \frac{1}{2}(D+i)}{\sin \frac{1}{2}i}$.

Practical example.—

	Deviation reading.	Direct reading.	Angle of minimum deviation. D
I	113° 10′	152° 7′	38° 57′
II	166° 45′	127° 46′	38° 59′
			Mean D = 38° 58′

Angle of the Prism = 59° 52′

$$\tfrac{1}{2}(D+i) = 49^\circ\, 25'$$
$$\tfrac{1}{2}i = 29^\circ\, 56'$$
$$\mu = \frac{\sin 49^\circ\, 25'}{\sin 29^\circ\, 56'} = 1{\cdot}522.$$

1. To which dark line of the solar spectrum does the above value belong ?

2. How do you proceed to determine the value with the solar light ?

3. Is the minimum deviation position of the prism, the same for all the different Fraunhofer lines of the solar spectrum ?

4. Determine the minimum deviation values for a few of the prominent dark lines of the solar spectrum.

Chapter XI

MAGNETISM

MAGNETS.

Magnets are of two kinds, namely, natural and artificial. Natural magnets are obtained in black stones and are composed largely of the magnetic oxide of iron (Fe_3O_4). These were known to the ancients; they knew that pieces of iron were attracted by them. These stones were found abundantly in Asia Minor at a place called Magnesia and hence the name magnet. When suspended freely the stone points north and south, one end always seeking north and the other seeking south. This was, therefore, of great use to mariners and was called the leading stone—hence the name loadstone or lodestone. Artificial magnets can be made of iron, nickel and cobalt. These metals acquire the property of a natural magnet when rubbed against it. Out of the three, iron acquires it best. Soft iron acquires it temporarily; it does not retain the property in the absence of the influencing magnet. Steel acquires it permanently. Hence all artificial magnets are made of tungsten or cobalt steel. They are made into different shapes to suit different purposes. Magnetic needles, bar and cylindrical magnets are generally used in the laboratory. The north-seeking end of a magnet is marked N or painted red and is briefly called the north end. The other end is marked S.

Suspend a magnet freely. Bring another magnet near it and present the ends to one another. Notice that like ends repel each other while the unlike ends attract each other. Present a soft iron nail to the suspended magnet. Note that both the ends of the magnet attract either end of the nail. It is thus clear that a soft iron piece behaves differently

from a magnetised piece of iron when placed in the neighbourhood of a suspended magnet. How then can you determine whether a given piece of iron is magnetised or not? Which is the test, repulsion or attraction?

Test the attracting force of the magnet along its length by noting how many small iron nails it can support at different points. This force is maximum at two regions near the ends, as shown in figure 84 and falls gradually towards the centre from either end. The central points within the regions at either end are called *poles* of the magnet. The line passing through the two poles is called the *axis* of the magnet. The distance between the poles is called the *magnetic length* of the magnet. The line through the centre at right angles to the axis is called the neutral line of the magnet and the attraction along this line is zero. The force of attraction at different points on the axis measures what is called *the free magnetism* of the magnet.

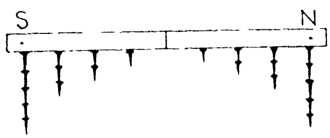


FIG. 84.

Why should a freely suspended magnet point only north and south? We naturally infer that this is due to the influence of the earth which is a big magnet. The space round the earth is a sphere of influence and is called the *earth's magnetic field*. This is a field of force and the earth pulls a suspended magnetic needle along the direction of this force and the axis of such a needle gives the direction of the earth's field at the point. If the direction is followed up by such a needle continuously, a *line of force of the earth's magnetic field* is obtained. The lines of force of the earth's magnetic field are arbitrarily supposed to run from the south towards the north, i.e. *the convention regarding the direction of the lines of force is, that they start from the south pole of the earth and go to the north pole.* This is, therefore, the direction which the north-seeking end of a freely suspended magnetic needle

points. The same considerations hold good in the case of any artificial magnet. Such a magnet has around it a space or sphere of influence within which a suspended magnetic needle is deflected out of its natural direction, i.e. out of the direction due to the influence of the earth alone. The direction of the axis of the deflected needle is therefore the resultant of the influence due to the magnet and that due to the earth. The convention regarding the direction of these magnetic lines of force is, from the north-seeking pole (N) (which is really the south pole) towards the south-seeking pole (S) (which is really the north pole). It must be clearly understood that a real south pole of the magnet is one that is attracted by the north pole of the earth and when freely suspended would seek the north magnetic pole of the earth. So the south pole of the magnet is its *north-seeking pole* which, for brevity, is called the *north pole.* Now the convention may be re-stated; the lines of force start out of the north-seeking or the north pole of a magnet and go towards its south-seeking or the south pole. All the lines starting at the N end of a magnet do not enter the S end but some of them go towards the north magnetic pole of the earth. Similarly some of the lines that enter the south pole, S, of the magnet, come from the sou~~th po~~le of the earth.

A line of magnetic force is the curve such that the tangent at any point on the curve gives the direction along which the resultant magnetic force at that point acts. A delicately suspended small magnetic needle, placed at any point in the field, lies with its axis parallel to the direction of the field at the point, and the axis of the magnet is a tangent to the magnetic line of force passing through that point.

The earth's field of force in a restricted area, say in the laboratory, can be determined with a fine pivoted magnetic needle. The needle stands with its length north and south at any point in the room, provided there are no magnets or magnetisable substances near by. When projected down and

produced, the directions at different points will be found to be parallel. Hence the lines of force of the earth's field in the room run northwards and are all parallel. Such a *parallel field* of force is called a *uniform field.*

Magnetisation.—There are different methods of magnetisation. We shall now describe the method of *single touch.*

Take a piece of steel watch spring *sn* (fig. 85) and lay it on the table. Keep a bar magnet NS as in the figure and draw it along from one end to the other and bring it back to the starting position as shown by the arrows and repeat the process a number of times. Present the end *s* which the north end of the magnet NS left last to a magnetic needle suspended on a vertical axis and it will repel the south-seeking end of the needle. The other end *n* will repel the north end of the needle. The spring is permanently magnetised. Note that the magnet is more and more powerfully magnetised as the number of strokes given by the magnet NS is increased. Note also that the power reaches a maximum after a certain number of strokes. This power or magnetic force can be judged by the deflection produced and the distance at which it is produced.

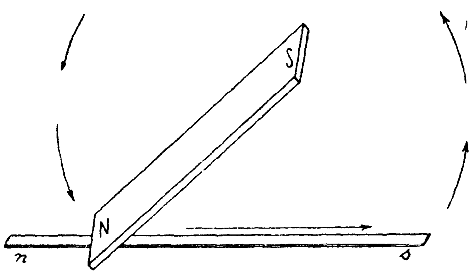


FIG. 85.

Cut the spring into pieces and keep them in position as cut, marking with chalk if necessary the right-hand end of each piece. Test the pieces one after the other with the pivoted needle. You will find that all the right ends are south poles and that each piece behaves exactly like a magnet. The ends of consecutive pieces in juxtaposition are of opposite polarities. This leads us to the idea that any small bit of the spring must be a complete magnet in itself. It can be easily

imagined that the same will be the case when the process of subdivision is further and further continued.

Magnetise another piece of the spring and heat it red-hot. Cool it by keeping it aside for a time. Examine it with the suspended magnetic needle and you find that there is no repulsion and that the spring has lost its magnetism or has been demagnetised completely. Red heat restores the spring to its original unmagnetised condition. Similarly, a magnet loses much of its magnetism by constant rough handling.

An alternative method of magnetisation is the one of *divided touch.* Take two magnets of the same size NS and N_1S_1 (fig. 86) and keep them on an unmagnetised piece of

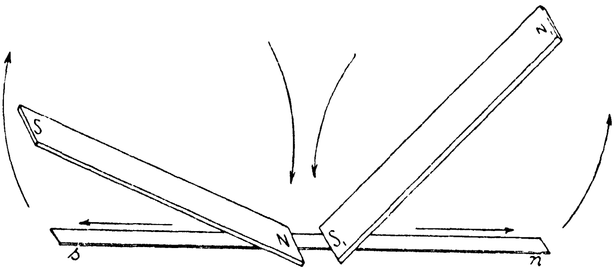


FIG. 86.

the spring *sn* with their opposite ends together at the centre of the spring. Draw them in opposite directions to either end of the spring and bring them back to the original positions, as shown by the arrows. Repeat the process a number of times. You find that the left end is a south pole and that the right end is a north pole. Note that the polarity of either end of the spring is opposed to that of the magnetising end of the moving magnet. A spring magnetised by this method will be found to be stronger or more powerful than that magnetised by the method of single touch.

Molecular Theory of Magnetisation.—Each molecule of a magnetisable substance like iron is by nature a magnet with a south and a north pole. In the unmagnetised state the

molecular magnets have their axes pointed in all possible directions. The south pole of one molecule is against the north of another and so on, until a number of the molecular magnets form a closed ring so that the ring has no external effect.

When a steel spring is magnetised by the method of single touch the molecular magnets are dislodged from the closed ring or circuit and all the south ends of the molecules are directed along the line of movement of the north end of the magnetising magnet. The north poles are repelled by the moving end into the opposite direction. This arrangement will be complete after a number of strokes and the spring is completely magnetised. The same explanation holds good in the case of magnetisation by the method of divided touch. One magnet helps the other in the arrangement of the molecules, and the process is more speedy and complete.

The axes of the molecules lie along the length of the spring, all the n ends pointing in one direction. The ends at either end of the spring are free, no opposite poles being in juxtaposition. Hence is the existence of poles near the ends.

If by any means the above arrangement is disturbed completely or partially, the magnetisation is also completely or partially destroyed. Heating the spring increases, according to the kinetic theory of matter, the activity of the molecules and endows them with greater energy of motion. When heated to redness the activity of the molecules is so great that the arrangement of the molecules, on which the magnetised condition depends, is not at all possible and hence the spring is completely demagnetised.

Dropping the magnet or handling it roughly causes a mechanical disturbance to the molecules resulting in partial disarrangement which depends on the magnitude of the mechanical disturbance caused. A hard blow has a great demagnetising effect.

The above theory and explanation of magnetisation of iron (and other materials) is further illustrated by the following experiment.

Fill a test-tube with iron filings. Cork it tight. Keep the test-tube horizontal on a table. Draw a bar magnet as in the case of magnetisation by single touch from one end of the test-tube to the other, a number of times. The filings arrange themselves visibly with their lengths parallel to the direction of the motion of the magnet and the tube behaves like a magnet. Shake the tube well and disturb the arrangement. You find that the tube ceases to be a magnet.

MAGNETIC FORCES.

Magnets employed in practical work are generally long and thin. The section may be a circle or a rectangle. In all such cases, the ends of the magnet may be regarded practically to be the poles and centres of magnetic action and the rest of the magnet may be considered to be inactive. The same idea may also be expressed by saying that a quantity of magnetism is concentrated at either pole and that the result of magnetisation is the concentration of equal and opposite quantities of magnetism at the two ends. This charge of magnetism at either end is called the *pole strength* of the magnet and is represented generally by the letter m. The strength of the north pole is considered positive and that of the south pole negative. Results that follow from these simple conceptions are found to be true in practice and observed facts are satisfactorily explained on their basis.

Coulomb's Law.—Though it was known from ancient times that two like poles repel each other and unlike poles attract each other, Coulomb was the first to enunciate quantitatively the law of mutual action between two magnetic poles in the following manner. *The force of action between two magnetic poles is directly proportional to the product of the strengths of the poles and varies inversely as the square of the distance*

between them. This may be symbolically expressed as $F \propto \frac{mm'}{d^2}$, where F is the force between the poles, m and m' are the strengths of the poles and d is the distance between them.

If all the quantities on the right-hand side remain the same, the force of action depends upon the nature of the medium between the two poles and the equation can be written as $F = \mu\frac{mm'}{d^2}$, where the value of μ, the constant of variation, depends on the nature of the medium. For air the factor of variation is unity. Ordinarily, therefore, the equation can be written as $F = \frac{mm'}{d^2}$.

In the C.G.S. system, the strength of a pole is unity when it repels an equal pole with a force of one dyne, when the poles are one centimetre apart.

The *intensity of a magnetic field* at any point is defined as the resultant magnetic force with which a unit positive pole is urged along the direction of the magnetic line of force at that point.

Let *ns* (fig. 87) be a magnet of pole strength m. O is

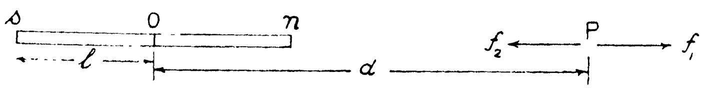


FIG. 87.

its middle point and $2l$ its length. Let P be a point *on the axial line*, nearer the north pole of the magnet, at a distance of d cm. from the middle point of the magnet. A force of repulsion $f_1 = \frac{m}{(d-l)^2}$ dynes, due to the pole n, acts on a unit north pole at P and a force of attraction $f_2 = \frac{m}{(d+l)^2}$, due to the pole s, also acts at P. The resultant force is evidently

one of repulsion as f_1 is greater than f_2. The direction of the resultant force is along $\overrightarrow{nP}$ and its magnitude F is obtained in the following manner:

$$F = f_1 - f_2 = m\left\{\frac{1}{(d-l)^2} - \frac{1}{(d+l)^2}\right\}$$

$$= \frac{m.4dl}{(d^2-l^2)^2} = \frac{4ml}{d^3}$$

if l^2 is small compared with d^2.

Let NS (fig. 88) be a magnet. OP is the *equatorial line* and we shall find the field at P. $OP = d$ and $NO = l$. P is symmetrically situated with respect to NS and so $NP = SP = \sqrt{d^2+l^2}$. A unit north pole placed at P is urged along $\overrightarrow{NP}$ by a force of repulsion $f = \frac{m}{(NP)^2} = \frac{m}{d^2+l^2}$ dynes and is attracted by the south pole S along $\overrightarrow{PS}$ by an equal force $f = \frac{m}{(SP)^2} = \frac{m}{d^2+l^2}$ dynes. Resolve these two forces acting at P along OP and at right angles to OP. The components of f along $\overrightarrow{NP}$ are $f \sin\theta$ along $\overrightarrow{OP}$ and $f\cos\theta$ parallel to $\overrightarrow{NS}$. The components of f along $\overrightarrow{PS}$ are $f\sin\theta$ along $\overrightarrow{PO}$ and $f\cos\theta$ parallel to $\overrightarrow{NS}$. Adding up the components, $f\sin\theta$ and $f\sin\theta$ being equal and opposite cancel each other, whereas $f\cos\theta$ and $f\cos\theta$ act together. So the resultant force or the intensity at P

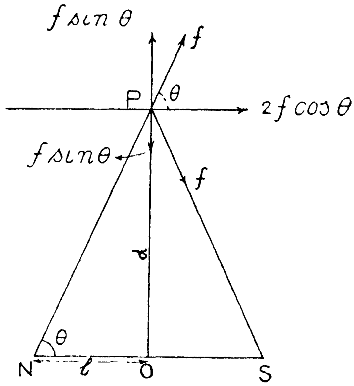


FIG. 88.

is along a direction through P parallel to the length of the magnet $\overrightarrow{NS}$ and its magnitude $F = 2f \cos\theta$, where $f = \frac{m}{d^2+l^2}$ and $\cos\theta = \frac{l}{\sqrt{d^2+l^2}}$.

$$\therefore\ F = \frac{2ml}{(d^2+l^2)^{\frac{3}{2}}} = \frac{2ml}{d^3} \text{ dynes,}$$

if d^2 is very big compared with l^2.

Intensity at any point in the field of force of a magnet can be calculated from the law of magnetic force. Let NS (fig. 89) be a thin long magnet of pole strength m. Let P be a point in the field. Let $NP = d_1$ cm. and $SP = d_2$ cm. A force of repulsion $f_1 = \frac{m}{d_1^2}$ dynes, due to the pole N, acts along $\overrightarrow{NP}$ and a force of attraction $f_2 = \frac{m}{d_2^2}$ dynes acts along $\overrightarrow{PS}$. Let f_1 and f_2 be represented by the adjacent sides PA and PB of a parallelogram. The resultant force at P is represented in magnitude and direction by the diagonal PC. This is a general case where P is any point and the direction of the resultant force makes an angle with the length NS of the magnet. The two cases where the resultant force is parallel to the length of the magnet are very simple and an expression for the intensity can be easily obtained as shown already. These two cases are (i) intensity at all points in the line in continuation of the axis (on either side of the magnets), i.e. along the axial line, and (ii) intensity at all points along the line drawn at right angles to the length of the magnet through its centre, i.e. along the equatorial line of the magnet.

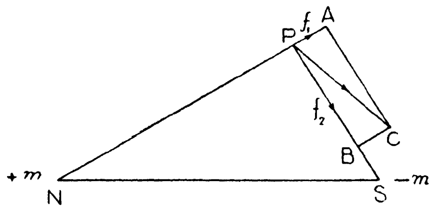


FIG. 89.

A magnetic needle delicately suspended on a vertical pivot, so as to be free to move in a horizontal plane will place itself with its length in the magnetic meridian.* If the needle is disturbed, it executes oscillations about the position of rest and comes back to it after a number of oscillations.

Magnetic Moment of a Magnet.—Suppose the needle is deflected through an angle θ from the position of rest (fig. 90). The north pole in its deflected position N_1 is acted on by a force mH dynes towards the north, where m is the strength of the pole and H is the intensity of the earth's field. The south pole S_1 is acted on by an equal force mH dynes in the opposite direction, towards the south (?). These two forces constitute a couple. Each force has a moment about the pivot in the clockwise direction, the magnitude of each moment $= m\mathrm{H} \times \mathrm{OB}$. $\mathrm{OB} = \mathrm{NN}_1 = l \sin\theta = \mathrm{OA}_1 = \mathrm{NA}$, where $l = \mathrm{ON}_1 = \mathrm{OS}_1$—half the length of the magnet. The total moment of the two forces about $\mathrm{O} = 2m\mathrm{H}l \sin\theta = m\mathrm{H}.2l \sin\theta = m\mathrm{H} \times \mathrm{N}_1\mathrm{A}$, and this is called the moment of the couple about O. The maximum angle of deflection is 90° and the maximum moment of the couple tending to restore the magnet to the position of rest is $m\mathrm{H} \times 2l \sin 90° = m\mathrm{H} \times 2l$. The maximum moment of this restoring couple when the needle is in a uniform field of unit intensity (H = 1) is therefore $m \times 2l$ and is a constant for the needle or any magnet of pole strength m and length $2l$. This constant is called the moment of a magnet and is represented by the

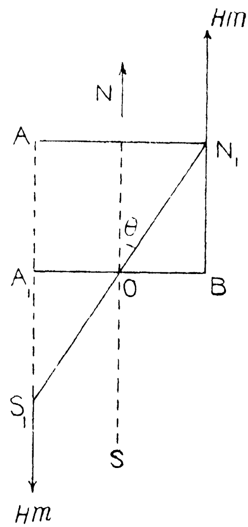


FIG. 90.

* The vertical plane passing through the axis of a freely suspended magnetic needle under the influence of the earth's magnetic field alone is called the *magnetic meridian*.

letter M and is numerically equal to $2ml$, the product of the length and pole strength of the magnet.

Tangent Law of Magnetic Forces.—The intensity of the earth's magnetic force H is found to be constant in any particular locality. The intensity of the field due to a magnet along its axial and equatorial lines varies from point to point but if the point chosen is far enough, the intensity F in a very small area round the point may be considered to be constant and the lines of force in that area to be parallel. In the case of the earth which is a very big magnet the area of uniform intensity is correspondingly big and hence is practically constant in any particular locality.

At any point round a magnet the field due to the magnet and that of the earth are superposed. The case where the two superposed fields are at right angles to each other is important and useful.

Let two such magnetic fields of force be superposed at O (fig. 91), the centre of a pivoted magnetic needle NOS. Let the intensity of the field due to the magnet be F and that due to the earth be H. So long as the disturbing force or deflecting force of intensity F is on, the needle will be deflected from its position of rest, i.e. from the magnetic meridian, by an angle θ. The earth's field tends to restore the needle back to the meridian and the moment of this restoring couple is $2ml\text{H} \sin \theta$. The field F acts on the north pole with a force $m\text{F}$ along the direction of the lines of force $\overrightarrow{\text{SA}}$ and the

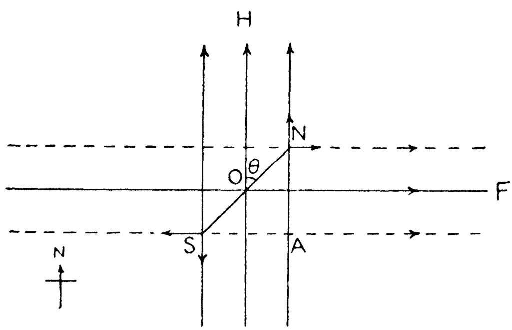


FIG. 91.

south pole is acted on by an equal and opposite force. These two forces constitute a couple and the moment of the deflecting couple about O is $m\text{F}\times\text{AN} = m\text{F}\times 2l\cos\theta = 2ml\text{F}\cos\theta$. The deflecting and restoring couples balance each other.

$$\therefore\ 2ml\text{F}\cos\theta = 2ml\text{H}\sin\theta$$

$$(\text{Deflecting couple}) = (\text{Restoring couple})$$

$$\therefore\ \text{F}\cos\theta = \text{H}\sin\theta \text{ or } \frac{\text{F}}{\text{H}} = \tan\theta.$$

The tangent of the angle of deflection measures the ratio of the deflecting force to the restoring force. The same law was studied and verified under tangent law of forces in chapter III. The forces there are mechanical and here magnetic.

LINES OF FORCE IN A MAGNETIC FIELD.

Apparatus required for tracing the lines of force in the field round a magnet are a drawing board, pins, a sheet of white paper, a *small, freely moving, pivoted, magnetic needle* and a magnet (about 10 cm. long). Pin the paper on the board. Keep one edge of the board coinciding with that of the working table. This secures the position of the board (?). Place the magnetic needle at the centre of the board, keeping the magnet away at a distance and project the ends vertically on the board with a pencil. Join the points thus marked and produce the line. This is a trace of the magnetic meridian at the place. Bring the magnet near and place it on the board with its axis lying over the line marked. Let the north end N point northwards. Trace the position of the magnet and take care to see that the position is not disturbed during the experiment. Put the needle near the left edge of the board, at the point A (fig. 92). Project the points s and n on the paper, move the needle such that the south end occupies the place which the north end occupied in the previous position.

Project the point n_1 again on the paper. Move it on and

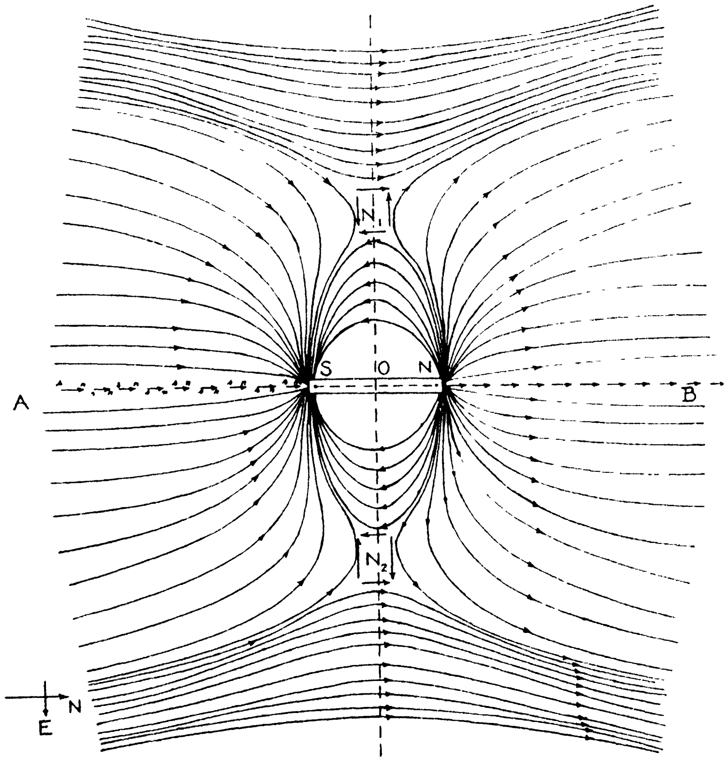


FIG. 92.

repeat the process until the needle is brought to the south-seeking pole of the magnet S.

In each position the axis of the needle gives the direction of the resultant magnetic force at its centre and hence is a tangent to the line of force passing through it. If these consecutive points are joined, a line of force is traced. Mark with an arrow-head the direction on the line along which the north end of the needle points. Starting again at a point above A trace another line of force. Having traced a number of lines on the left, trace a corresponding set of lines on the

right. Now bring the needle to the north-seeking end of the magnet N and trace a number of lines which start at N and go to S of the magnet. Explore the other half of the field similarly. Trace completely the lines of force in the region round the magnet, which lie in the plane of the paper as shown in the figure. Mark the regions of space N_1 and N_2 which are symmetrically situated on either side of the magnet. N_1ON_2 is a line at right angles to the length of the magnet, passing through its centre. This is the equatorial line of the magnet and AOB is the axial line. Place the needle at different points along N_1ON_2 and note the direction which the north end of the needle points. Note the change as the needle passes through regions N_1 and N_2. What are these regions called, and why? Move the needle up along the line passing through O at right angles to the plane of the board. What do you notice regarding the direction of the axis of the magnetic needle? How does it compare with that observed along the line ON_1 or ON_2? What do you infer? Are the lines of force confined to a single plane? Note that a movement of the needle along the lines AS and BN would not produce any change in the direction of the axis of the needle.

Note that the tracing of the lines of force about the regions N_1 and N_2 is not so easy as at others. Why?

Note also the crowding in of the lines near the poles of the magnet.

Trace the lines of force, keeping the north-seeking end of the magnet southward. Where will be the neutral regions situated, and why?

1. How do you trace the lines of force due to the earth's magnetic field in a horizontal plane, and how do they look like?
2. What is the field traced in the above exercise due to?

An alternative method of tracing the lines of force is to use iron filings.

Apparatus required are some fine iron filings in a cylindrical box with a perforated top for sprinkling the filings, a glass plate of large size, magnet, and a few cork pieces of about the same thickness as the magnet.

Place the magnet in the desired position on the table and place the glass plate on it so that the magnet lies at the centre. Insert the cork pieces at the corners of the plate. Sprinkle iron filings lightly and uniformly on the plate and tap gently with the forefinger at a corner of the plate. Magnetism is induced into the filings, they temporarily behave like magnets and turn their axes parallel to the lines of force. The resultant magnetic field in the plane of the glass plate is thus visualized and a map obtained. Note how the filings appear in the neutral regions of the field.

MAGNETOMETERS.

Deflection Magnetometer.—If a freely suspended magnetic needle is simultaneously under the influence of two uniform magnetic fields whose directions are at right angles to each other, the needle occupies a resultant direction running between the directions of the lines of force due to each field. If θ be the deflection of the needle (fig. 91), i.e. the angle which the axis of the magnet makes with the magnetic meridian, and if one of the fields be that of the earth, of horizontal intensity H and the other be that due to a magnet of intensity F at the needle, the ratio of the intensities of the two fields, the latter to the former, is equa to the tangent of the angle of deflection or $\frac{F}{H} = \tan\theta$.

Methods of arranging a given magnet in different positions round a pivoted magnetic needle, such that the field due to the magnet at the needle is at right angles to the earth's field, will now be given.

Apparatus required are a magnet, a freely suspended magnetic needle, or a magnetometer box, drawing board, etc.

The magnetometer box, a top view of which is given in fig. 93,

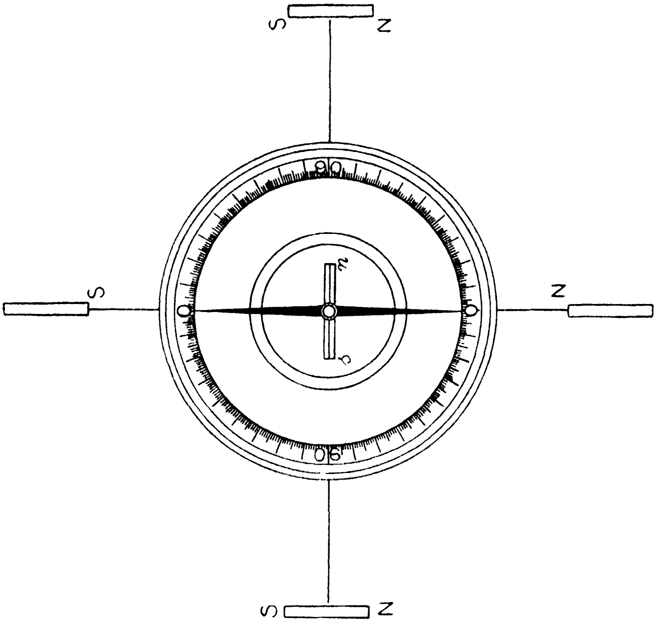


Fig. 93.

consists of a finely suspended needle (jewel suspension on a steel pivot) at the centre of a graduated circle reading to degrees of an arc. Over the divisions of the scale moves an aluminium pointer which is attached to the centre of the needle at right angles to its length. The whole arrangement is enclosed in a circular, flat-bottomed brass case. The bottom of the case is partly covered by an annular, anti-parallax mirror and the top is closed with a glass plate.

Let the edge of the drawing board coincide with that of the working table (?). Place the magnetometer box at the centre of the board; keep the magnet off and project the magnetic east and west line on the board by placing the edge of a set square over the glass top, along the pointer. Remove

the magnetometer and complete the line. A line at right angles to this at any point gives the north and south line. With the point of intersection O (fig. 94) as the centre describe a circle of radius equal to that of the bottom of the case. Fit in the box into the circle. Place the magnet *with its axis* always lying *east and west*, as in the figure. Only four typical positions are possible. The fields are at right angles to each

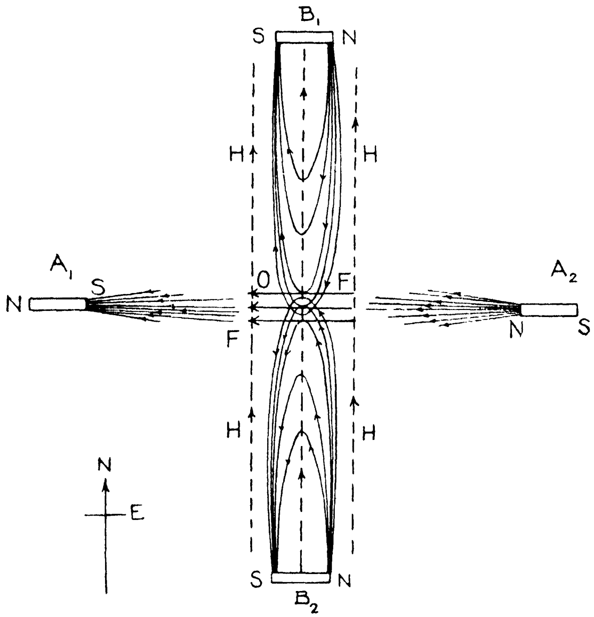


FIG. 94.

other at O the centre of the needle. The direction of lines of force due to the magnet is always east to west for the four positions of the magnet.

In the two positions marked A_1 and A_2 which are similar, the point O is on the axial line of the magnet and since one end of the magnet is on the point O these positions are called the *end-on positions* and also the *tangent A positions*

of Gauss *. The point O lies on the equatorial line of the magnet in the positions marked B_1 and B_2 and these positions are called the *broad side-on positions* or *tangent B positions of Gauss.*

The intensities of the field due to the magnet at the centre of the needle, in the two positions A and B, may be compared in the following manner.

Mark the points A_1, B_1, A_2 and B_2 (fig. 94) equidistant from O. Place the magnetometer box in position and turn the box about O so that the ends of the pointer read zero on the scale. Put the magnet NS with its length east and west and its centre coinciding with one of the points marked. Observe the deflections of both the ends of the pointer. Reverse the magnet and note the deflections again. Take the mean deflection. Proceed similarly in the other three positions and tabulate thus :

Position.	Mean Deflection.	Intensity.
A_1	θ_1	$F_1 = H \tan \theta_1$
A_2	θ_2	$F_2 = H \tan \theta_2$
B_1	θ_3	$F_3 = H \tan \theta_3$
B_2	θ_4	$F_4 = H \tan \theta_4$

It will be noticed that $F_1 = F_2$, and $F_3 = F_4$ very nearly. Generally, $\frac{F_1}{F_3}$ will be greater than 2 and this ratio can be calculated theoretically.

$$F_1 = \frac{2Md}{(d^2-l^2)^2}$$

$$F_3 = \frac{M}{(d^2+l^2)^{\frac{3}{2}}}$$

$$\frac{F_1}{F_3} = \frac{2d(d^2+l^2)^{\frac{3}{2}}}{(d^2-l^2)^2}$$

and this ratio will be equal to *2* when l^2 is very small compared with d^2. This calculated ratio will be found to be nearly equal to the

* Gauss was the German physicist who used these two positions in doing experiments on magnetic measurements. The unit of magnetic intensity is called the *gauss.*

observed one. Since the above formulae were deduced by assuming the truth of the inverse square law of magnetic attraction, this agreement in the two ratios *verifies indirectly the inverse square law.*

The magnetic moments of two magnets may be compared by two alternative methods.

(1) Replace the magnet in the above experiment (fig. 94) by another and take observations similarly. In the position A_1, let the intensity be F_1' and the mean deflection observed be θ_1'. The ratio of the intensities of *the two magnets at the same distance from O and in the same position* $A_1 = F_1/F_1'$ $= \dfrac{\tan\theta_1}{\tan\theta_1'}$

$$\therefore \quad \frac{2M_1 d}{(d^2-l_1^2)^2} \Big/ \frac{2M_2 d}{(d^2-l_2^2)^2} = \tan\theta_1/\tan\theta_1'$$

where the moments of the magnets are M_1 and M_2, and l_1 and l_2 are their half lengths. If l_1 and l_2 are nearly the same, since d is the same, $M_1/M_2 = \tan\theta_1/\tan\theta_1'$ nearly. Calculate the ratio from the observations in the other three positions also and take the mean ratio $\dfrac{M_1}{M_2}$.

(2) This is a *null method.* Place the two magnets *in the same tangent position*, say B, one on either side of the needle at such distances that the deflections produced are equal and opposite. The resultant deflection observed will be zero. Since the intensities at O are equal,

$$\frac{M_1}{(d_1^2+l_1^2)^{\frac{3}{2}}} = \frac{M_2}{(d_2^2+l_2^2)^{\frac{3}{2}}}$$

and if d^2 is large compared with l^2, we have

$$\frac{M_1}{d_1^3} = \frac{M_2}{d_2^3}, \text{ i.e., } \frac{M_1}{M_2} = \left(\frac{d_1}{d_2}\right)^3$$

This ratio will be nearly the same as that obtained in the first method.

Practical example.—

Comparison of intensities in the tangent A and tangent B positions of a magnet.

NOTE : *4°S means 4° deflection towards the south, etc.*

Magnet No. 1. Length $2l_1 = 10{\cdot}4$ cm.

Position of magnet.	Zero reading of pointer.		Deflected reading.		Deflection.
	W. end.	E. end.	W. end.	E. end.	
A_1	4° S	0	51° N	55°	55°·0
,, poles reversed	,,	,,	56° S	53°	52°·5
					Mean θ_1 = 53°·8
A_2	,,	,,	55° S	52°	51°·5
,, poles reversed	,,	,,	52° N	55°	55°·5
					Mean θ_2 = 53°·5
B_1	,,	,,	28°·5 S	25°·5	25°·0
,, poles reversed	,,	,,	23°·5 N	27°	27°·2
					Mean θ_3 = 26°·1
B_2	,,	,,	24° N	27°·5	27°·8
,, poles reversed	,,	,,	29°·5 S	26°·5	26°·0
					Mean θ_4 = 26°·9

The mean deflection for A positions, $\theta_A = 53°{\cdot}6$

,, ,, B ,, $\theta_B = 26°{\cdot}5$

$$\frac{F_A}{F_B} = \frac{\tan 53°{\cdot}6}{\tan 26°{\cdot}5} = 2{\cdot}72 \text{ (experimentally).}$$

In this experiment $d = 15$ cm. and $l_1 = 5{\cdot}2$ cm.

$$\therefore \quad \frac{F_A}{F_B} = \frac{2d\left(d^2+l_1^2\right)^{\frac{3}{2}}}{\left(d^2-l_1^2\right)^2} = 3{\cdot}06 \text{ (theoretically).}$$

Comparison of magnetic moments.—Deflection method.

Magnet No. 2. $2l_2 = 7{\cdot}6$ cm.

Position.	Zero reading.		Deflected reading.		Deflection.
	W. end.	E. end.	W. end.	E. end.	
A_1	4° S	0	28° S	25°	24°·5
,, poles reversed	,,	,,	22° N	25°	25°·5
					Mean $\theta_1' = 25°{\cdot}0$
B_2	,,	,,	15° S	11°·5	11°·3
,, poles reversed	,,	,,	8° N	11°·5	11°·7
					Mean $\theta_4' = 11°{\cdot}5$

$$\frac{M_1}{M_2} = \frac{\tan 53°{\cdot}8}{\tan 25°} = 2{\cdot}93 \text{ in position } A_1$$

$$,, = \frac{\tan 26°{\cdot}9}{\tan 11°{\cdot}5} = 3{\cdot}12 \quad ,, \quad B_2$$

$$\text{Mean ratio } M_1/M_2 = 3{\cdot}03$$

Find, for this magnet also, the experimental and theoretical values of $\frac{F_A}{F_B}$.

When the lines of force of the magnet of moment M_1 were traced with the north end pointing southward, it was observed that the neutral field was about 28 cm. from the centre of the magnet on the axial line. The value of H at the place is 0·36 C.G.S. unit. The magnetic moment M_1 of the magnet can be calculated from these data.

$$F = H = \frac{2M_1 d}{(d^2 - l^2)^2} = 0{\cdot}36 = \frac{2.M_1.28}{(28^2 - 5.2^2)^2}$$

$$\therefore \quad M_1 = 3682 \text{ C.G.S. units.}$$

Vibration Magnetometer.—The oscillations of a magnet in a magnetic field furnish us with an instrument called a vibration magnetometer.

A freely suspended magnet keeps its axis parallel to the lines of force of the magnetic field in which it is placed and when disturbed from its position of rest through a small angle,

it oscillates isochronously about the position of rest. The time of oscillation $T = 2\pi\sqrt{\frac{K}{MH}}$, where K is a quantity depending upon the mass and dimensions of the magnet, M its magnetic moment and H the intensity of the magnetic field influencing the magnet.

For a given magnet $T \propto \sqrt{\frac{1}{H}}$

$$\therefore \quad \frac{1}{n} \propto \sqrt{\frac{1}{H}} \text{ or } n^2 \propto H$$

where n is the number of complete oscillations which the magnet makes in one second.

The strength (F) of the field due to a bar magnet at a point on its axial line may be compared with that of the earth's field (H) in the following manner:

Apparatus required are a vibration magnetometer, a bar magnet, stop-watch, pivoted needle, drawing board, paper, etc.

The vibration magnetometer (fig. 95) consists of a short and heavy magnet suspended by means of an unspun (?) silk thread from a support. The magnet is enclosed in a glass vessel to avoid air currents.

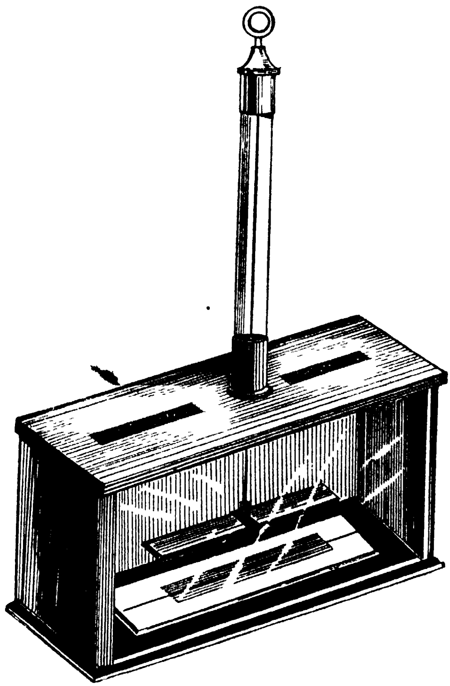


FIG. 95.

Keep the drawing board with its edge coinciding with that of the working table. Trace the magnetic meridian on the paper attached to the board with the help of a magnetic needle. Place the bar magnet flat on the board with its axis on the meridian line. Let the north-seeking end point northward. Mark off two points on either side of the magnet on the axial line at some convenient distance from the ends of the magnet. Place the vibrating magnetometer so that the centre of the suspended magnet is vertically over one of these points and that its axis is along the axial line of the bar magnet. This can be secured by adjusting the level of the bar magnet with blocks of wood of different thickness. Disturb the suspended magnet slightly from its position of rest. Start the clock when the magnet passes its position of rest towards one side and count zero. When the magnet passes the same position next time in the same direction, count one and so on till a number n of oscillations are just finished and stop the clock as n is counted. Divide the interval of time by the number of oscillations and note down the time of an oscillation. Repeat and take the mean. Calculate from the mean, the number of oscillations the needle would make in one second. Place the magnetometer over the point on the other side and calculate similarly the mean time of oscillation and the number of oscillations in one second. This number will practically be the same as in the previous case (why ?). Let the mean number of oscillations be n_1.

Reverse the bar magnet which is causing the field, keeping it in the same place as before and you might notice that the magnetometer magnet changes sides; this would happen if the bar magnet is sufficiently near. In such a case, the intensity of the field due to the magnet at the magnetometer would be stronger than that of the earth's field and the neutral point would be situated a few centimetres still further away from the magnet than the magnetometer (why ?). If otherwise, the north-seeking end of the magnetometer magnet maintains to

point northward even after reversing the bar magnet and it is thus clear that the earth's field at the magnetometer is stronger than that of the magnet. Where will be the position of the neutral field in this case? With the position of the magnet reversed, determine again the times of oscillation of the needle on either side of the magnet and let the mean number of oscillations per second be n_2.

It is assumed in this exercise that, in the region in which the magnetometer oscillates, the strength of the magnet is uniform, i.e. that the field due to the magnet is a parallel field of intensity F.*

Remove the bar magnet to a distance and find the mean time of oscillation of the magnetometer under the influence of the earth's horizontal field (H) alone and calculate the number of oscillations per second n.

In the first case, the intensity of the combined field acting at the needle is F+H (why?).

$\mathrm{F}+\mathrm{H} \propto n_1^2$ and $\mathrm{H} \propto n^2$ the constant of variation being the same (why?).

$$\therefore \quad \frac{\mathrm{F}+\mathrm{H}}{\mathrm{H}}=\frac{n_1^2}{n^2} \quad \text{or} \quad \frac{\mathrm{F}}{\mathrm{H}}=\frac{n_1^2-n^2}{n^2} \quad \ldots \quad \ldots \quad \ldots \quad \text{(I)}$$

In the second case when the north end points southward, and *when the magnetometer magnet changes sides*, i.e. when F is greater than H,

$\mathrm{F}-\mathrm{H} \propto n_2^2$ and $\mathrm{H} \propto n^2$

$$\therefore \quad \frac{\mathrm{F}}{\mathrm{H}}=\frac{n_2^2+n^2}{n^2} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \text{(II)}$$

The mean of the ratios (I) and (II) gives the required value.

Note.—If the value of H at the place is known, F is known and from the equation $\mathrm{F}=\dfrac{2\mathrm{M}d}{(d^2-l^2)^2}$ and from the known

* This is the reason why the suspended magnet should be as short as possible. Again unless the magnet is also heavy the time of oscillation will not be conveniently large. This may be secured if necessary by loading the magnet symmetrically about the axis of oscillation with a non-magnetic material.

values of d, l and F, the magnetic moment of the magnet can be calculated and therefore the pole strength.

Practical example.—

A steel bar magnet 8 inches long was taken. The magnetometer was placed at 10 cm. from either end of the magnet.

A. N. end of bar magnet points northward. Intensity of field in which the magnetometer oscillates is $F+H$.

Position of magnetometer.	Time of oscillation.	
(I) N. of magnet	$\frac{57}{20} = 2{\cdot}85$ sec.	
	$\frac{57}{20} = 2{\cdot}85$,,	
	Mean $= 2{\cdot}85$ sec.	Mean time of oscillation $= 2{\cdot}75$ sec.
(II) S. of magnet	$\frac{53}{20} = 2{\cdot}65$ sec.	$n_1 = \frac{1}{2{\cdot}75}$ per sec.
	$\frac{53}{20} = 2{\cdot}65$,,	
	Mean $= 2{\cdot}65$ sec.	

B. N. end of bar magnet points south and N. end of magnetometer also points southward. Intensity of field is $F-H$.

Position of magnetometer.	Time of oscillation.	
(I) S. of magnet	$\frac{70}{20} = 3{\cdot}5$ sec.	
	$\frac{70}{20} = 3{\cdot}5$,,	
	Mean $= 3{\cdot}5$ sec.	Mean time of oscillation $= 3{\cdot}58$ sec.
(II) N. of magnet	$\frac{72}{20} = 3{\cdot}6$ sec.	$n_2 = \frac{1}{3{\cdot}58}$ per sec.
	$\frac{74}{20} = 3{\cdot}7$,,	
	Mean $= 3{\cdot}65$ sec.	

C. Intensity of the Field is H only.

Time of oscillation.

$$\frac{61}{10} = 6\cdot1 \text{ sec.}$$

$$\frac{61}{10} = 6\cdot1 \text{ ,,} \qquad n = \frac{1}{6\cdot1} \text{ per sec.}$$

$$\text{Mean} = 6\cdot1 \text{ sec.}$$

A. $$\frac{F}{H} = \frac{n_1^2 - n^2}{n^2} = \frac{\frac{1}{2\cdot75^2} - \frac{1}{6\cdot1^2}}{\frac{1}{6\cdot1^2}} = 3\cdot92$$

B. $$\frac{F}{H} = \frac{n_2^2 + n^2}{n^2} = \frac{\frac{1}{3\cdot58^2} + \frac{1}{6\cdot1^2}}{\frac{1}{6\cdot1^2}} = 3\cdot90$$

Again, $$\frac{F+H}{F-H} = \frac{3\cdot58^2}{2\cdot75^2} \text{ and } \frac{F}{H} = \frac{3\cdot58^2 + 2\cdot75^2}{3\cdot58^2 - 2\cdot75^2} = 3\cdot88$$

$$\therefore \quad \text{Mean } \frac{F}{H} = 3\cdot9$$

The value of $H = 0\cdot36$ gauss, and so the intensity, F, due to the bar magnet, on its axial line at a point *10* cm. from either end, is $3\cdot9 \times 0\cdot36 = 1\cdot4(04)$ gauss, i.e. a unit north magnetic pole placed at either of the points would experience a force of 1·4 dynes along the axial line. From the relation $F = \frac{2Md}{(d^2 - l^2)^2}$, where $F = 1\cdot4$ gauss, $l = 4$ in. or 10 cm., and $d = 20$ cm., M can be calculated.

$$M = \frac{F(d^2 - l^2)^2}{2d} = 3150 \text{ C.G.S. units, and } m = 157\cdot5 \text{ C.G.S. units.}$$

1. In the positions A and B in the above experiment, what do you observe when the magnetometer is gradually moved away from one end of the magnet, along the axial line?

2. How does the time of oscillation of the magnetometer change, when its position is changed as in 1 above?

3. How do you proceed with the experiment in order to locate the neutral points on the equatorial line of the magnet?

4. How do you determine by the method of oscillations the ratio of the intensities of the magnetic field due to the magnet at points on the equatorial and axial lines of the magnet, at the same distance from its centre?

TERRESTRIAL MAGNETISM.

Suspend an un-magnetised steel needle through its centre of gravity by means of a torsionless silk fibre. The needle will hang with its axis horizontal. Magnetise the needle and observe. The needle now hangs with its axis inclined to the horizontal. The north end or the north-seeking end of the needle dips downwards in the northern hemisphere and the south end dips downwards in the southern hemisphere.

Let OA (fig. 96) be the direction of the axis of the needle. This is the direction along which a free unit north pole will move and the magnitude of the force with which it moves along OA is the magnitude of the *total intensity I* of the earth's magnetic field at the place. The angle BOA (θ) is called the *angle of the dip.* The force I can be resolved along the horizontal and the vertical. The *horizontal component H* of the earth's magnetic intensity is I cos θ and the *vertical component V* is I sin θ.

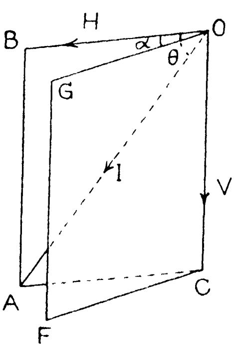


FIG. 96.

The vertical plane containing the axis of the needle is called the magnetic meridian and the vertical plane passing through the geographical north and south poles is called *the geographical meridian* at any place. OBAC represents the magnetic meridian and OGFC represents the geographical meridian. The angle BOG (α) is the angle between the two meridians and is called *the variation or the declination.* OB is the direction of the horizontal component of the earth's magnetic field and OG is the horizontal indicating the geographical north. The angle α is the angle between these two horizontals. The geographical meridian at any place can be very accurately determined by astronomical instruments and is taken as the standard plane of reference. Therefore

a knowledge of α and θ determines the direction of the earth's field completely. If the value of H, the horizontal component, is also known the magnitude of I ($= \mathrm{H} \sec \theta$) is known. Therefore a knowledge of α, θ and H, completely determines the total intensity of the earth's magnetic field both in direction and in magnitude. Therefore *the three quantities, viz. the variation, the dip and the horizontal intensity are called the magnetic elements of the earth's field at any place.*

The earth's horizontal component *H may be determined* in the following manner. Find the time of oscillation of a magnet accurately, by hanging it freely in the earth's field.

$$\mathrm{T} = 2\pi\sqrt{\frac{\mathrm{K}}{\mathrm{MH}}}$$

$$\therefore \quad \mathrm{MH} = \frac{4\pi^2\mathrm{K}}{\mathrm{T}^2}$$

T = time of oscillation.
K = moment of inertia; a quantity depending on the mass of the magnet and its dimensions.*
M = magnetic moment of the magnet.
H = horizontal intensity of the earth's magnetic field.

Place the same magnet in the tangent A position and using a deflection magnetometer, find the mean value of $\frac{\mathrm{M}}{\mathrm{H}}$.† Then by dividing MH by M/H the value of H is determined. (By multiplying MH and M/H, ~~M~~ is obtained and if the length of the magnet is measured, the pole strength of the magnet can be calculated.)

The magnetic elements of the earth's field generally differ from place to place and each element has the same value at certain different localities. A line can be drawn on the map

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* If the magnet used is cylindrical and if it oscillates about the vertical axis through the centre of the magnet perpendicular to the length, $\mathrm{K} = \mathrm{M}\left(\frac{l^2}{12} + \frac{r^2}{4}\right)$ where M = mass of magnet, l its length and r the radius.

† $\frac{\mathrm{M}}{\mathrm{H}} = \frac{(d^2 - l^2)^2}{2d} \tan \theta.$

of the globe connecting the localities where the values of the elements are the same. Many such lines, each line corresponding to a particular value of the element, can be drawn and a *magnetic map* for any one of the elements can be thus obtained.

The three elements of the earth's magnetic field are subject to *three periodic changes* of different magnitudes, namely *the diurnal, the annual and the secular*. The secular change is one of a long period of nearly 900 years and the magnitude of variation is large. In addition to these three variations, rapid and big changes in the three elements are simultaneously observed once in eleven years which is the period of solar activity or the maximum frequency of sun spots. These sudden and violent changes are called *magnetic storms* and they are frequently associated with the maximum display of the polar light called the aurora.

Dip Circle and Determination of Magnetic Dip.—In the ordinary method of supporting a magnetic needle over a vertical pivot, a magnetic force in the vertical plane cannot influence the needle as freely as a force in the horizontal plane. But if the needle is suspended from its centre of gravity by a delicate unspun silk thread, any force in the vertical plane as well as any in the horizontal, can act upon it equally freely. Such a needle puts itself, when suspended in the earth's field, with its axis inclined to the horizontal, at almost all points on the surface of the earth. This direction of the axis of the needle gives the direction of the total intensity of the earth's magnetic field at a place, whereas that of a needle suspended on a vertical pivot gives only the direction of the horizontal component. Hence a pivoted needle is not strictly speaking a freely suspended magnetic needle. However, in both these suspensions the needles keep their axes in the same vertical plane, viz. the plane of the resultant magnetic force of the earth's field, which is called the magnetic meridian. It is already stated that the inclination of the axis of a freely

suspended magnetic needle to the horizontal is called the 'dip' or *inclination* and varies from place to place.

The dip circle (fig. 97a) consists of a magnetic needle NS

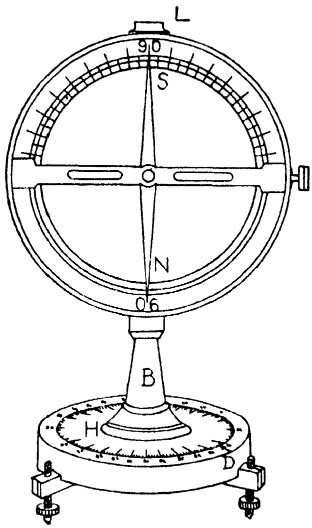

FIG. 97a.

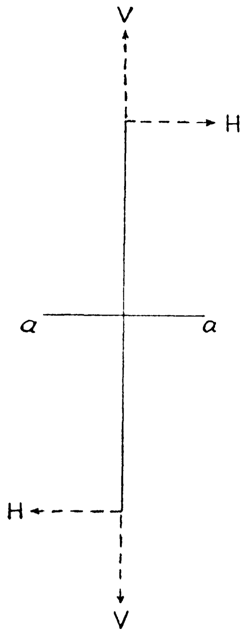

FIG. 97b.

suspended on a horizontal axis through its centre of gravity. The needle can therefore rotate in a vertical plane and the position of the ends of the needle can be read on a graduated vertical circle. The centre of the circle coincides with that of the needle. The circle is enclosed in a brass frame, the sides of which are closed by plane glass. The frame is supported on a vertical brass stand B which passes through the centre of a graduated horizontal circle H at the bottom. The stand and frame can be rotated about the vertical axis passing through the stand and the position can be read on the horizontal circle by means of an index mark D. The three supporting legs are provided with levelling screws and a spirit level L is fixed at the top of the instrument.

Level the instrument. Rotate it about the vertical axis such that the needle takes up a vertical position as shown in the figure. The ends N and S read 90 on the vertical circle. In this position the vertical component only of the earth's magnetic force is acting on the needle. This would be the case when the axis of suspension of the needle lies exactly in the magnetic meridian. In Fig. 97(*b*), *a a* is the axis lying in the plane of the paper * and represents the magnetic meridian. The plane in which the broad face of the needle NS lies is at right angles to that of the paper and is also the plane in which the vertical circle lies. V and H are the vertical and horizontal components of the earth's field, in the plane of the paper. It is clear that the couple due to the horizontal components tends to turn the needle in the plane of the paper in which the axis lies and hence the couple wastes itself on the pivots of the axis and has no effect. This is why, in this position, the needle is acted on only by the vertical component. The axis of suspension of the needle is in the plane of the paper and the plane of the vertical circle therefore coincides with the magnetic east and west plane. Read the position of the index D (fig. 97*a*) on the horizontal circle and turn the instrument through a right angle. The vertical circle now lies exactly in the magnetic meridian. In this position both the components V and H freely act on the needle, and the direction of the axis of the needle is along that of the total intensity of the earth's field. The vertical circle is so graduated that the diameter of the circle passing through the two zero marks is horizontal. Therefore the reading of either end of the needle on the vertical circle gives the value of the dip at the place. Take the mean of a few readings (θ). $H = I \cos \theta$, I being the total intensity. Note that the north-seeking end dips downwards.

* Note that *a a* lies in a plane at right angles to that of the paper in fig. 97*a* and the plane of the paper contains the broad face of the needle NS.

CHAPTER XII.

ELECTRIC CURRENT AND MEASUREMENT.

ELECTRIC CELLS.

The voltaic cell is the simplest type of electric cell. Such a cell has two defects and they are (1) local action and the consequent wastage of zinc when the circuit outside the cell is not complete and (2) polarization or the fall in the difference of potential * set up between the copper and the zinc plates immersed in dilute sulphuric acid. This fall in potential is due to the accumulation of hydrogen at the copper plate.

Local action is prevented by amalgamating the zinc plate and polarization by preventing hydrogen to collect on the copper plate, the best method being that of oxidation. In any form of an improved cell therefore, some oxidising agent is used.

The *apparatus required to study the nature of polarization in a simple voltaic cell* are a simple voltaic cell with an additional copper plate, a current detector or tangent galvanometer, a two-way key, and a few pieces of insulated copper wire. In fig. 98(*a*), 1, 2, 3, are the binding screws of the two-way key and 4, 5, 6 are metal plugs. 1 and 4 and 3 and 6 are permanently connected by a metal wire. By means of a metal tounge which rotates about 2, connection can be made between 2 and 4 or 2 and 6. A is the active copper plate and D the additional or dummy copper plate. K and L are the terminals of the galvanometer. (I) Place the coil of the galvanometer in the magnetic meridian and connect up as in

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* Explanation of the terms potential, difference of potential and electromotive force is given under Ohm's law.

the figure. Connect 2 and 6 of the key. The circuit A, 2, 6, 3, D is complete through the galvanometer. The

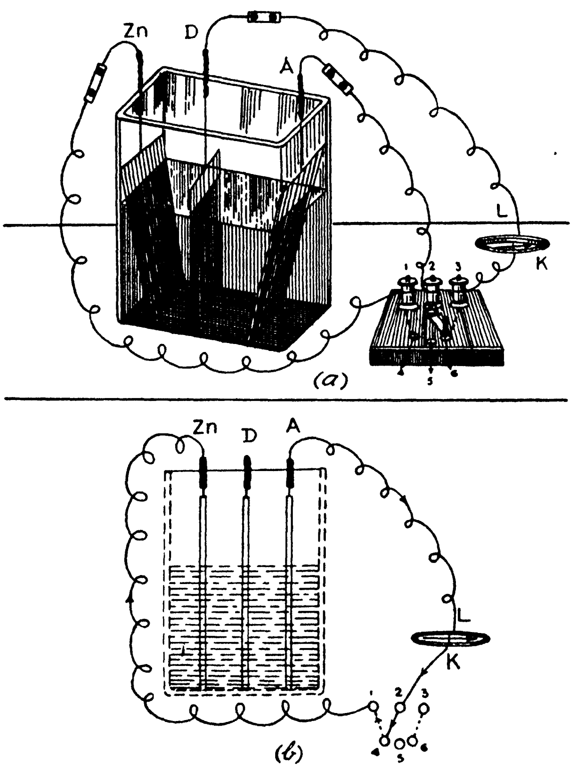

FIG. 98.

galvanometer does not show any deflection. Hence no current passes through it and there is no difference of potential between the two copper plates. (II) Connect 2 and 4. The cell is now active and the galvanometer is out of the circuit. Wait a minute or two. Note that the hydrogen bubbles collect on the copper plate A. Suddenly connect 2 and 6 as in (I) above. The galvanometer needle will now be deflected say towards K. This shows that a difference of potential is set

up between the two copper plates. (III) To determine the direction of the fall of potential between the two copper plates, include the galvanometer in the circuit of the cell as shown in fig. 98(*b*). Connect 2 and 4 just for a moment and note the direction of deflection. The needle will be deflected towards K. The direction of the current outside the cell is from copper to zinc and therefore, for the needle to be deflected towards K, the current must enter at L, the terminal to which the copper plate A at the higher potential is connected. Therefore in the second stage of experiment (II) above, since the deflection is towards the same terminal, the dummy plate must have been higher in potential than A. As there is no chemical action going on at the plate D, the potential of D must have been the same in (I) and (II). Hence it is clear that *the copper plate A in experiment II has fallen in potential after being active for some time.*

Connect once again as in fig. 98(*b*). Watch the readings of the deflection. You will observe that the current falls off rapidly and becomes very small in a short time. You will also note that hydrogen bubbles cover the copper plate A. If the bubbles are brushed off, the deflection will increase but will fall off again quickly. This shows clearly that the potential of the copper plate A falls off on account of the accumulation of hydrogen bubbles on it. This layer of hydrogen prevents contact between the acid and the copper plate. Hence the action of the cell rapidly falls off and therefore the potential difference between the zinc and the copper plates.

Set up a Daniell cell with two copper plates in the copper sulphate solution. Repeat the experiment in its first two stages (I) and (II) as above. What do you observe and how do you explain it?

Which gives a steady current, a Leclanche or Daniell cell?

MAGNETIC EFFECT OF ELECTRIC CURRENT.

To study the magnetic effect of an electric current, the apparatus required are a Daniell cell, commutator, a thick copper rod with binding screws at either end, compass-needle, simple key, connecting wires, etc.

A and B are the two binding screws of the commutator, fig. 99(*a*). They are fixed one each to the two circular brass

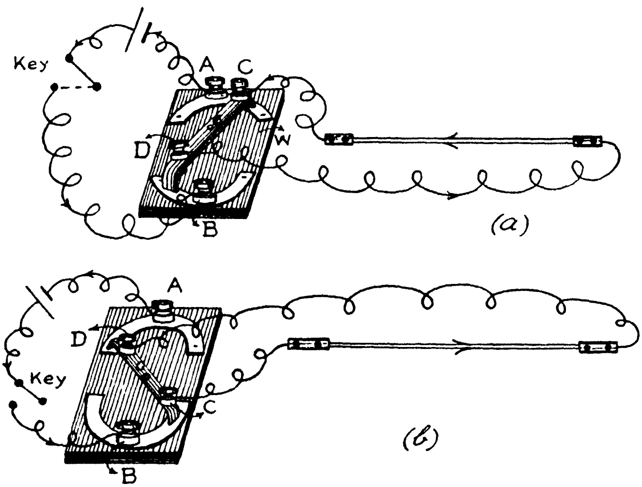

FIG. 99.

strips which are screwed on to a wooden block W. Through the centre of the block O passes a screw about which the wooden arm COD rotates. C and D are the binding screws with metal brushes attached at the bottom and contact with the metal strips A and B is thus secured. The terminals of the cell are generally connected to the screws A and B and the terminals of the portion of the circuit in which the current has to be reversed are connected to C and D.

Connect the apparatus as shown in fig. 99(*a*). Keep the length of the rod in the magnetic meridian and note the direction of the deflection of the needle when it is above and below the rod. Reverse the current by turning the handle COD

such that C is now connected to the metal strip B as shown in fig. 99(*b*). Note down the observations again. Keep the rod vertical, near and against the north-seeking end of the needle and then against the other end and note the direction of the deflection of the north end of the needle. Reverse the current and repeat. Tabulate the readings.

Deduce the direction of deflection of the north end of the needle in each case according to Maxwell's corkscrew rule or Ampere's rule and verify.

Maxwell's corkscrew rule states that if a corkscrew is imagined to be driven along the rod in the direction of the current, then the direction in which the thumb rotates gives the direction along which the north-seeking end of a needle is deflected. (The corkscrew is a right-handed screw).

Ampere's rule states that if we imagine a person swimming along the current with the face turned towards the needle, the north-seeking pole is deflected towards the left hand.

Practical example.—

Observation No.	Position of the rod as regards the needle.	Direction of current.	Deflection of N end of needle towards.
	Rod horizontal in the magnetic meridian.		
1	Above	N—S (North to South)	East
2	,,	S—N (reversed)	West
3	Below	S—N	East
4	,,	N—S (reversed)	West
	Rod vertical.		
5	Against N end	Top to bottom	West
6	,,	Reversed	East
7	Against S end	Bottom to top	West
8	,,	Reversed	East

Imagine yourself driving a right handed screw horizontally into the southern wall of a room. You will be facing the south with the screw between yourself and the wall. The direction of translation of the screw is from north to south and corresponds to the direction of the current in observations Nos. 1 and 4 above. The thumb, in the act of screwing, rotates in a circle round the length of the screw in a vertical plane in the clockwise direction as seen by you. The direction at the topmost point of the circle is towards the west and at the lowest point towards the east and hence according to the rule if the north seeking end of the needle is situated at the top point of the circle (4 above) it will be deflected towards the west and if at the lowest point i.e. below the rod the deflection will be towards the east (1, above). Again if the screw is imagined to be driven into the top of a working bench and the cork-screw rule applied it will be clear that as shown in the fig. 100(a) the north-seeking end of the needle will be deflected towards the east, at A (compare observation 8 above) and towards the west, at B, (compare observation 5). Verify the other observations similarly. Verify also by applying Ampere's rule. The above observations clearly show that the field round a wire conducting a current would influence a freely suspended magnetic needle in different directions at different points. This region within which a needle is deflected is the magnetic field due to the current in the wire and the direction of the lines of force at any point in the field is indicated by the

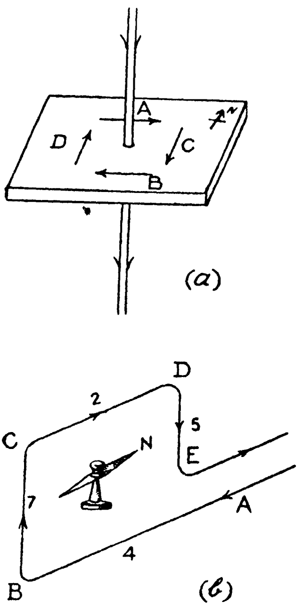

FIG. 100.

direction of the deflection of the north-seeking end of the needle. Thus in fig. 100(*a*) the lines of force due to the current run north to south at C and south to north at D. Move the needle, round the conducting rod, keeping the needle very close to it (?) and note that the northseeking end traces a circular line of magnetic force. (Use two Daniell's cells if necessary.) Reverse the current and note how the direction of the lines of force changes.

To study the magnetic field at the centre of a coil of wire through which a current is passing substitute a long insulated (?) copper wire for the bare rod used in the above experiment and connect up as before. Give the wire the form shown in the fig. 100(*b*) and considering the elements AB, BC, CD and DE separately which correspond to observation numbers 4, 7, 2 and 5 respectively of the previous exercise, note that the effect in each case is to direct the north-seeking end of the needle towards the west. The plane of the coil is in the magnetic meridian. Note that the effect is the same if the shape of the coil is circular. Reverse the current. What do you observe? *Verify the statement that if the coil is held so that its face is perpendicular to the line of sight, and if the current passes round in the clockwise direction then the lines of force at the centre of the coil pass away from you along the line of sight and vice versa.*

Coil up the wire into two, three and four turns and keeping the needle at the centre of the coil note that the effect is accumulative and the strength of the field increases. Put the coil in the magnetic east and west plane, reverse the current and note the effect. Explain the observations clearly. Try intermediate positions keeping the current and the number of turns in the coil constant. What do you observe? Why is an insulated wire doubled and wound round a wooden bobbin, in a resistance coil? Why are the insulated leads to a sensitive galvanometer run parallel to each other?

TANGENT GALVANOMETER.

The lines of force at the centre of a circular coil of wire carrying a current with its plane in the magnetic meridian pass through the centre in a direction at right angles to the plane of the coil and a compass needle placed at the centre of the coil would be deflected from the magnetic meridian in the direction of the lines of force. The lines of force at the centre of the coil are practically parallel to each other in the small circular area in which the needle moves. (See fig. 101(*a*).)

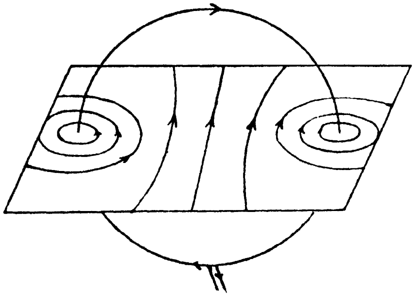

FIG. 101(*a*).

The needle at the centre of the coil is simultaneously under the action of two uniform magnetic fields—one due to the earth along the magnetic meridian and the other due to the current, in the plane at right angles to the meridian. Consequently the needle will be deflected through an angle θ from the magnetic meridian and $F = H \tan\theta$ (see fig. 101(*b*)). F is the intensity of the deflecting field due to the circular current. H is the directing field and represents the horizontal component of the earth's magnetic field.

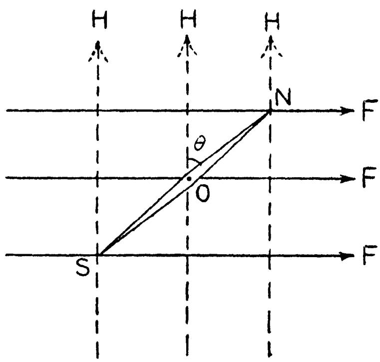

FIG. 101(*b*).

The intensity F of the magnetic field at the centre of a circular coil of wire carrying a current C depends on—

(1) the strength of the current—the larger the current, the greater the effect.
(2) the length l of the wire forming the coils—the greater the number of turns of coil the greater the effect.
(3) the radius r of the coil—inversely as the square of the radius as in the case of inverse square law of magnetic action.

So
$$F \propto \frac{lC}{r^2}$$

If the unit current is defined such that $F = 1$, when l and r are each 1 cm., then we can write $F = \frac{lC}{r^2}$. *The unit current in the C.G.S. system is the current flowing through a centimetre length of wire bent into the arc of a circle of one centimetre radius and causing a magnetic field of intensity one gauss, at the centre.*

This current has been found to be too much for practical purposes and so one tenth of it is used in practice and is called an *ampere* after the famous French physicist André Marie Ampére. If n is the number of circular turns of coil, $l = 2\pi r \times n$

and
$$F = \frac{2\pi r n C}{r^2} = \frac{2\pi n C}{r} = H \tan\theta$$

Further $C = \frac{rH}{2\pi n} \times \tan\theta = K \tan\theta$, where $K = \frac{rH}{2\pi n}$ and is called *the reduction factor* of the galvanometer.

If the above arrangement is included in any electric circuit the presence of the current can be detected and measured. The circular coil, with its plane in the magnetic meridian, and with a delicately suspended small magnetic needle at its centre, essentially constitutes a *tangent galvanometer.* Since the intensity of the field due to the coil and hence the current (passing through the coil) are proportional

to the *tangent* of the angle of deflection the arrangement is called a tangent galvanometer.

To verify that the intensity of the field (*F*) *at the centre of the coil varies directly as the number of turns* (*n*) *and inversely as the radius of the coil* (*r*), the apparatus required are a Daniell cell, tangent galvanometer, commutator, connecting wires, etc.

In the tangent galvanometer (fig. 102) six different lengths of insulated copper wire are wound into two sets of three concentric circles one on either side of a vertical board V. The ends of each turn are secured to separate binding screws fixed to the horizontal base board H on which the vertical board is supported. Current can be passed into one or two turns of the coil at a time as desired. A grove is cut at the centre of the vertical board, into which a compass box is fitted so that the needle is situated at the centre of the coils of wire.

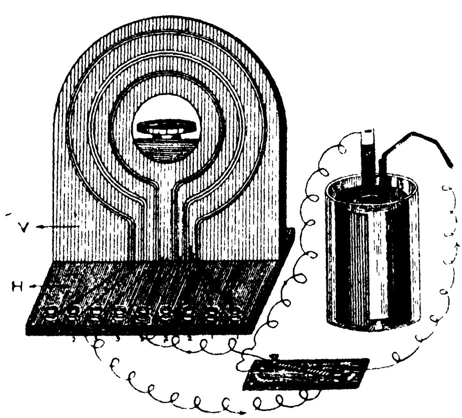

FIG. 102.

Turn the galvanometer such that the plane of the vertical board is parallel to the length of the needle (?). Turn the compass box about itself such that the aluminium pointer attached at right angles to the needle reads zero on the graduated circular scale of degrees.

Connect the apparatus as shown in the figure, using single and double turns of the three coils separately. Note down the deflections. Reverse the current in each case and take the mean deflection. Measure the diameters of the coils

and tabulate the results. You will find that the expression $\frac{r \tan \theta}{n}$, in all the six different cases, is practically constant.

$$\text{Since} \qquad \frac{r \tan \theta}{n} \text{ is constant, } \tan \theta \propto \frac{n}{r},$$

but $F = H \tan \theta$, i.e. $F \propto \tan \theta$, since H is constant at a place.

Therefore, the intensity F varies directly as the number of turns and inversely as the radius of the coil. This has been experimentally verified.

Here it is assumed that the strength of the current in the six different cases is constant. Since the Daniell cell is one of constant electromotive force and since the resistances of the coils and more so their differences, are practically negligible when compared with the total resistance of the circuit, the assumption is justified.

Practical example.—

Coil number.	Number of turns. n	Radius r in cm.	Mean deflection θ in degrees.	$\frac{r \tan \theta}{n}$
1	1	15	8	2·11
1	2	15	17	2·30
2	1	12	11	2·33
2	2	12	21	2·30
3	1	8	16	2·30
3	2	8	30	2·31

Mean = 2·28

Measurement of Current.

$$F = \frac{2\pi nc}{r} \quad \text{or} \quad C = \frac{rH}{2\pi n} \times \tan \theta = \frac{r \tan \theta}{n} \times \frac{H}{2\pi}$$

$$\text{But} \qquad \frac{r \tan \theta}{n} = 2{\cdot}28 \text{ as found above}$$

$$\therefore \qquad C = 2{\cdot}28 \times \frac{H}{2\pi}$$

and taking $H = 0{\cdot}36$ gauss at the place,

$$C = 2{\cdot}28 \times \frac{0{\cdot}36}{2\pi} = 0{\cdot}131 \text{ C.G.S. unit of current.}$$

$$\therefore \qquad C = 1{\cdot}31 \text{ amperes.}$$

This is the current in each of the above six cases.

To find the reduction factor of the galvanometer (K). The current K that produces a deflection of 45° of the needle is defined as the reduction factor of a tangent galvanometer and is numerically equal to $\frac{rH}{2\pi n}$. From the expression $C = K \tan\theta$ it is clear that the current is proportional to the tangent of the angle of deflection and since a current of 1·31 amperes produces a deflection of 30° in the last of the above observations, the reduction factor K for that case (2 turns of coil of radius 8 cm.) is $\frac{1{\cdot}31}{\tan 30} = 2{\cdot}26$ amps.

and by direct calculation $\left(\frac{rH}{2\pi n}\right) \times 10(?) = 2{\cdot}29$ amps.

1. Is the reduction factor the same for each of the above coils? How does it change with n and r?
2. How would its value for any given coil change from place to place?
3. Define the constant of a tangent galvanometer. Why is it so called? Calculate the value, in each of the above six cases.

Ohm's law

Liquids flow from a higher to a lower level, i.e. from points at higher to points at lower potential. The difference in levels measures the difference in potential. The greater the difference of potential between any two given points the greater is the flow or the current. In this case the difference in potential or pressure is hydraulic. In the case of heat flow the difference is thermal. Similarly, in the case of flow of electricity the electric current is said to be due to an electric *Potential Difference.* (P.D.)

When the circuit of a Daniell cell is closed the potential difference (P.D.) between the poles of the cell maintains a current of electricity in the circuit. The potential difference

between the terminals of the cell when it does not send a current is called its *electromotive force* (E.M.F.). The practical unit of E.M.F. is a *volt* named after Volta an Italian physicist who invented the voltaic cell and battery in 1800. The E.M.F. of a cell measured in volts is called its *voltage.*

The relation between the potential difference (P.D.) applied to a circuit and current (C) maintained in it, was first investigated by George Simon Ohm a German physicist of Munich, in 1827 and as a result he enunciated the following law known as *Ohm's law. The ratio of the potential difference between any two points in a circuit, through which a steady current flows, to the current passing through, is a constant, so long as the physical state of circuit (as regards temperature, etc.) remains the same.* This constant, under the conditions, is called the *resistance* of the circuit between the points. The law can be expressed thus $E/C = R =$ constant, where E is the potential drop between any two points, C the current and R the constant or resistance of the circuit between the points.

Ohm's law can also be expressed thus: *the current flowing between any two points of a conductor at a constant temperature is proportional to the applied E.M.F.* This may be expressed as $C = kE$. The constant $k = C/E = 1/R$, the reciprocal of the resistance of the conductor between the points and is called its *conductance.*

As already stated the practical unit of E.M.F. is a volt and the practical unit of current is an ampere. As $R = E/C$, $R = 1$, if $E = 1$ and $C = 1$; *thus a conductor has a unit resistance if a difference of potential of one volt between its ends produces a current of one ampere.* This unit of resistance in the practical system is called the *Ohm.* Resistances,—one ohm, multiples of an ohm and fractions of an ohm—are made of coils of wire of *manganin* (copper, nickel and manganese alloy) or of *constantan* (copper, nickel alloy) because the resistance of these alloys does not change appreciably with moderate changes of temperature. These resistance coils

must have a standard to compare with so that the resistances may be accurately known whenever necessary. For this purpose a *Standard Ohm* is kept at the Standards Office of the Board of Trade, Westminster. It is defined as the resistance of a column of mercury, of uniform cross-section, 14·4521 gm. in mass and 106·3 cm. in length, at 0°C. The cross-section is practically equal to one sq. mm.

The resistance of a conductor between any two points is independent of the applied potential difference and is a constant so long as the physical state of the conductor is the same. However, the resistance of a conducting wire depends on the material of which it is made, on the length of the wire and on the sectional area. Copper and silver offer very small resistance and are very good conductors whereas german silver, manganin and constantan (eureka) offer greater resistance. It has been found by experiment that the resistance of a wire directly varies as the length (l) and inversely as the sectional area (A). So $R = \rho l/A$ where ρ is a constant. If $l = 1$ cm. and $A = 1$ sq. cm. then $\rho = R$ and is the resistance between the opposite sides of a unit cube of the material of the wire and is called its *specific resistance.*

The *apparatus required to verify Ohm's law* are a Daniell cell, tangent galvanometer (50 turns of coil), commutator, connecting wires and a few coils of various resistances r_1, r_2, etc.

Put the plane of the coil of the galvanometer in the magnetic meridian and connect the apparatus in series as shown in fig. 103. Gradually increase the resistance (R) of the circuit by introducing the resistance coils (r_1, r_2, etc.) one after the other, and note down the deflections in each case. Reverse the current and take the mean. R includes B, the resistance of the cell and G the resistance of the galvanometer. Hence $R = B+G+r$ where r is the resistance of the coil introduced into the circuit. If C is the current in

the circuit, $\frac{E}{K \tan \theta} = R$ (K = reduction factor of the galvanometer).

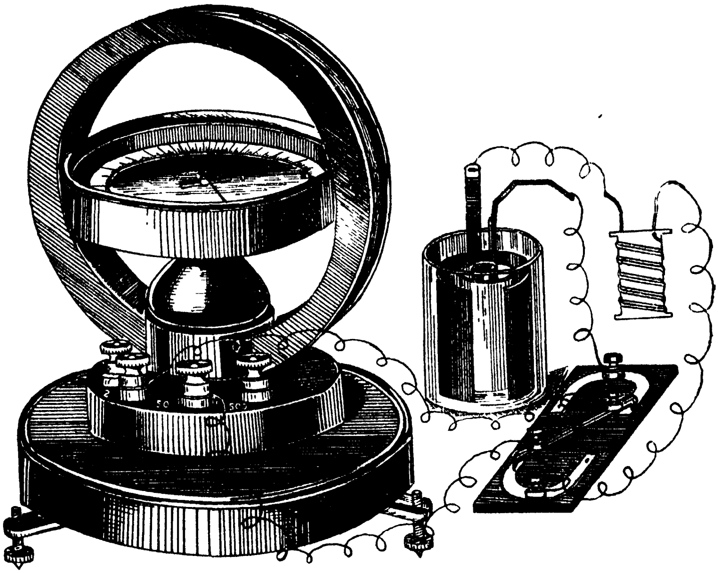

FIG. 103.

$\therefore$ $E/K = R \tan \theta =$ constant for the given cell and galvanometer.

So if $R \tan \theta$ can be experimentally shown to be constant Ohm's law can be verified. But R cannot be known if B and G are not given. Generally these values may not be known. In such a case the combined resistance $B+G$ can be calculated from any two of the observations taken.

Let θ_1 and θ_2 be the deflections when resistances r_1 and r_2 are introduced into the circuit.

Assuming Ohm's law we have

$$(B+G+r_1) \tan \theta_1 = (B+G+r_2) \tan \theta_2$$

$$\therefore \ (B+G) \times (\tan \theta_1 - \tan \theta_2) = r_2 \tan \theta_2 - r_1 \tan \theta_1$$

$$\therefore \ B+G = \frac{r_2 \tan \theta_2 - r_1 \tan \theta_1}{\tan \theta_1 - \tan \theta_2}$$

The total resistance R can now be easily calculated and R $\tan\theta$ will be found to be nearly constant.

Practical example.—

A Pye tangent galvanometer with three coils of 2, 50 and 500 turns of insulated wire provided with separate binding screws was used and the middle coil was the one employed in the experiment. Four resistance coils, 2, 2, 5 and 10 ohms, were also used.

Observations.

Number.	External resistance r in ohms.	B+G (calculated) in ohms.	(B+G+r) = R, in ohms.		θ in degrees of arc.		Mean θ	R $\tan\theta$
1	2		5		70·5	70	70	13·7
				(Reversed)	70	69·5		
2	4		7		61·5	61	61	12·6
				(Reversed)	60·5	60·5		
3	5		8		57	57	57·5	12·6
				(Reversed)	58	57·5		
4	7	3 ohms	10		51	51	50·5	12·1
				(Reversed)	50·5	50		
5	10		13		43	42·5	43	12·1
				(Reversed)	43·5	43·5		
6	12		15		39·5	39·5	39	12·1
				(Reversed)	39	38·5		
7	15		18		34	33·5	34	12·1
				(Reversed)	34	34		
8	19		22		28·5	28·5	28·5	12·0
				(Reversed)	28·5	28·5		

Mean = 12·4

B+G was calculated *from the observations 4 and 6, where the deflections produced lie on either side of 45°(?).* The values of *R* $\tan\theta$ are nearly constant and hence the law is verified.

We can now determine the resistance of an unknown coil of wire with the tangent galvanometer.

Use the same arrangement as above. Insert the coil into the circuit and note the mean deflection θ. Taking the mean value of R tan θ, (12·4) obtained above, we have $R = \frac{12 \cdot 4}{\tan \theta}$

$$\therefore\ r, \text{ the external resistance} = \frac{12 \cdot 4}{\tan \theta} - (B+G) = \frac{12 \cdot 4}{\tan \theta} - 3 \text{ ohms.}$$

When a coil marked 10 ohms was introduced into the circuit a deflection of 45° was observed. Hence its calculated resistance is $12 \cdot 4 - 3 =$ *9·4 ohms*. Check the value thus obtained by the method of substitution (?) with a resistance box in series with the galvanometer. You will find that this is not an accurate method of finding the resistance of a coil and that the values obtained will be correct to within an ohm.

1. Insert fractions of an ohm into the circuit and observe how the needle of the galvanometer is affected; what do you infer?

2. Calculate the reduction factor of the galvanometer used in the above exercise, given that the diameter of the coil is 15 cms. and H = 0·36 C.G.S. unit. (0·086 amp.)

3. The value of H is 0·18 gauss at London. Calculate the reduction factor for the galvanometer. Where is the galvanometer more sensitive, at Madras or London?

4. Calculate the E.M.F. of the Daniell cell, using the mean value of R tan θ obtained above. (1·07 volt.)

An *alternative method of verifying Ohm's law* is given below. In the method given above the value of B+G was calculated by assuming Ohm's law and this value of B+G thus obtained was employed to get the total resistance R and R tan θ was found to be nearly constant. It might be therefore said that the method is not very satisfactory. So an alternative method by which the law is directly verified is now described.

Apparatus required are five Daniell cells of the same size, Pye tangent galvanometer (50 turns of coil), commutator, connecting wires and a resistance box.

Join the cells in series and connect up the apparatus as in the previous exercise using a suitable resistance from the box. Note the deflection, reverse the current in the galvanometer and take the mean deflection. The E.M.F. in the circuit is 5E, where E is the E.M.F. of each of the Daniell cells. Now

connect the cells so that the E.M.F. of one of them opposes the combined E.M.F. of the other four. This is the same as saying that one of the cells is reversed. The effective E.M.F. in the circuit is therefore (4E-E) or 3E. Note the mean deflection again. Reverse another cell also so that the effective E.M.F. is only (3E-2E) or E and note the mean deflection. You will find that in each case the ratio of the E.M.F. to the tangent of the angle of deflection will be found to be nearly constant. This verifies Ohm's law. The total resistance in the circuit is made up of (*a*) internal resistance of the five cells, (*b*) the resistance of the tangent galvanometer (50 turns of coil), (*c*) the resistance plugged out from the box and (*d*) other resistances chiefly due to bad contacts in the circuit. The resistance due to bad contacts is to be avoided by taking care that the terminals at each contact of metal with metal are kept clean and bright.

Practical example.—

I. Resistance in box = 40 ohms.

E.M.F. of the circuit in volts.	Mean Deflection. θ	Tan θ	$\frac{\text{E.M.F.}}{\tan\theta}$
5 E	50°	1·1918	4·195 E
3 E	35°	0·7002	4·284 E
1 E	14°	0·2493	4·012 E

Mean = 4·164 E

II. Resistance in box = 30 ohms.

E.M.F. in volts.	Mean Deflection. θ	Tan θ	$\frac{\text{E.M.F.}}{\tan\theta}$
5 E	56°·5	1·5108	3·311 E
3 E	41°	0·8693	3·491 E
1 E	16°·5	0·2962	3·376 E

Mean = 3·393 E

The E.M.F. of a Daniell cell is 1·07 volts and the reduction factor of the galvanometer used is 0·09 amperes.

$$\therefore \quad \frac{\text{E.M.F.}}{\text{C}} = \frac{\text{E.M.F.}}{\text{K} \tan \theta} = \text{R} = \text{total resistance of the circuit.}$$

Hence in case I, $\text{R} = \dfrac{4{\cdot}164 \times 1{\cdot}07}{0\ 09} = 49{\cdot}55$ ohms. i.e. 50·0 ohms (practically).

And in case II, $\text{R} = \dfrac{3{\cdot}393 \times 1{\cdot}07}{0\ 09.} = 40{\cdot}33$ ohms, i.e. 40·0 ohms (practically).

The difference between the two resistances is nearly 10 ohms as it ought to be. It may be noted that in each of the cases I and II the total resistance of the circuit is kept constant; the applied E.M.F. has been varied and the corresponding variation in the current has been observed.

The tangent galvanometer may be used to compare the electromotive-force of a Leclanche cell (E_1) with that of a Daniel cell (E_2) in the following manner.

Apparatus required are Leclanche cell, Daniell cell, tangent galvanometer of high resistance (500 turns of coil), commutator and connecting wires.

To get consistent and least varying results the solutions used in the cells are to be made afresh.

Put the coil of the galvanometer in the magnetic meridian; connect the Leclanche cell in series with the commutator and the galvanometer so that the current can be reversed only in the galvanometer circuit. Note the deflection, reverse the current and note again. Let the mean deflection be θ_1. Repeat the same with the Daniell cell and let the mean deflection be θ_2.

According to Ohm's law

$$\frac{\text{E}_1}{\text{B}_1 + \text{G}} = \text{K} \tan \theta_1 \text{ and } \frac{\text{E}_2}{\text{B}_2 + \text{G}} = \text{K} \tan \theta_2.$$

If G is big compared with B_1 and B_2, the resistance of the cells, then

$$\frac{\text{E}_1}{\text{E}_2} = \frac{\tan \theta_1}{\tan \theta_2} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

Connect the two cells in series and in opposition and note the mean deflections θ_3 and θ_4. Then

$$\frac{E_1+E_2}{E_1-E_2}=\frac{\tan\theta_3}{\tan\theta_4}$$

$$\therefore \quad \frac{E_1}{E_2}=\frac{\tan\theta_3+\tan\theta_4}{\tan\theta_3-\tan\theta_4} \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

Calculate $\frac{E_1}{E_2}$ from (1) and (2) and compare; take the mean.

Practical example :—

Pye tangent galvanometer with 500 turns of coil was used.

Cell.	Deflection.			Mean.
Daniell		33°	33°	32°
		32°	32°	
	Reversed	31·5	31·5	
	,,	32·5	32·5	
	,,	31·5	31·5	
	,,	31·3	31·3	
Leclanche		38·2	38·2	37°·7
	Reversed	37·5	37·0	
	,,	38·0	38·0	
Two in series		54·5	54·5	53°·6
	Reversed	52·5	53·0	
	,,	54·5	54·5	
	,,	52·5	53·0	
Two in opposition		8·5	8·5	8°·3
	Reversed	8·0	8·0	

$$\frac{E_1}{E_2}=\frac{\tan 37\cdot 7}{\tan 32}=1\cdot 24 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

Again

$$\frac{E_1+E_2}{E_1-E_2}=\frac{\tan 53\cdot 6}{\tan 8\cdot 3}$$

$$\therefore \quad \frac{E_1}{E_2}=\frac{\tan 53\cdot 6+\tan 8\cdot 3}{\tan 53\cdot 6-\tan 8\cdot 3}=\frac{1\cdot 497}{1\cdot 216}=1\cdot 23. \ldots \quad \ldots \quad (2)$$

Mean ratio $\quad \frac{E_1}{E_2}=1\cdot 235$

WHEATSTONE BRIDGE.

The determination of the resistance of a coil of wire with a tangent galvanometer was indicated in the previous exercise. You would have noticed that the tangent galvanometer was not sensitive to small variations in the resistance of the circuit and that therefore the resistances obtained did not compare well with the values marked on them. An accurate measurement of resistance therefore requires a more sensitive type of galvanometer. Two different types of delicate galvanometers are in general use, (I) with fixed coil and movable magnetic needle and (II) with moving coil and fixed magnet. A galvanometer of the second type, i.e. of the suspended-coil type, is more convenient and handy for many purposes. Sensitive galvanometers should never be put in series with a cell with resistances such as were used with the tangent galvanometer (?)

We will now describe a *suspended-coil galvanometer* (R. W. Paul, Unipivot-A-type).

A fine insulated copper wire is wound into a small circular coil C fig. 104 (*b*) of many turns. It is delicately suspended between the poles of a powerful ring-form magnet NS, fig. 104 (*a*), so that it can rotate about its vertical diameter, just clearing the circular ends of the magnet. The plane of the coil is vertical. The normal position of the plane of the coil is symmetrically situated between the ends of the magnet, parallel to the lines of force between them. If a current is

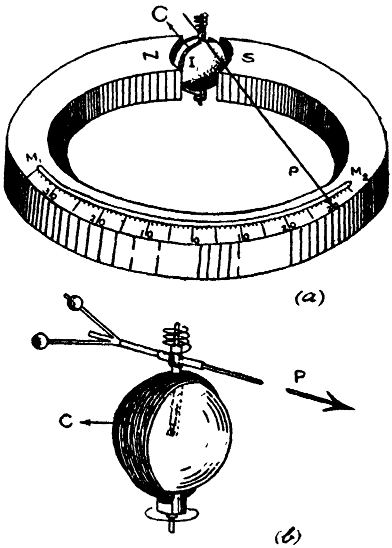

FIG. 104.

passed through the coil a magnetic field is set up. In the horizontal plane passing through the centre of the coil, these lines of force run at right angles to those due to the magnet and the coil is therefore deflected. The deflection is read by means of the pointer P attached to the coil, on a scale graduated on either side of the zero marked at its centre. An anti-parallax mirror strip M_1 M_2 is provided to facilitate accurate reading of the pointer. The free motion of the coil can be arrested, when required, by a clamping arrangement. The ends of the coil are soldered to two terminals not shown in figure. To intensify the magnetic effect and to render the magnetic field uniform in all positions of the coil, a spherical soft iron core (I) is symmetrically secured inside the coil. The coil when freely suspended just clears the core and when clamped, rests on it.

Let us now consider once again the method already described for the determination of electric resistance. The method assumed that the relation between the deflection of the galvanometer and the current producing it could be known ($C = K \tan \theta$). In the case of a tangent galvanometer the reduction factor K at any given place could be calculated from the dimensions of the coil. Moreover, currents of values ranging between 0·15 and 0·025 amp. could be measured with the type of the tangent galvanometer used. But with a suspended coil sensitive galvanometer such big currents can never be measured, the maximum that can be measured being of the order of $\frac{1}{1000}$ amp. The reduction factor for the suspended coil cannot be accurately determined because the dimensions of the coil cannot be accurately measured. Moreover, the plane of the coil, when the current is passing through is not confined to the magnetic meridian or to any single plane. Hence the method employed with the tangent galvanometer is not applicable in the case of a sensitive galvanometer of the suspended coil type. Again we assumed in the tangent galvanometer method that the E.M.F. of the

cell used was constant. This is not true with all cells and even in the case of a Danfell cell, beyond a measure. It is therefore desirable to employ a different method in which these difficulties are avoided. Such a method is the Wheatstone bridge method.

The Wheatstone bridge is an arrangement * of four resistances P, Q, R, S as shown in fig. 105 (a). An E.M.F. is applied between the points A and B by means of a voltaic cell. G is a sensitive galvanometer. ACB and ADB are two parallel circuits. The main current i from the cell enters at A and divides itself into two branches i_1 along ADB and i_2 along ACB. The currents rejoin at B. The fall of potential between any two points in a circuit is proportional to the resistance between the points.

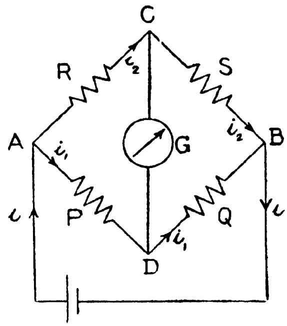

FIG. 105 (a).

$E \propto R$ if C is constant. So $\dfrac{\text{P.D. in AD}}{\text{P.D. in DB}} = \dfrac{P}{Q}$ and similarly $\dfrac{\text{P.D. in AC}}{\text{P.D. in CB}} = \dfrac{R}{S}$. Again the potentials at A and B are equal, i.e. the fall of potential in ADB is equal to that in ACB. Therefore, for every point in ADB there must be a corresponding point of the same potential in ACB. So there must be a point in ACB which must be at the same potential as D in the branch ADB. Suppose the resistances P and Q are fixed in value, then the potential at D will have a definite fixed value. Arguing similarly it is obvious that the potential of C can be varied by fixing the value of R and changing S. Therefore, for a particular value of S there will be no difference

* This network of conductors was invented by Sir Charles Wheatstone, Professor of Experimental Philosophy, King's College, London, and one of the inventors of electric telegraph.

of potential between the points C and D and a sensitive galvanometer connected between the points will show no deflection *. Under these conditions it follows that P.D. in AC is equal to P.D. in AD and that P.D. in CB is therefore equal to P.D. in DB.

Therefore, $$\frac{P}{Q} = \frac{R}{S} \text{ and } R = S \times \frac{P}{Q}$$

An unknown resistance R can therefore be found if the ratio of two resistances (P and Q) and the value of the third resistance (S) is known. One great advantage of this method is that the E.M.F. of the cell need not be steady. It can be easily seen that the positions of the cell and the galvanometer are interchangeable. When so interchanged we find from the diagram that

$$\frac{R}{P} = \frac{S}{Q} \text{ and } R = S\,\frac{P}{Q} \text{ as before.}$$

The *Metre Bridge* and the *Post Office Box* are the two forms of Wheatstone bridge used in the laboratory.

A metre bridge is so called because a wire one metre long is used. For convenience the wire used is sometimes only half metre in length and is used for less accurate (?) work.

We now proceed *to determine the resistance of a coil of wire. Apparatus required* are half-metre bridge, moving-coil galvanometer, Leclanche cell, an unknown resistance, some known resistance coils, connecting wires and a simple key.

In the half-metre bridge, a bare platinoid (?) wire AB, fig. 105 (*b*), of uniform cross-section is stretched parallel to the edge of a half-metre scale mounted on a wooden base board. The ends of the wire are soldered to thick copper plates provided with binding screws. A copper plate C

* This is why this method is a null method, little or no current passes through the galvanometer when the arms of the bridge are very nearly or exactly balanced.

with three binding screws, is fixed to the board on the other side of the scale. D is a sliding metal contact provided with a binding screw and an index mark.

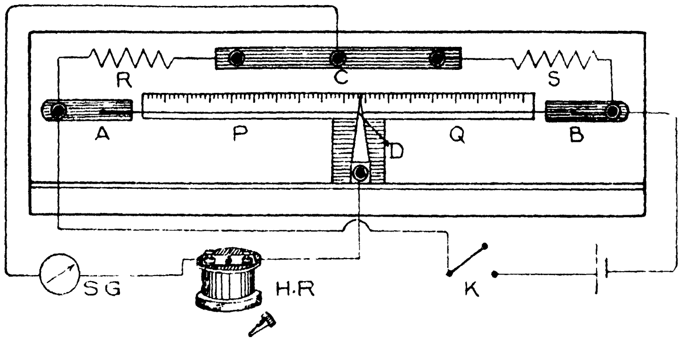

FIG. 105 (*b*).

Connect the apparatus supplied as shown in the figure. R is the unknown coil and S a known coil. H.R., is a high resistance introduced into the galvanometer circuit and the resistance can be cut off by introducing a plug between its terminals. Keep the contact key D at the middle of the wire AB to begin with and move it to the right or left making contact at successive points, with the battery key K pressed. When a point is found, at which the galvanometer shows no appreciable deflection, plug the high resistance. The galvanometer is now much more sensitive (?) Determine carefully once again the point on the wire at which the deflection of the galvanometer is zero. The key D divides the length of the wire into two parts AD of resistance P and DB of resistance Q. As the resistances are proportional to the lengths

$$\frac{P}{Q} = \frac{AD}{BD} = \frac{R}{S}$$

$$\therefore \quad R = S\,\frac{AD}{DB} \text{ ohms.}$$

Interchange the position of the coils R and S and repeat the observations. The mean value of R gives the resistance required.

When two resistances r_1 and r_2 are connected in parallel their *equivalent resistance* r is given by the relation $\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2}$. This relation can be verified with the apparatus, thus.

In the above arrangement of the bridge fig. 105 (b), shunt the given unknown coil with a known one of a smaller resistance (?) and determine, as before, the mean equivalent resistance. The reciprocal of this equivalent resistance would be found to be very nearly equal to the sum of the reciprocals of the resistances of the two parallel conductors.

Practical example.—

R. W. Paul's unipivot galvanometer, type A, was used in the experiment and H.R. = 15,000 ohms. S = 5 ohms.

Resistance.	Position of D, cm.	Deflection in scale divisions.	Whether H.R. in circuit.	Balancing point, D cm.
	25	1 left	Yes	
	20	2 ,,	,,	
	15	3 ,,	,,	
	30	0·5 ,,	,,	
	35	Inappreciable	,,	Near 35
R	40	1 right	,,	
	35	off the scale right	No	
	34	26 right	,,	Between 33 and 34.
	33	23 left	,,	
	33·5	2 left	,,	33·53 by interpolation
	33·6	4 right	,,	
	R and S interchanged 16·7	Zero	,,	16·7
R in parallel with 2 ohms.	37·1	Zero	,,	37·1
	Interchanged 13·0	1 left	,,	13·02
	Interchanged 13·1	3 right	,,	

(I) From the observations it is clear that

$$R : S :: 33{\cdot}53 : 16{\cdot}47$$

and when interchanged $R : S :: (50-16{\cdot}7) : 16{\cdot}7$

$$\therefore \quad R = \frac{5 \times 33{\cdot}53}{16{\cdot}47} = 10{\cdot}18 \text{ ohms.}$$

and when interchanged $R = \dfrac{5 \times 33{\cdot}3}{16{\cdot}7} = 9{\cdot}97$ ohms.

mean value of $R = 10{\cdot}1$ ohms.

How do you account for the difference in the value when interchanged ?

(II) since $R = 10{\cdot}1$ ohms, the equivalent resistance according to the formula

$$= \frac{10{\cdot}1 \times 2}{12{\cdot}1} = 1{\cdot}67 \textit{ ohms (calculated value).}$$

But the observed value is $\dfrac{5 \times 12{\cdot}9}{37{\cdot}1} = 1{\cdot}739$ ohms.

and when interchanged is $\dfrac{5 \times 13{\cdot}02}{36{\cdot}98} = 1{\cdot}76$ ohms.

*The mean observed value is 1·75 ohms.**

The *Post-Office Box* fig. 106 is a portable, compact and

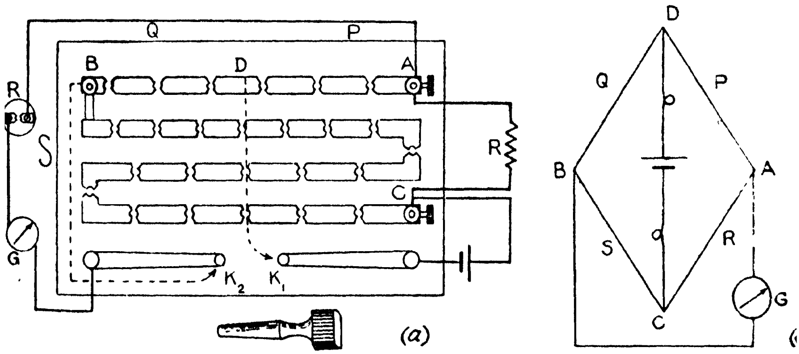

FIG. 106.

convenient form of Wheatstone bridge originally devised for

* The ends of the connecting wires and the connecting surfaces of the binding screws are to be bright and clean; otherwise they offer considerable resistance and vitiate the results.

the post-office for finding the resistance of telegraph wires. This form is often used in the laboratory to determine resistances accurately. A number of resistance coils are arranged in series as in a resistance box and the ends of each coil are connected to brass blocks with gaps between them. These gaps can be bridged by brass plugs with ebonite handles. A plug inserted between two blocks short-circuits the gap and cuts off the resistance between them. A top view of such a box with the lid removed and the plugs taken off, is shown in the figure. As before ADB and ACB are the two circuits in parallel and the points A and B are connected to the terminals of a sensitive moving-coil galvanometer G through a high resistance, H.R. AD and DB are called the "proportional arms" or "the ratio arms" of the bridge and correspond to the two lengths of the bridge wire AD and DB into which it is divided by the moving contact D. In the metre bridge the value of S was fixed and the corresponding proportion of the arms AD and DB was determined. In the post-office box, the proportion is fixed at one of three ratios provided, $\left(\frac{P}{Q} = 1 \text{ or } \frac{1}{10} \text{ or } \frac{1}{100}\right)$ and the particular value of S that balances the bridge is determined. K_1 is the battery key and K_2 is the galvanometer key.

The *specific resistance of a german silver wire can be determined with the post-office box as follows.* Connect the apparatus as shown in fig. 106. Screw down all the plugs, to secure good conduct. The plugs and the corresponding gaps must be bright and clean so that they offer no resistance between them. Remove the plug 1000 ohms in each of the proportional arms. With no resistance in the arm S press down the battery key K_1 and then K_2 (?) for a moment with the high resistance in the galvanometer circuit. Note the direction of the deflection of the needle. Remove the infinity plug in the S arm (the resistance is simply the air gap between) and

note that the deflection is in the opposite direction. This indicates that the connections are all right (?) The following observations made in an actual experiment show how to proceed further.

Practical example.—

$$R = S\frac{P}{Q}$$

Q Ohms.	P Ohms.	S Ohms.	Deflection in scale divs.	Whether H.R. is in the galvanometer circuit.	Resistance lies between Ohms.
1000	1000	0	Zero		
,,	,,	infinity	off the scale right	Yes	
,,	,,	1	Zero		
,,	,,	1	4 left	No	1 and 2
,,	,,	2	11 right		
,,	100	12	2 right	,,	1·1 and 1·2
,,	,,	11	14 left		
,,	10	116	11 ,,		
,,	,,	117	6 ,,		1 18 and
,,	,,	118	2 ,,	,,	1·19
,,	,,	119	2 right		

Interpolating, R = 1·185 ohms.

Remove the wire R, whose specific resistance is being determined marking out with a file the points at each end where it just enters into the binding screws. Stretch the wire tight and measure the length between the file marks. Lay bare the insulation at 5 or 6 points along the length of the wire and find the mean diameter with the screw gauge. Take measurements of two cross diameters at each point and tabulate thus:

Length .... 89·1 cm.

Diameter.	
mm.	mm.
⟷	↕
0·55	0·55
0·555	0·545
0·545	0·555
0·555	0·545

Mean diameter *0·055* cm.
Mean radius *0·0275* cm.

$$\therefore \text{ Specific resistance } \rho = \frac{R \times A}{l} = \frac{R \times \pi r^2}{l} = \frac{1 \cdot 185 \times \pi \times 0 \cdot 0275^2}{89 \cdot 1}$$

$$= 31 \cdot 6 \times 10^{-6} \text{ ohms per cm. cube.}$$

$$= 31 \cdot 6 \text{ micro-ohms per cm. cube,}$$

for the given specimen of the *german silver wire*.

The P.O. box can be used to study the following laws relating to resistances and will be found to be more accurate than the metre bridge.

1. The resistance of a given wire varies directly as the length and inversely as the cross-section.

2. $\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2}$.

POTENTIOMETER—(PYE).

The instrument shown in fig. 107 (*a*) is handy and accurate enough for all ordinary potentiometer work. "The instrument consists of a base board T with the necessary terminals and plates to carry in continuity ten wires each one metre long, lying evenly on a plate of opal glass. A raised metre scale divided into millimetres, is provided on the back edge of the base and one edge of the triangular brass sliding contact maker K nearly touches this scale". The contact maker slides along the whole length of the scale on three brass legs, one of which sliding on a brass strip M near the front edge of the base. The contact maker is provided with a plunger P, which can slide on the triangular frame at right angles to the length of the wires and thus can make contact at any point on any wire.

The apparatus required for *comparing the E.M.F. of two cells with the potentiometer* are a Laboratory Wire Potentiometer, moving-coil galvanometer, an accumulator or a secondary cell, Leclanche and Daniell cells, high resistance with short circuiting plug key, and some connecting wires.

If a cell C fig. 107(*b*) sends a *steady current* through a circuit CAB the drop of potential (P.D.) between any two points of the circuit is, according to Ohm's law, proportional

to the resistance between them and the direction of the fall is from A to B. (?) If AB is wire of uniform cross-section then the P.D. between any two points is proportional to the length between them.

Connect the apparatus as shown in the fig. 107(a). The positive poles of all the three cells are connected to the end—A of the wire potentiometer and the E.M.F. of the cell C is such that the P.D. between A and B due to it is greater than the E.M.F. of either of the two other cells L and D. The direction of the current in the galvanometer G due to C is

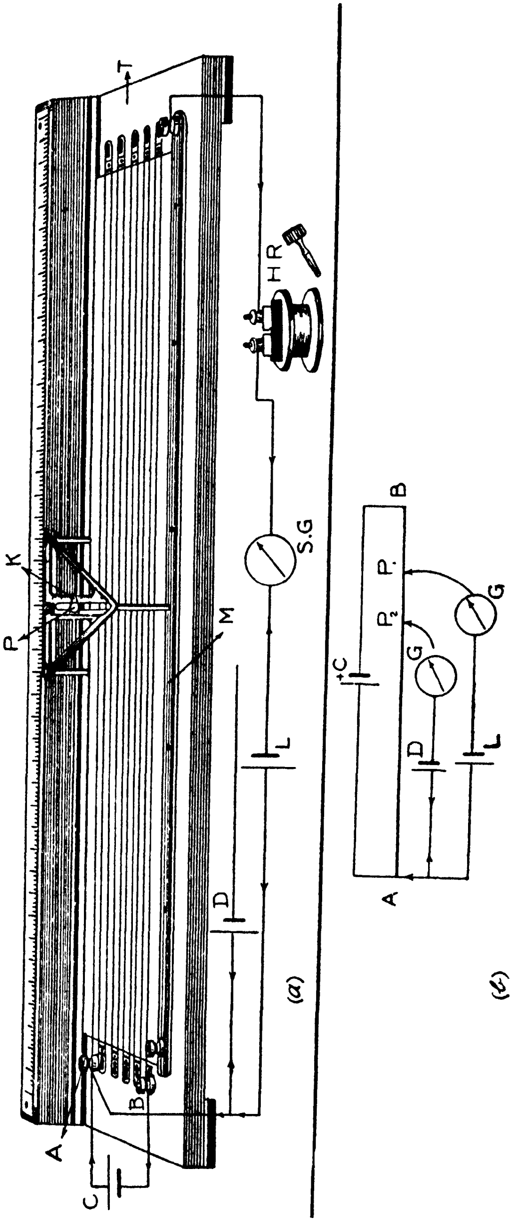

FIG. 107.

opposed to that of L or D. Connect the negative pole of the Leclanche cell as shown in the figure and find out a point P_1 on the wire AB such that no current passes through the galvanometer. Then the P.D. on the wire from A to P_1 is equal and opposite to the E.M.F. of the cell L. Now disconnect L and connect D and find a point P_2 similarly, cutting off the high resistance from the galvanometer circuit only for the final adjustments in each case.

We have
$$\frac{E_1}{E_2} = \frac{\text{P.D. in } AP_1}{\text{P.D. in } AP_2} = \frac{AP_1}{AP_2}.$$

Take another set of readings and take the mean ratio. When the balancing point is obtained, press the plunger for a time continuously and note the changes in the deflection of the galvanometer needle in both the cases. What difference do you notice between the two cases ?

Practical example.—

A secondary cell was used for C and the galvanometer used was the same as that used in the Wheatstone bridge.

Cell.	Index reading of contact maker. cm.	H.R. in or out of circuit.	Deflection in scale divisions.
(1) L	550	In	13·3 left.
	650	,,	5·5 ,,
	750	,,	5·5 right.
	700	,,	0·5 ,,
	690	,,	0·2 left.
	690	Out	Off the scale.
	695	,,	4 right and gradually off the scale to the left.
	700	,,	Off the scale, right.
	697	,,	27 left.
	698	,,	19 ,,
	699	,,	Zero.
	½ a minute after.	,,	Pressed the plunger for a time; 25 left, then 4·0 right and then at 0: needle oscillates about 0 always.

Cell.	Index reading of contact maker. cm.	H.R. in or out of circuit.	Deflection in scale divisions.
D	650	In	6·0 right.
	550	,,	3·0 left.
	570	,,	1·5 ,,
	580	,,	0·2 ,,
	590	,,	0·5 right.
	583	,,	Zero.
	583	Out	Off the scale left.
	584	,,	21 left.
	585	,,	3·5 right.
	584·9	,,	1·5 ,,
	584·8	,,	Zero.
	½ a minute after.	,,	Moved about zero ½ a division on either side slowly.
	Repeated again.	,,	,,
(2) L	699	Out	Zero; immediately moved about zero quickly and went off the scale to the left.
	705	,,	Off the scale, left.
	710	,,	15 left.
	711	,,	4 right.
	710·9	,,	0·5 left.
	½ a minute after.	,,	Deflection increased gradually to left.
D	583·8	,,	Zero.

$$\therefore \quad \frac{E_1}{E_2} = \frac{699}{584 \cdot 8} = 1 \cdot 195 \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

Again, $$\frac{E_1}{E_2} = \frac{710 \cdot 9}{583 \cdot 8} = 1 \cdot 218 \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

Mean ratio, $$\frac{E_1}{E_2} = 1 \cdot 206$$

1. What inferences do you draw regarding the constancy of the E.M.F. of the two cells L and D from the observations given above.

2. What is meant by the E.M.F. of a cell (i) in closed circuit and (ii) in open circuit? How do they differ and why?

3. Why is a storage battery preferable to a Bunsen or Daniell battery in potentiometer work?

CHAPTER XIII.

ELECTROLYTIC AND THERMAL EFFECTS.

FARADAY'S LAWS OF ELECTROLYSIS.

Apparatus required to verify the laws of electrolysis are a water voltameter, tangent galvanometer, commutator, a battery of cells (the laboratory D.C. supply) adjustable resistance and connecting wires.

Keep the coil of the galvanometer in the magnetic meridian. Connect the apparatus as shown in fig. 108; + − are the terminals of the battery. R is the adjustable resistance. O and H stand for oxygen and hydrogen liberated. m, m are projections or glass cups containing mercury into which the leads from the binding screws dip. p_1, p_2 are the platinum anode and cathode respectively. C is a commutator and G a tangent galvanometer.

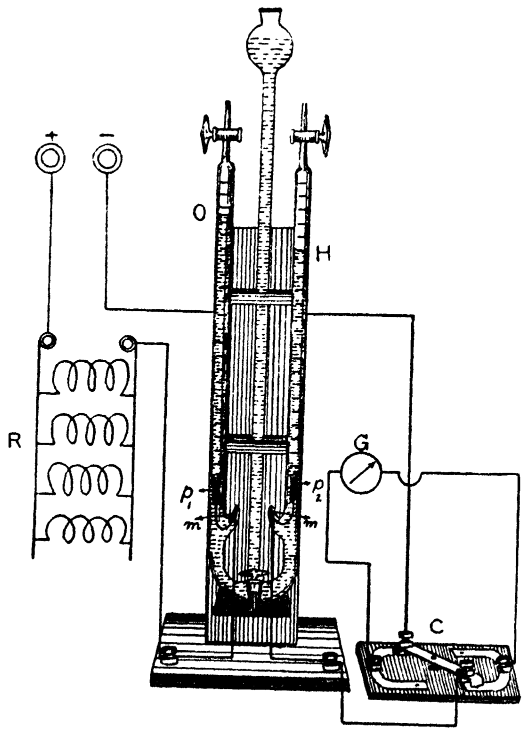

FIG. 108.

Switch on the current and adjust the resistance such

that the deflection of the galvanometer is nearly 45°. Stop the current. Allow the gases to collect and let them escape. Switch on the current again and with the help of a watch provided with a seconds hand, switch off the current at the end of 30 seconds. Meanwhile note the deflection of the needle. Wait for a few minutes till the gases collect and take readings of the levels O and H on the graduated tubes. Reverse the current in the galvanometer circuit only (?) and start again. Proceed similarly and take six sets of readings. Note the head of the dilute acid in each limb or tube at the end. Let off the liberated gases. Alter the resistance suitably, start again and proceed as before. Tabulate the observations.

Practical example.—

A sixty volt storage battery was used. Incandescent electric lamps in parallel were employed for the adjustable resistance. The Pye tangent galvanometer *with two-turns* coil (?) was used.

Observed barometric height .. .. 75·95 cm.
Reading on the attached thermometer .. .. 88° F. (31° C.)

I. R_1—4 lamps in parallel.

Time of passage of current.	Deflection—degrees of arc.	Mean deflection.	Vol. of gas. c.c.		Vol. collected per minute. c.c.	
			H	O	H	O
30 sec.	Direct 40 40		7·8	3·8	15·6	7·6
1 min.	Reversed 39 39		15·1	7·6	14·6	7·6
1·5 ,,	,, 39 39	39°·5	22·4	11·2	14·6	7·2
2·0 ,,	Direct 40 40		29·6	14·8	14·4	7·2
2·5 ,,	,, 40 40		36·7	18·5	14·2	7·4
3·0 ,,	Reversed 39 39		43·9	22·2	14·4	7·4
				Mean ..	14·6	7·4

Acid head in cm. for H limb is 50·5 and for O limb is 33.

II. R_2—3 lamps in parrallel.

Time of passage of current.	Deflection—degrees of arc.	Mean deflection.	Vol. of gas. c.c.		Vol. collected per minute. c.c.	
			H	O	H	O
0·5 min.	35° 35°		6·2	3·1	12·4	6·0
1·0 „	35 35		12·4	6·1	12·4	6·0
1·5 „	35 35	35°	18·3	9·1	11·8	6·0
2·0 „	35 35		24·2	12·0	11·8	5·8
2·5 „	35 35		30·2	15·0	12·0	6·0
3·0 „	35 35		36·1	18·1	11·8	6·2
				Mean ..	12·0	6·0

Acid head in cm. for H limb is 44 and for O limb is 29.

The following are the inferences from the experiment.

(I) The volumes of the gases collected in a minute during successive stages are nearly constant when the current passing through is steady. But the masses of the gasos collected are proportional to their volumes if they are subject to the same conditions regarding pressure and temperature. Therefore *the masses* (m) *of the gases set free above are proportional to the time* (t) *when the current* (C) *is constant.*

Are the gases soluble in the liquid? If so which is more soluble? Is the pressure of each gas the same at different stages of the experiment? How then would the observed volumes for each gas be changed when reduced to a common pressure? In which gas would the variation be greater?

(II) If C_1 and C_2 are the strengths of the current in I and II respectively $\frac{C_1}{C_2} = \frac{\tan 39^\circ\cdot 5}{\tan 35^\circ} = 1\cdot 18$

Again the ratio of the mean volume of hydrogen collected per minute in I to that in II is $\frac{14\cdot 6}{12\cdot 0} = 1\cdot 22$

and of oxygen is .. $\frac{7\cdot 4}{6\cdot 0} = 1\cdot 23$

Under the limitations discussed above the ratios 1·22 and 1·23 are nearly equal to the ratio 1·18 of the currents. Combining the two inferences, since $m \propto t$ when C is constant (I) and

$m \propto C$ when t is constant (II), we have $m \propto Ct$ when both C and t vary. But Ct is the quantity (Q) of electricity that flows through a circuit when a current C passes for a time t.

$$\therefore \quad m \propto Q$$

Hence *Faraday's First Law of Electrolysis which states that the masses of ions (?) liberated during electrolysis are directly proportional to the quantity of electricity passing through the electrolyte*, is verified.

(III) In each of the above cases I and II the volume of oxygen collected at each stage is nearly half of the corresponding volume of hydrogen and the mass of oxygen collected is nearly eight times that of hydrogen at any instant of time because oxygen is 16 times heavier bulk for bulk than hydrogen under the same condition of temperature and pressure. The ratio of the masses is therefore the ratio of the chemical equivalents. Hence *Faraday's Second Law of Electrolysis* viz. *the masses of the ions liberated in electrolysis, when the same quantity of electricity passes through, are proportional to their chemical equivalents*, is also verified.

We proceed now to calculate the electro-chemical equivalent ϵ of hydrogen from the observations made above.

Since $m \propto Ct$, we may write $m = \epsilon.C.t$ where ϵ is a constant, and this constant ϵ *is equal to the mass of the ion liberated when unit quantity of electricity passes through and is called the electro-chemical equivalent of the ion.*

$$\therefore \quad \epsilon = \frac{m}{Ct} = \frac{m}{K \tan \theta . t}$$

From the observations in I above, m of hydrogen is thus calculated:

Observed barometric height	75·95 cm.
Attached thermometer reading	88°F. or 31°C.
Pressure corrected for temperature ..	75·57 cm.
Pressure due to 50·5 cm. head of dilute acid of sp. gr. 1·125	4·18 cm.
Total pressure of hydrogen collected at 28°C ..	79·75 cm.

Partial pressure of dry hydrogen (?)
$79{\cdot}75 - 2{\cdot}83$ .. .. .. $76{\cdot}92$ cm.
($2{\cdot}83$ = the max. tension of aqueous vapour at 28°C.)
Temperature of gas .. .. .. 301° abs.
Vol. of hydrogen collected .. .. $43{\cdot}9$ c.c.

$$\text{Vol. at N.T.P.} = \frac{43{\cdot}9 \times 273 \times 76{\cdot}92}{301 \times 76{\cdot}0} = 40{\cdot}3 \text{ c.c.}$$

m of hydrogen collected $= 40{\cdot}3 \times 0{\cdot}00009 = $ *0·003627 gm.*

The reduction factor of the galvanometer with 2 turns of coil of radius 7·1 cm. at the place of the experiment where H = 0·36 gauss is—

$$K = \frac{rH}{2\pi n} = \frac{7{\cdot}1 \times 0{\cdot}36}{2\pi \,.\, 2} \times 10 \ (?) = 2{\cdot}034 \textit{ amps.}$$

$$\therefore \quad \epsilon = \frac{0{\cdot}003627}{2{\cdot}034 \times \tan 39{\cdot}5 \times 180} = 0{\cdot}000012 \text{ gm.}$$

or

$$\therefore \quad \epsilon = 0{\cdot}012 \text{ gm. per kilo-coulomb.*}$$

The accepted value for hydrogen is 0·01044 gm. per kilo-coulomb.

When a current is passed through the electrolyte (dilute sulphuric acid) the sulphuric acid molecule dissociates into two hydrogen ions—hydrions—and one sulphion, SO_4. Hydrogen travels with the current and is liberated at the cathode, the negative electrode and the sulphion travels up the current to the anode, the positive electrode. *Hydrogen* is therefore *electropositive,* like metals. The sulphion combines with H_2 of a water molecule, forms a molecule of H_2SO_4 again and O of the water molecule is liberated at the anode after a *secondary reaction. Oxygen* is therefore *electronegative* as it is liberated at the positive plate.

It has been stated above that hydrions of mass 0·00001044 gm. carry a charge of a coulomb of electricity. The atomic

* Coulomb is the unit quantity of electricity conveyed by a current of 1 ampere in 1 second. This is called after Charles Augustin de Coulomb (1736–1806) a French physicist who enunciated Coulomb's law—the inverse square law of action between magnetic poles and electric charges.

weight of hydrogen is 1·008 gm. and therefore 1·008 gm. of hydrogen is liberated in electrolysis by the passage of $\frac{1\cdot008}{0\cdot00001044} = 96{,}550$ coulombs. This is the same thing as saying that a gram-equivalent of hydrogen ions carry 96,550 coulombs of electricity. This quantity of electricity is called a *Faraday.** Now the second law of Faraday can be stated thus. "*Masses of substances proportional to their chemical equivalents carry equal charges of electricity.*" Therefore 1·008 gm. of hydrogen or 108 gm. of silver or $\frac{63}{2}$ gm. of copper carry 96,550 coulombs of positive charge as ions. The least quantity of electricity that can be carried during electrolysis is therefore that carried by a monovalent ion positive or negative. This quantity has been evaluated many years after Faraday to be $96{,}550/6\cdot06 \times 10^{23} = 1\cdot59 \times 10^{-19}$ coulombs, $6\cdot06 \times 10^{23}$ being Avogadro's constant. This quantity $1\cdot59 \times 10^{-19}$ coulombs is also found to be the free negative charge associated with what is known as an "Electron" the fundamental unit of electricity.

Electrolysis affords a very accurate method of measuring current strength.

From the standard value of ϵ for hydrogen the reduction factor can be calculated backwards (?) This is the method generally adopted for determining the value K of a tangent galvanometer. A copper voltameter (?) can better be employed for the purpose, the strength of the current being calculated from the increase in weight of the cathode copper plate ($C = m/\epsilon t$). The method of copper deposition by electrolysis is a reliable method of finding the current strength in a circuit and is much more accurate than that of the tangent galvanometer which was employed in a previous exercise.

---

* Michael Faraday was an English physicist and chemist (1791–1867). He was a great discoverer. He discovered the laws of electrolysis, and the laws of electro-magnetic induction.

Measurement of current by silver deposition is even more accurate than by copper deposition. Hence the Board of Trade has defined a coulomb, the unit quantity of electricity, as that which deposits 0·001118 gm. of silver from a solution of silver nitrate.

1. Calculate from the data given above the mass of oxygen collected in I and find the ratio of the masses of the gases obtained for the same flow of electricity.
2. Can a single Daniell or Leclanche cell electrolyse water ?

JOULE'S LAW.

The E.M.F. of a cell is measured by the work done in the circuit of the cell when a unit quantity of electricity passes round and is expressed in practical units in Joules per coulomb. A volt is the unit of E.M.F. in practical units and is equal to a Joule per coulomb. If E = E.M.F. of a cell or the drop in potential (D.P.) in any part of a circuit and W is the work done in mechanical units and Q, the quantity of electricity when a current C passes for t seconds; then

$$E = \frac{W}{Q} \text{ or } E.C.t = W.$$

It is an experimental fact that the temperature of a coil rises, when a quantity of electricity flows through it. The heat energy thus liberated is equal to the mechanical work done in the circuit.

$\therefore$ $E.C.t = W = J.H.$, where J = mechanical equivalent of heat. If R is the resistance of the circuit, $E = CR$ (Ohm's law) and $C^2Rt = JH$, i.e. the heat liberated in a circuit is directly proportional (*a*) to the square of the current flowing through, (*b*) to the resistance of the circuit and (*c*) to the time of flow. This statement is called Joule's Law.

The apparatus required *to verify Joule's law* are two insulated coils of wire of resistances R_1 and R_2, two similar calorimeters (B) with stirrers (S), two specific heat thermo-

meters (T) reading to 0·1°C, tangent galvanometer (G), commutator, a battery of cells, adjustable resistance (R), measuring jar, connecting wires, etc.

Put the coil of the galvanometer in position.

I. Connect the apparatus as shown in fig. 109. Find the weight of the calorimeter with stirrer correct to a tenth of a gram; add sufficient amount of water from the measuring jar. Adjust the resistance to give a good deflection. Stir the water in the calorimeter and note the initial temperature. Start the current and note the time with a watch provided with a seconds hand. Stir the water gently and constantly (?). Note the temperature at the end of each minute. Note down the mean deflection by reversing the current in the galvanometer. Pass the current for a few minutes until the rise in temperature in the calorimeter is considerable. Stop the current, noting the maximum observed temperature. Let the calorimeter cool for half the time of the experiment and add the fall to the observed maximum temperature. This gives the final temperature corrected for cooling.

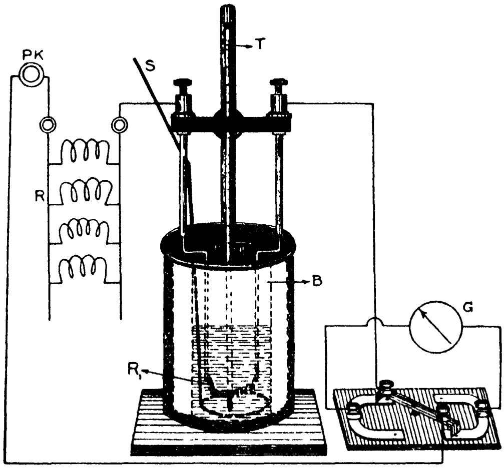

FIG. 109.

II. Cool the calorimeter nearly to its initial temperature and include in series with the first calorimeter the other calorimeter containing the coil R_2. Alter the resistance R if necessary to get a different galvanometer reading. Proceed

as before and take observations similarly. Determine the resistances of the coils R_1 and R_2 with the Post-Office Box.

Calculate from the data in I, the rates of rise of temperature. You find that they are nearly constant. This verifies that the heat liberated is directly proportional to time when the resistance and the current are constant. Note that the heat evolved in the calorimeter is proportional to the rise in temperature. Again in the case of the first calorimeter through which two different currents were passed you find that $\frac{C_1^2}{T_1} = \frac{C_2^2}{T_2}$ where T_1 and T_2 are the rises of temperature per minute, i.e. you find that $\frac{\tan^2\theta_1}{T_1} = \frac{\tan^2\theta_2}{T_2}$. This therefore verifies the statement that the heat liberated is directly proportional to the square of the current flowing through the calorimeter.

Moreover, from the second set of observations where the same current C_2 is flowing through the two calorimeters B_1 and B_2 you find that $\frac{\text{rate of rise of temperature in } B_1}{\text{rate of rise of temperature in } B_2} = \frac{R_1}{R_2}$. This verifies the statement that the heat liberated is directly proportional to the resistance, if the current and time are kept constant. Note that the total mass of water in the two calorimeters is taken to be the same because they are equivalent or similar. Thus you find that Joule's Law is completely verified.

The mechanical equivalent of heat (J) can be calculated from any one of the above sets of observations.

$$JH = C^2Rt$$

$$J = \frac{C^2Rt}{H} = \frac{K^2 (\tan\theta)^2 Rt}{m \times b}.$$

K is the reduction factor of the galvanometer, m the total mass of water and b the rise in temperature.

Practical example.—

60 volts main was used to send a steady current. Incandescent electric lamps in parallel were used for the variable resistance. A Pye tangent galvanometer with a *coil of 2 turns* was used.	Wt. of calorimeter $B_1 = 28{\cdot}0$ gm. ,, $B_2 = 29{\cdot}1$,, Vol. of water in each $= 116$ c.c. Wt. of water in each $= 116$ gm. $R_1 = 4{\cdot}10$ ohm. $R_2 = 10{\cdot}07$,,

I. Calorimeter B_1 with the resistance coil R_1 is used.

$$C_1 = K \tan \theta_1, \quad R = 3 \text{ lamps in parallel.}$$

Time of passage of current in minutes.	Temp. of calorimeter. °C.	Deflection observed. θ_1	Mean θ_1	Rate of rise of temp. in °C. per min. T.
4 hr. 1′ 0″	31·1	46° 46°		
2′ 30″	34·7		47°	2·4
3′ 30″	37·1	Reversed 48° 48°		2·4
4′ 30″	39·2			2·1
6′ 0″	42·5			2·2
7′ 0″	44·6			2·1
8′ 0″	46·7			2·1
current stopped				
11′ 30″	45·2			

Corrected maximum temperature $= 46{\cdot}7 + (46{\cdot}7 - 45{\cdot}2) = 48^\circ{\cdot}2$C.

Mean corrected rate of rise $T_1 = \dfrac{48{\cdot}2 - 31{\cdot}1}{7} = 2^\circ{\cdot}44$C per minute.

II. B_2 with the resistance coil R_2 is put in series with the above arrangement and the number of incandescent lamps in parallel is reduced to *2*. $C_2 = K \tan \theta_2$.

Time.	Temperature. °C. B_1	B_2	Deflection. θ_2	Mean θ_2
4 hr. 41′	35·5	29·7	32·5 32·5	
			Reversed	32
4 45′	38·2	37·2	31·5 31·5	
4 50′	41·2	45·8		
	Current stopped			
4 54′ 30″	40·0	44·2		
Corrected max. temp.	42·4	47·4		
Rate of rise of temp.	T_2	T_2'		
	$\dfrac{6{\cdot}9}{9}$	$\dfrac{17{\cdot}7}{9}$ °C per min.		

Inferences.—

(*a*) $$\frac{(\tan\theta_1)^2}{T_1} = \frac{(\tan 47)^2}{2{\cdot}44} = 0{\cdot}471 \quad .. \quad .. \quad .. \quad \mathrm{I}$$

and $$\frac{(\tan\theta_2)^2}{T_2} = \frac{(\tan 32)^2\times 9}{6{\cdot}9} = 0{\cdot}509 \quad .. \quad .. \quad \mathrm{II}$$

$\therefore$ for the same calorimeter B_1, $\dfrac{\tan^2\theta_1}{T_1} = \dfrac{\tan^2\theta_2}{T_2}$ nearly (7% error)

Hence $H \propto C^2$ when C and *t* are constant, is verified.

(*b*) The calorimeters are made of aluminium and the total mass of water in each is $\begin{cases} m_1 = 28\times 0{\cdot}22+116 = 122{\cdot}2 \text{ gm.} \\ m_2 = 29{\cdot}1\times 0{\cdot}22+116 = 122{\cdot}4 \text{ gm.}\end{cases}$

The total mass is thus practically the same.

$$\frac{R_1}{R_2} = \frac{4{\cdot}10}{10{\cdot}07} = 0{\cdot}407 \text{ and } \frac{T_2}{T_2'} = \frac{6{\cdot}9}{17{\cdot}7} = 0{\cdot}390$$

These two ratios are nearly the same (4% error).

Hence $H \propto R$ when C and *t* are constant is verified.

(*c*) In I the figures in the last column are nearly constant and hence, *$H \propto t$, when R and C are constant.* Therefore conjointly *$H \propto C^2Rt$* and the law is completely verified.

From the observations in I, since K = 2·034 amps.,

$$J = \frac{K^2 \tan^2\theta\, Rt}{m.b}$$

$$= \frac{2{\cdot}034^2 \times \tan^2 47 \times 4{\cdot}1 \times 7 \times 60}{122{\cdot}2 \times 17{\cdot}1} = 3{\cdot}92 \text{ Joules per calorie.}$$

The following calculations will be interesting.

$$E = \frac{J\times H}{c\times t} = \frac{JH}{K\cdot\tan\theta\times t}$$

Therefore the drop of potential E between the ends of a resistance coil can be calculated by assuming the value of J which can be independently determined (chapt. VI). For example, taking the coil R_1 into consideration, the potential fall can be calculated as well as the ratio of $\dfrac{\text{D.P.}}{C}$ in I and II separately.

D.P.		Current flowing through R_1		$\frac{\text{D.P.}}{\text{current}}$ in ohms.	
I.	II.	I.	II.	I.	II.
$\frac{\text{J.H.}}{K \tan \theta_1 \times t_1}$	$\frac{\text{J.H.}}{K \tan \theta_2 \times t_2}$	$K \tan \theta_1$	$K \tan \theta_2$	$\frac{9 \cdot 58}{2 \cdot 18}$	$\frac{\text{J.H.}}{(K \tan \theta_2)^2 \times t}$
$= \frac{4 \cdot 2 \times 122 \cdot 2 \times 17 \cdot 1}{2 \cdot 034 \times \tan 47 \times 420}$		$= 2 \cdot 034 \times \tan 47$		$= 4 \cdot 4$	$= \frac{4 \cdot 2 \times 122 \cdot 2 \times 6 \cdot 9}{2 \cdot 034^2 \times \tan^2 32 \times 540}$
$= 9 \cdot 58$ volts.		$= 2 \cdot 18$ amps.			$= 4 \cdot 07$
					$= 4 \cdot 1$

The two ratios 4·4 and 4·1 give the resistance of the coil R_1 which must be the same and was found to be 4·1 ohms.

What is the effect of temperature on the resistance of a coil? How does this altered resistance change the current in the circuit?

CHAPTER XIV

SOUND

GENERAL.

Sound is caused by a vibrating system. After production at the source, sound is transmitted by a material medium, ordinarily air, and is perceived by the ear. If the mechanical disturbances caused in the sounding body are irregular, *noises* are produced and are unpleasant to the ear. They are sometimes abrupt and short lived. If the disturbances caused are regular, a *musical note* is produced and each particle of the sounding body periodically goes through a cycle of disturbances which repeats in equal intervals of time. This interval is called the period of the vibration. The quicker the motion, the smaller the periodic time and the greater the number of vibrations or oscillations per second. This number is called the *frequency* of the sound produced.

The *pitch* of a note rises or falls with its frequency and the *loudness* with the amplitude of vibration.

A sounding body generally vibrates in more than one mode and emits sounds of different frequencies at the same time. The sound with the lowest frequency will be prominent and loud and is called the *fundamental note*. When we talk of the frequency of a sounding body we mean that of the fundamental. The attendant notes of higher frequencies are not loud enough and die away quickly while the fundamental note persists. Two sounding bodies may have the same fundamental note but the attendant sub-notes may be different. Such sounds are said to differ in *quality* or *timbre* and the ear is able to distinguish them. The greater the number of sub-notes the richer is the quality or *character*. The attendant notes are called overtones or upper partials.

The less the number of overtones present the purer is the note. A tuning fork is generally employed to produce a pure note.

The following are the three characteristics of a musical sound :—

1. Intensity or loudness, which depends on the amplitude of vibration.
2. Pitch, which depends on frequency.
3. Quality or timbre, which depends on the overtones produced along with the fundamental.

To understand the mechanism of transmission of sound in an elastic medium, let us consider the case of a vibrating tuning fork (fig. 110). We will describe for convenience only four prominent positions of each prong. Let us start with one prong moving through its position of rest or its mean position, towards the right and count time from this point of reference, *a*. The four prominent states of vibration are described in the tabular statement given below. The same state or phase of oscillation repeats itself every t seconds where t is the time of a complete oscillation.

FIG. 110.

Direction of Motion.	Position.	Velocity.	Displacement.	Time. Secs.
⟶	(*a*) Mean.	Maximum.	Zero.	0
Momentary rest.	(*b*) Extreme right.	Zero.	Maximum right.	$\frac{t}{4}$
⟵	(*c*) Mean.	Maximum.	Zero.	$\frac{t}{2}$
Momentary rest.	(*d*) Extreme left.	Zero.	Maximum left.	$\frac{3t}{4}$
⟶	(*e*) Mean.	Maximum.	Zero.	t

Let us now consider what is happening in the medium, air. In the position (*a*) layers of air to the right of the fork are compressed and as the air is elastic, the energy of compression is transmitted to the successive layers to the right. The compression travels in the air from point to point with a definite velocity depending on the elasticity, density and temperature of the medium. Under a given set of conditions the velocity will be constant and the compression travels equal distances in equal intervals of time. Let d be the distance travelled in $\frac{t}{4}$ sec. Let the points in air A, B, C, D, etc., be in the same line d cm. apart. The compression that starts at A will be at B after $\frac{t}{4}$ sec., at C after $\frac{t}{2}$ sec. and so on. But at the end of $\frac{t}{4}$ sec. the prong will be in state (*b*) and hence momentarily at rest; so the layer of air next to the prong will be in the normal or undisturbed state marked N as given in the table which shows the condition in space and time of the elastic medium, air.

	A	B	C	D	E	F	G
	$\longleftarrow$	d $\longrightarrow$	$\longleftarrow$	d $\longrightarrow$	$\longleftarrow$	d $\longrightarrow$	
t, 0	C						
$t/4$	N	C					
$t/2$	R	N	C				
$3t/4$	N	R	N	C			
t	C	N	R	N	C		
$5t/4$	N	C	N	R	N	C	
$3t/2$	R	N	C	N	R	N	C

Rarefaction ..	R.	Point of max. decrease of pressure.	Points of zero displacement or Nodes.
Compression ..	C.	Point of max. increase of pressure.	
Normal state ..	N.	Point of zero variation of pressure.	Point of max. displacement or Antinode.

Between 0 and $\frac{t}{4}$ sec. the compressed condition of air at A next to the prong, gradually becomes less and less compressed until it is normal. This normal condition N will also travel in the air and will appear at the point B, $\frac{t}{4}$ sec. afterwards, i.e. after a total time of $\frac{t}{2}$ sec. Thus N will be the condition at C, D, etc. at the end of $\frac{3t}{4}$, t, etc. From the table you find that the point A goes through a cycle of compression, normal state, rarefaction, normal state and once again compression in each period of time of t sec. The point B goes through exactly the same cycle $\frac{t}{4}$ sec. afterwards, the point C $\frac{t}{2}$ sec. afterwards and so on. The condition of air at A, B, C, D, at the same instant, is different but each point goes through the same cycle in the same order in the same periodic time, the only difference being that the starting point in each is progressively behind by a time $\frac{t}{4}$ sec. The phase of vibration of air at points along any direction, whose distance apart is such that the sound takes t, $2t$, $3t$ to travel, is the same, i.e. points which are $4d$, $8d$, $12d$ away from the source are in the same phase. $4d$ is the least distance between two points which are in the same phase and is called the *wave length* and is represented by the letter λ. If V is the velocity of sound in the medium $V.t = 4d = \lambda$. So $V = \lambda/t = n\lambda$ where n is the frequency of the sound. It may be noted that the air at any point does not travel far from its mean position and transmits the motion to the next layer on account of its elasticity Thus the disturbance travels in the medium with the velocity V. Every point of the medium is in a state of vibratory motion about its mean position and the medium is said to undulate or to be in a state of wave motion. The

line along which each point oscillates about its position of rest is along the direction of propagation of the wave in the medium and hence this wave motion is called *longitudinal wave motion.*

The state of the medium when a wave is progressing can be graphically represented as shown in the figure 111.

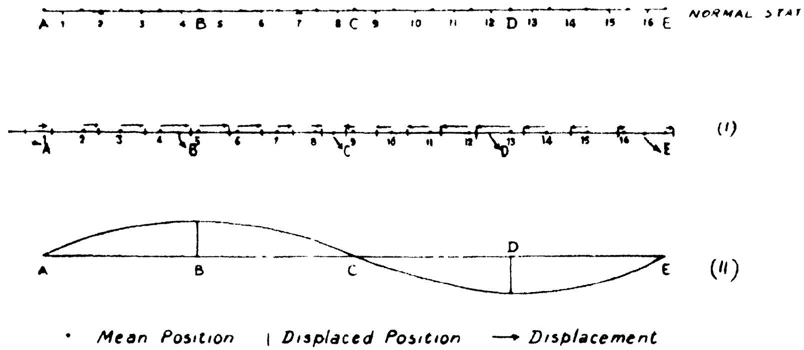

FIG. 111.

ABCDE is a line along which a sound wave progresses in air. AE is divided into 16 equal parts and the middle points of these parts are marked 1, 2, 3, 4, ... 16. In the normal state, all the layers of air are equally apart. At a given instant when the wave progresses along, the layers of air are displaced from their normal positions as shown in fig. 111 (i). The displacements of the middle points 1, 2, ... 16 are shown in the figure. C is the region of maximum compression; A and E are regions of maximum rarefaction. AC = CE = $\lambda/2$ and AE = λ. Every point goes through the successive phases of displacement in each period of time *t.*

In fig. 111 (ii) the displacement of each layer of air is represented on the Y axis and the position on the X axis. Displacements towards the right are marked above the line ABCDE and displacements to the left, below the line. This curve is called the *displacement curve.* Note that C is the point of maximum compression and that the displacement

there is zero. Such a point of zero displacement is called a node. C is also the region of maximum increase of pressure and A or E is the region of maximum decrease of pressure as well as a node. B and D are regions of no change in pressure and the displacements here are maximum.

On a breezy evening waves are observed on the surface of a big sheet of water (river or tank). The waves appear to progress in the direction of the breeze. Small boats and rafters tied to their places dance up and down regularly, about their mean positions, whereas some wave form appears to approach the shore with the wind. Every point on the surface is periodically displaced up and down and the state of vibration at different points on the surface at any instant in the direction of propagation of the wave is different; all the different phases which each point passes through are observed in succession along the surface. *The displacement of the points is in a direction at right angles to that of the progress or propagation of the water wave.* Hence the waves are called *transverse waves.* The point of maximum upward displacement is called the *crest* of the wave and the point which has the maximum downward displacement is called the *trough* of the wave. Midway between a trough and a crest lies the point where the displacement is zero, i.e. the surface at this point is neither higher nor lower than the mean level of the whole surface. This point corresponds to the node of the longitudinal wave referred to already. The points of maximum displacement are called antinodes.

When a stretched string is plucked it vibrates and takes up the form shown in fig. 112. A and B are points at rest.

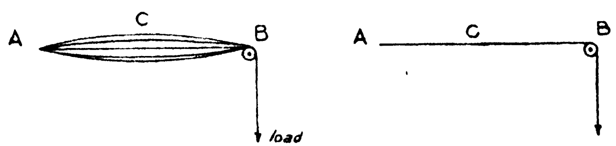

FIG. 112.

On account of the quickness of vibration and the persistency of vision the string appears in the form of a spindle bulging out most at the middle point C where the displacement is maximum. Every point of the string is successively passing through all the phases of vibration, position of maximum upward displacement, mean position, position of maximum downward displacement, mean position, and once again position of maximum upward displacement. If at any instant we can glance at the upper half of the spindle from A to B and at the lower half from B to A all the stages of the periodic motion are simultaneously seen. The displacements are transverse to the length of the string and hence the vibration is called transverse vibration. The distance AB is that between consecutive nodes and is therefore half the wave length. The undulatory disturbance of the vibrating string is unlike that of the wave motion observed on a sheet of water in one important respect. In this case the wave motion is steady or stationary, whereas on the surface of water the waves are progressive. Except for this main difference the nature of the motion is the same. If a long rope is tied to a post and the other end is held in hand and suitably moved periodically at right angles to its length, transverse waves are seen to progress towards the tied end. In the case of a vibrating string of a finite length, the wave is unable to progress beyond the fixed points A and B and is reflected back from B towards A and from A towards B. As a result the string takes up the stationary form of vibration.

VIBRATIONS OF AIR COLUMNS.

Long and narrow columns of air enclosed in tubes of uniform section when set into vibration give out musical notes. Organ pipe, clarionet and flute are familiar examples. The columns may be of two kinds, (i) open at one end and (ii) open at both ends. In both the cases longitudinal waves of a permanent or stationary type are set up and the condition of vibration of air

at definite points is always of a permanent or steady type. These definite points are called Nodes and Antinodes. The condition of air at the node is one of rest and at the antinode one of maximum displacement. At the closed end the layers of air cannot have any free motion and therefore the displacement of air at the closed end is always zero. The closed end is therefore a node. At the open end the air is free to move about and hence the condition there is one of maximum displacement. The open end is therefore always an antinode.

Two closed tubes of different lengths l_1 and l_2 are shown in the fig. 113. The simplest mode of vibration in each case is

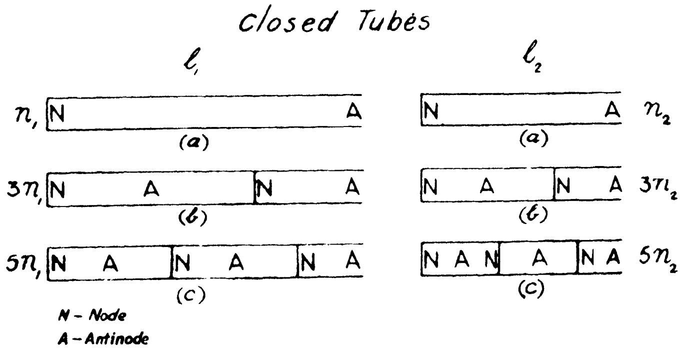

FIG. 113.

shown in (a). As l_1 is the distance between a point of rest and the next point of maximum displacement, $l_1 = \lambda_1/4$ and $l_2 = \lambda_2/4$, λ_1 and λ_2 being the corresponding wave lengths of the notes given out by the tubes.

$$V = n\lambda = n_1 \times 4l_1 = n_2 \times 4l_2$$

$$\therefore \quad \frac{n_1}{n_2} = \frac{l_2}{l_1}.$$

n_1 and n_2 are the frequencies of the tubes when they are vibrating in the simplest mode and so are the lowest notes they give out, viz. their fundamental notes. It is possible to set up different modes of stationary vibration in the

closed tubes. The next two possible modes of vibration are shown in (*b*) and (*c*) of the figure. In (*b*) another node is formed at one-third the length of the tube from the open end and midway between the nodes is an antinode. So the wave length is reduced to a third of the former value. Hence the frequencies of the notes are $3n_1$ and $3n_2$. Similarly, the frequencies in the next possible type of vibration (*c*), are $5n_1$ and $5n_2$. In mode (*a*) the air column is vibrating in one segment with one antinode, the segment being the distance between N and A. In (*b*) and (*c*) the column is vibrating in three and five segments with two and three antinodes respectively. The mode (*a*) is the fundamental mode and (*b*) and (*c*) are the next two possible *harmonic* modes of vibration. The frequencies of the harmonics in a closed tube are therefore 3, 5, 7, etc. times the frequency of the fundamental note.

The condition of air in a vibrating column open at both ends is shown in fig. 114, for the first three possible modes. $l = \dfrac{\lambda_1}{2} = \lambda_2 = \dfrac{3\lambda_3}{2}$ as could be seen from the figure. Hence the frequencies are in the ratio 1: 2: 3 and the fundamental and the harmonics have their frequencies in the ratio 1 : 2 : 3 : 4, etc. In the case of an open tube all the harmonics are possible, whereas in the case of a closed tube the odd harmonics only are possible.

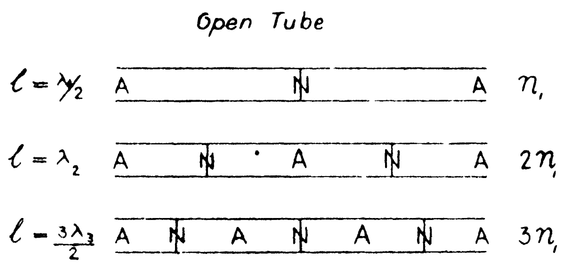

FIG. 114.

The condition of air in a vibrating column can be studied as described below. Mount an organ pipe OB vertically, (fig. 115). Connect the bottom end B to bellows and gently blow in compressed air. The fundamental note of the open

tube is sounded. Lower a scale pan P which has a bottom made of tightly stretched fine paper over which some fine sand is sprinkled. At the antinodes A, i.e. at the top and the bottom of the open tube the sand particles jump up and down vigorously and are thrown out of the pan. The stretched paper bottom of the pan is set in rapid vibration and emits a buzzing sound. This clearly indicates that the displacement at the open end is maximum. As the pan is brought nearer the middle of the tube the dancing of the sand decreases to a minimum at N, the node. If one side of the organ pipe is made of glass as shown in the figure the motion of the sand particles can be easily observed.

By blowing the air through more strongly, the next harmonic can be sounded and the middle of the pipe can be shown to be an antinode.

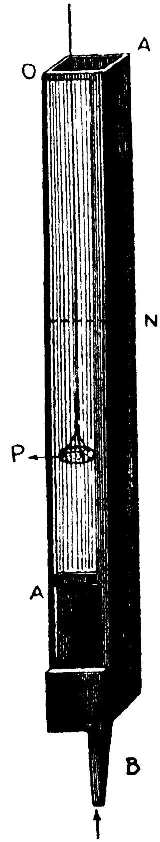

Fig. 115.

VELOCITY OF SOUND.

Common experience shows that sound takes time to travel. The report of a gun is heard after the flash is seen. The lightning flash is seen before the thunder is heard. The display of fireworks which occurs at the end of a rocket's flight high into the air is seen before the report of the bursting of the rocket is heard. Observe the washerman on the river side; the beating of the cloth against the stone is heard some time after the actual contact is seen.

Light travels at a great velocity of 1,86,000 miles per second. Therefore the interval of time between the seeing and the hearing in all the cases mentioned above is practically the time taken by the sound to travel. If the distance between

the source of sound and the observer is known, then the ratio of the distance to the time gives the velocity. The velocity of sound in air was found to be nearly 332 metres per second or nearly 1090 ft. per second at 0°C. The velocity increases with rise of temperature nearly at the rate of 60 cms. per second per 1°C. Therefore velocity of sound in dry air at t°C is given by

$$V = (332 + 0{\cdot}6t) \text{ metres per second.}$$

Velocity of sound in air can be determined by the method of echoes. Echo is repetition of sound caused by reflection at an obstacle. Sound waves, like light waves, obey the two laws of reflection. But sound waves are very long and light waves are very short. Hence a fairly big obstacle is required to reflect a sound wave. Echoes are generally observed at a deep well, against a row of trees or buildings and inside the compound of temples enclosed by high walls. If sharp monosyllabic sounds are uttered they are clearly and distinctly reflected back and echoed. The sharpest monosyllabic sound takes 0·2 sec. to be uttered. If therefore the echo is to be heard immediately after the uttering of the sound, the sound has to reach the obstacle in front of the observer and has to travel back the same distance after reflection. So the distance between the observer and the obstacle has to be travelled twice in 0·2 second. Hence the shortest distance for an echo to be heard is the distance through which sound takes 0·1 sec. to travel, i.e. nearly 110 feet or 33 metres.

If sharp sounds are produced at regular intervals of time, say by clapping hands, such that a sound coincides with the echo of the previous one, the velocity of sound in air can easily be determined. Let n sounds be produced in t seconds and let d ft. be the distance between the observer and the obstacle.

Time between two consecutive sounds is $\frac{t}{n}$ sec.

Time between a sound and its echo is also $\frac{t}{n}$ sec.

The distance travelled by sound in this $\frac{t}{n}$ sec. is $2d$ ft.

$$\therefore \text{ Velocity of sound} = \frac{2d \times n}{t} = \frac{2nd}{t} \text{ ft. per sec.}$$

A mechanical arrangement like a metronome by means of which sounds at equal intervals of time can be produced, is employed for accurate work and t can be observed with a stop-watch. This interesting method can give only an approximate value for the velocity of sound.

A more accurate value for the velocity of sound can be indirectly determined with the *resonance tube.* The *apparatus required* for the purpose are a tall glass jar about 50 cm. high containing some water, a long *narrow* glass tube open at both ends, a set of tuning forks, metre scale, cork-hammer, retort stand, etc.

Arrange the apparatus as shown in fig. 116. Sound a tuning fork of frequency n with the cork-hammer and hold it near the mouth of the glass tube and find the length of the resonating * column of air, moving the tube slowly up and down. As the length of the tube sinks into the water or rises out of it, notice how the intensity of the note given out by the air column in the tube gradually increases, how it reaches a maximum and how it gradually dies away. Take at least three or four observations in each case at the point of maximum intensity. Measure the length of the air column with the metre stick laid alongside the tube. Repeat the same

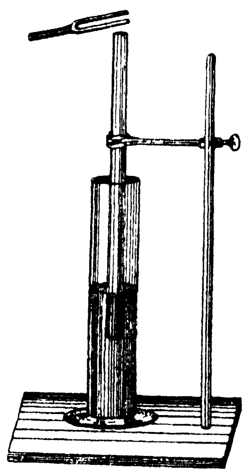

FIG. 116.

* When the frequency of the induced vibration of the column of air is that of the inducing fork the column of air is said to resonate to or be in resonance with the fork.

with tuning forks of different frequencies and tabulate. Calculate the velocity of sound in air from the relation $V = 4ln$.

Practical example.—

The mean of four observation of l is given; $r = 2$ cm.
Temperature of air in the tube = 30°C.

n	l cm.	$V = 4ln$ metres per second.	Corrected velocity $V = 4n(l+1{\cdot}2)$ metres per second.
256	32	327·7	340·0
320	26·5	339·2	354·6
384	22·7	348·7	367·1
	Mean ..	338·5	353·9

The point of maximum displacement is experimentally found to be slightly above the mouth of the glass tube. So $\frac{\lambda}{4}$ is slightly longer than l and is equal to $(l+0{\cdot}6r)$ where r is the radius of the tube. The mean corrected velocity as given above compares well with the calculated velocity 350 metres per second $(332+0{\cdot}6\times30)$.

1. Why is a narrow tube mentioned in the exercise?

2. Is the length of the resonance-box on which the fork ($n = 256$) is mounted, the same as that obtained above? Explain.

3. How do you proceed to determine the velocity of sound in carbonic-acid gas when the gas-producing Kipp is supplied?

MEASUREMENT OF FREQUENCY.

The frequency of a tuning fork can be determined by the *falling plate* method. This method is the same in principle as that of the Fletcher's trolley.

A tuning fork is fixed to the vertical wooden board BC of the apparatus as shown in fig. 117 (a), so that the prongs vibrate horizontally. A short length of thick horse hair is attached to one of the prongs by a piece of wax. In front of the fork is arranged a vertical glass plate P whose surface is coated with a thin layer of smoke. The glass plate is

supported by means of a string from a hook at the top A of the vertical stand. The bristle attached to the prong is carefully adjusted so that it sufficiently touches the smoked surface at the bottom of the plate P. The fork is set vibrating and the string is burnt. The plate falls with the acceleration 'g' due to gravity and moves in a direction at right angles to that of the vibration of the horse hair. A wavy curve is cut into the smoked surface. Select a point p (fig. 117, b) on the wavy trace. Count n wave lengths along and mark, with a fine steel point, the point q. Proceed further and count another n waves and mark the point r. Let $pq = l_1$ and $qr = l_2$. If u is the velocity of the plate when the point p passes the bristle, the velocity when the point q passes it is $u+gt$ where t is the time of fall through qp.

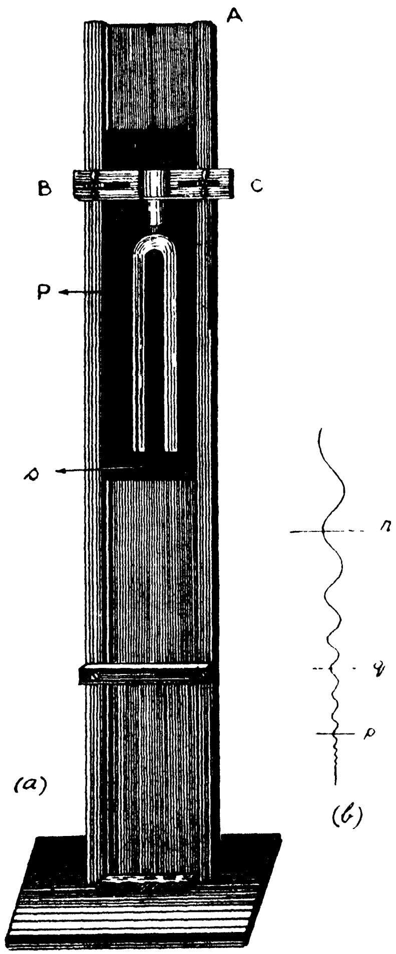

FIG. 117.

The time taken for a fall over a wave length is $\frac{1}{N}$ sec. where N is the frequency of the fork. Hence t the time of fall through n wave lengths is $\frac{n}{N}$ sec.

$$\therefore \quad l_1 = ut+\tfrac{1}{2}gt^2$$
$$l_2 = (u+gt)t+\tfrac{1}{2}gt^2$$
$$\therefore \quad l_2-l_1 = gt^2$$
$$\therefore \quad \frac{1}{t^2} = \frac{g}{l_2-l_1}$$
$$\therefore \quad \frac{N^2}{n^2} = \frac{g}{l_2-l_1}$$
$$\therefore \quad N = n\sqrt{\frac{g}{l_2-l_1}}$$

Practical example.—

A tuning fork of frequency C is used.

$l_1 = 2{\cdot}1$ cm., $l_2 = 3{\cdot}6$ cm., $g = 978$ cm. sec^{-2} and $n = 10$.

$$\therefore \quad N = 10\sqrt{\frac{978}{1{\cdot}5}} = 255{\cdot}4$$

The ratio of frequencies of two notes is called the interval between them. This interval determines the musical value of the notes. The degree of the pleasing effect produced on the ear when different notes are sounded depends on this interval. This pleasing effect is of two kinds, viz. melody and harmony. Two or more notes sounded in succession produce an agreeable effect called *melody* and the pleasing sensation produced when a number of notes are sounded together is called *harmony.* The interval 2 is the most pleasing to the ear and is called an *octave.* This sensation is the same whatever be the individual frequencies of the notes, so long as the ratio is 2. Starting with any note of a convenient frequency N which is called the fundamental its octave has a frequency 2N. Between the fundamental and its octave the ear recognises a number of notes of musical value of frequencies less than 2N. These notes can individually form with the fundamental a harmonious combination. Such notes are said to be in accord, or in consonance, or in harmony with the fundamental.

The following table gives some of the musical intervals between the fundamental or the tonic and its octave.

Interval.	Name and symbolic representation in different notations.						
	Carnatic or South Indian.		Hindustani.		English.		
1	Shadja ..	sa	Shadja ..	sa	Tonic ..	*Doh,*	C
16/15	Dvisruti Rishabha ..	ri	Ati Kōmala Rishabha ..	ri	Semi tone ..	*Ray,*	
10/9	Trisruti Rishabha ..	,,	Kōmala Rishabha ..	,,	Minor tone ..	,,	
9/8	Chatusruti Rishabha ..	,,	Suddha Rishabha ..	,,	Major tone ..	,,	D
6/5	Sādhāraṇa Gāndhāra ..	ga	Kōmala Gāndhāra ..	ga	Minor Third ..	*Me,*	
5/4	Antara Gāndhāra ..	,,	Suddha Gāndhāra ..	,,	Major Third ..	,,	E
4/3	Suddha Madhyama ..	ma	Suddha Madhyama ..	ma	True Fourth ..	*Fah,*	F
45/32	Prati Madhyama ..	,,	Tīvra Madhyama ..	,,	Lydian Fourth ..	,,	
3/2	Panchama ..	pa	Panchama ..	pa	True Fifth ..	*Soh,*	G
8/5	Dvisruti Dhaivata ..	dha	Ati Kōmala Dhaivata ..	dha	Minor Sixth ..	*Lah,*	
5/3	Trisruti Dhaivata ..	,,	Kōmala Dhaivata ..	,,	Major Sixth ..	,,	A
27/16	Chatusruti Dhaivata ..	,,	Suddha Dhaivata ..	,,	Lydian Sixth ..	,,	
9/5	Kaisiki Nishāda ..	ni	Kōmala Nishāda ..	ni	Minor Seventh ..	*Te,*	
15/8	Kākali Nishāda ..	,,	Suddha Nishāda ..	,,	Major Seventh ..	,,	B
2	Tāra Shadja ..	ṡa	Tāra Shadja ..	ṡa	Octave ..	*doh*	c

In the table there are three intervals under each of the names ri and dha, two intervals under each of the names ga, ma and ni whereas there is only one interval under each of the names sa and pa. These fourteen intervals are thus represented by seven *Swaras* or the *Sapta Swaras* of the musical scale whether Carnatic, Hindustani or English.

Different Musical Scales or Modes or Ragas.—The succession of notes of intervals $1, \frac{9}{8}, \frac{5}{4}, \frac{4}{3}, \frac{3}{2}, \frac{5}{3}, \frac{15}{8}, 2$ is called the Major Diatonic scale and forms the raga called Sankarābharana in Carnatic music. The succession $1, \frac{10}{9}, \frac{6}{5}, \frac{4}{3}, \frac{3}{2}, \frac{8}{5}, \frac{9}{5}, 2$ is the Minor Diatonic scale and forms the raga Nata-Bhairavi in Carnatic music. It is with reference to these two musical scales the names major tone, minor tone, major third, minor third, major and minor sixth and major and minor seventh in the table, are given. Similarly, the fourth and the sixth notes in the Lydian scale are called the Lydian fourth and sixth. The succession of notes in the Lydian mode or scale is $1, \frac{9}{8}, \frac{5}{4}, \frac{45}{32}, \frac{3}{2}, \frac{27}{16}, \frac{15}{8}, 2.$ These are the Sapta Swaras of the Kalyāni raga in the ascending order—ārōhana krama.

TRANSVERSE VIBRATIONS OF STRINGS.

When a wire stretched between two points is gently plucked at the centre, a musical note is given out. The wire vibrates in its simplest form, with a loop or antinode at the centre and a node at either end. The vibration frequency N of the note can be expressed thus.

$$N = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

where l = length between two nodes.

T = tension of the string.

m = mass per unit length of wire.

$\therefore$ (I) $lN = \frac{1}{2}\sqrt{\frac{T}{m}}$ = constant, if T and m are constant.

(II) $\sqrt{\frac{T}{l}} = 2\,N\sqrt{m}$ = constant, if N and m are constant.

and (III) $\frac{1}{l\sqrt{m}} = \frac{2N}{\sqrt{T}}$ = constant, if T and N are constant.

From the equation $N = \frac{1}{2l}\sqrt{\frac{T}{m}}$ the *three* following *laws of transverse vibration of strings* follow :—

I. The vibration frequency of the note varies inversely as the length of the vibrating segment of the string, when the tension is constant; $\left(N \propto \frac{1}{l}\right)$.

II. The frequency of the note varies directly as the square-root of the tension of the wire, for the same vibrating length; $(N \propto \sqrt{T})$.

III. The rate of vibration varies inversely as the square-root of the mass per unit length of the string, when the tension is constant; $\left(N \propto \sqrt{\frac{1}{m}}\right)$.

The three relations (I), (II) and (III) are very useful and convenient in verifying successively the three laws of transverse vibrations of strings. The three relations can as well be deduced, as shown below, successively from the three laws of vibrations.

1. $N \propto \frac{1}{l}$ when T and m are constant

$\therefore$ *Nl = constant when T and m are constant.* .. (I)

2. $N \propto \sqrt{T}$ when l and m are constant

but $N \propto \frac{1}{l}$ when T and m are constant

$\therefore \quad N \propto \dfrac{\sqrt{T}}{l}$ when m alone is constant, i.e.

$\dfrac{\sqrt{T}}{lN}$ is constant if m is constant and

$\dfrac{\sqrt{T}}{l}$ *is constant when N and m are constant.* .. (II)

3. $N \propto \dfrac{1}{\sqrt{m}}$ when l and T are constant

$N \propto \dfrac{1}{l}$ when m and T are constant

$\therefore \quad N \propto \dfrac{1}{l\sqrt{m}}$ when T alone is constant

So $\dfrac{1}{Nl\sqrt{m}}$ is constant when T is constant

and $\dfrac{1}{l\sqrt{m}}$ *is constant if T and N are constant.* .. (III)

Corollary—

If r is the radius of the wire and d the density of the material of the wire the mass m per unit length (*linear density*) is $\pi r^2 d$. Substituting, we get

$$N = \frac{1}{2l}\sqrt{\frac{T}{\pi r^2 d}}$$

$$= \frac{1}{2lr}\sqrt{\frac{T}{\pi d}}$$

Hence the third law can be split up into two subsidiary laws:

(i) $N \propto \dfrac{1}{r}$ when l, T and d are constant, or lr is constant when N, T, and d are constant.

(ii) $N \propto \sqrt{\dfrac{1}{d}}$ when all other variables are constant.

We now proceed *to verify the laws of transverse vibrations of strings. Apparatus required* are monochord, a set of tuning-forks, a cork-hammer, a set of kilogram weights, two wires, and a metre scale.

The monochord or sonometer (fig. 118) consists of a sounding

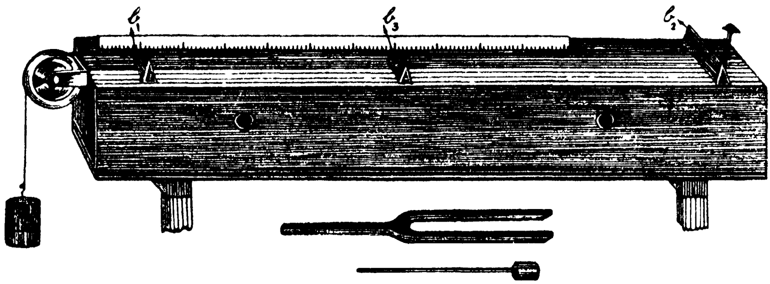

FIG. 118.

box (?) over which are fixed at either end two bridges b_1 and b_2. At one end of the box is driven an iron peg, to which a wire is fastened and is stretched parallel to the length of the box over the two bridges. The loaded end of the string passes over a pulley fixed to the box. A movable bridge b_3 is provided, so that any desired portion of the wire can be set in vibration. Strike a tuning-fork with the cork-hammer. Tune a length b_1b_3 of the wire to the fork. The ear is the best guide. Having secured a position which is very near the right one as judged by the ear, you find that a peculiar throbbing sound is produced, when the fork and the string are sounding together (?). The quicker the sensation felt, the farther is the position of the movable bridge from the correct one (?). This effect is produced on either side of the correct position where the two notes blend into each other and are in complete unison. If you are not able to appreciate this method, the following (a more mechanical and less reliable) method may be adopted. Cut paper riders of the form ʌ and mount them on the vibrating portion of the wire, some-

where in the middle. Sound the fork and put it on the sonometer box. If the rider is not affected, change the position of b_3 in regular steps until a length is found for which the rider is thrown out. Find the limits of the length between which this occurs and take the mean. This length of the string is said to resonate to the tuning-fork (?).

I. Note down the different lengths which resonate to the different forks for the same tension of the wire and the products Nl will be found to be nearly constant and the first law is thus verified.

II. Alter the load and take another set of readings. Calculate $\frac{\sqrt{T}}{l}$ in each case for the two sets of values observed. $\frac{\sqrt{T}}{l}$ will be found to be nearly constant, for the same tuning-fork in the two sets of observations. The second law is thus verified. The vibrating length may be kept constant for the string and the tension may be varied so that the string vibrates in unison with different forks and the ratio $\frac{N}{\sqrt{T}}$ may be calculated in each case and found to be nearly constant. This is a direct method of verification of the law, viz. $N \propto \sqrt{T}$ when m and l are constant. The difficulty in this method is in adjusting the tension in steps small enough for the accurate tuning of the string. A different method (Practical example II*a*) may sometimes be followed.

Calculate the value $\sqrt{T}/lN$ for the string in both the cases and you find that it has very nearly the same value.

III. Mark off any two points on the wire and measure the length. Cut the wire at the points and weigh it accurately. The mass per unit length is obtained. Mount another wire and stretch it with one of the tensions used already and take observations as before. Calculate the values $\frac{1}{l\sqrt{m}}$ and com-

pare the values corresponding to the same fork. You find them to be nearly the same and that the third law is verified.

Practical example.—

A steel wire weighing 0·760 gm. per metre was used; mean diameter = 0·347 mm.

I. Load = 5 Kilos.

N	l cm.		N × l	$\frac{\sqrt{T}}{l}$	$\frac{1}{l\sqrt{m}}$	$\frac{\sqrt{T}}{lN}$
256	48·2, 48·6	48·4	1·244 × 10⁴	0·04621	0·237(0)	0·000180
320	38·3, 39·2	38·8	1·236 × 10⁴	0·05764	0·2957	0·000180
384	32·3, 32·1	32·2	1·2415 × 10⁴	0·06943		0·000181
512	24·2, 24·4	24·3	1·239 × 10⁴	0·09202		0·000180

N	II. Load = 4 Kilos						II(a). Load = 1 Kilo	
	l cm. Method				$\frac{\sqrt{T}}{l}$	$\frac{\sqrt{T}}{lN}$	l cm.	
	Beats.		Rider.					
256	43·6, 44·0, 44·0	43·9	45·7, 42·9	44·3	0·04514	0·000176	22·1	
320	35, 35	35	34·6, 35·5	35	0·05714	0·000180	18·0	
384	29·1, 29·3	29·2			0·06848	0·000179	14·6, 15·4	15·0
512	22, 22	22	21·9, 22·1	22	0·0909	0·000177	11·6, 11·0	11·3

A thicker steel wire weighing 1·900 gm. per metre was substituted and the following observations were taken with two tuning forks.

Mean diameter = 0·545 mm.

III. Load = 5 Kilos

N	l cm.		$\dfrac{1}{l\sqrt{m}}$
256	31·0 31·2	31·1	0·233
320	25·1 24·9	25	0·290

I. In table I, the values of $N \times l$ are nearly equal.

II. The values of $\dfrac{\sqrt{T}}{l}$ in I and II corresponding to the same fork are nearly equal and the values of $\dfrac{\sqrt{T}}{lN}$ for all the observations in I and II are very nearly equal. You note in II(a) that when T has been reduced 4 times and $\sqrt{T}$ reduced to half, the corresponding resonating length l for each fork is also very nearly reduced to half its previous value.

III. Comparing the tables I and III for the first two forks the values of $\dfrac{1}{l\sqrt{m}}$ corresponding to the same fork are nearly the same.

1. In table I for fork E (N = 320), when the length of the wire is increased to 78 cm., the rider is observed to be thrown out vigorously at two points nearly 19 cm. from either end of the wire and when the rider is placed in the middle of the wire it is not at all disturbed. Explain.

2. If the experiment in I is repeated with the fork N = 512 using 73 cm. of the same wire, at what points on the wire do you expect the rider to be thrown out vigorously ?

3. Calculate from the observations the frequency of two or three forks and compare them with the standard values.

4. From the two samples of steel wire used above verify the relation $l \times r$ = constant.

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